Logic, Epistemology, and the Unity of Science 47

Raffaele Pisano
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Murray Peake

Essay *on Machines in General* (1786)

Text, Translations and Commentaries. Lazare Carnot's Mechanics—Volume 1

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Volume 47

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ISSN 2214-9775 ISSN 2214-9783 (electronic)
Logic, Epistemology, and the Unity of Science
ISBN 978-3-030-44384-9 ISBN 978-3-030-44385-6 (eBook)
https://doi.org/10.1007/978-3-030-44385-6

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Preface

It is never easy to conceive and to realize a vast editorial project. This volume represents the first successful step in exactly such a broad editorial project. It presents a faithful English translation of Lazare Carnot's *Essai sur les machines en général* (1786), enriched by a long and accurate *Introduction* and by a detailed technical and linguistic apparatus of *End Notes*. The *Introduction* draws a very good picture of the epoch in which Lazare Carnot lived and worked, including his engagement both as a scientist and as an important politician. The authors reconstruct Carnot's cultural milieu with great care and offer a translation of Carnot's *Essay* on the machines. I congratulate the authors on their excellent work!

This book and its research programme is to be interpreted as the natural continuation of the masterly work *Lazare and Sadi Carnot. A Scientific and Filial Relationship* (Springer, 2014, 2nd ed.) by Charles C. Gillispie and Raffaele Pisano.

The ambitious editorial research enterprise I signalled at the beginning of this Preface, of which the *Essai sur les machines en general* is the first volume, consists of the analysis, translation, and commentary of Lazare Carnot's three Mechanical works. The other two essays whose publication is anticipated by Springer are the *Principes fondamentaux de l'équilibre et du mouvement* (1803) and *Géométrie de position* (1803).

It is a pleasure to observe and to emphasise that all the possible difficulties of such an extensive editorial programme have been overcome in the best possible manner. In particular, the authors have been able to render the eighteenth-century French in modern English without any scientific ambiguity and with great precision. The authors often add—in square brackets—explicative terms, which are useful for a precise reading of the text. Finally, the *End Notes* clarify all aspects of Carnot's scientific work, in reference to both mechanical and mathematical issues. It also seems appropriate to me to stress the high quality of both the scientific and the historical apparatuses of this book, which helps the reader to follow Lazare Carnot's Mechanics.

I also commend to the reader the new insights it offers regarding the relations between pure and applied science and the links between mechanics, mathematics, and engineering. The conceptual, empirical, and methodological aspects of Carnot's works are appropriately underlined by the authors in the *Introduction* and in the *End Notes*.

I am confident that this research project, including the other two forthcoming volumes, can be considered a milestone for the diffusion and comprehension of Lazare Carnot's science.

Paolo Bussotti
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Acknowledgments

We express our gratitude to the directors and staff members of libraries and archives that we met; we express our profound appreciation for their collaboration. Special thanks to distinguished scholar and friend Paolo Bussotti (University of Udine, Italy) for his accurate *Preface* and suggestion.

We are particularly grateful to Monsieur Dr. Gaetan Carnot, member of Carnot's family and Founding Member and Executive Committee Member of *Fondation Carnot*, who was so generous for again removing (as already done for Lazare and Sadi Carnot, 2014 by Gillispie and Pisano) all restrictions and authorizing us to publish portraits, from the original and special collection of Carnot's family. The images are conserved in France at *Académie François Bourdon, Le Creusot—Archives Lazare Carnot*. We acknowledge the kind permission and rights to use the images, particularly of the *Essai sur les machines en général* (1786), *Gallica National French Library* (BnF), *Archives et patrimoine historique de l'Académie des sciences, Paris* (France), *Académie François Bourdon, Le Creusot—Archives Lazare Carnot* (France; in particular M. François-Yves Julien), *Collections archives de la bibliothèque de l'École polytechnique de Palaiseau* (Essonne, France; in particular M. Olivier Azzola). We warmly acknowledge M. Thierry et Mme Caroline Carnot for the extraordinary opportunity to visit (RP) private documents and manuscripts of Carnot's Family archived at their Château de Presles, South of Paris. We address our gratitude to Mme Sylvie Carnot for all information concerning her private Lazare Carnot’s Nolay Archives. We thank M. Jean Le Bret, member of Carnot’s family who gave to one of us (RP) copies of Lazare Carnot’s Ascending (1719–1489) and Descendants (1753–2020) Genealogies and Mme Pascale Cordier, another member of Carnot’s family to have made accessible her private Lazare and Sadi Carnot's family documents. We are grateful to M. Alain Stouvenel (BDFF—*Base de Données de Films Français avec Images*) for sending us (RP) a rare documentary film on *Lazare Carnot ou Le glaive de la Révolution* (Antenne 2 by Jean–Francois Delassus, January 2nd, 1978).

We thank *Springer Nature* for its kind authorisation to allow us to adapt some pages of the *Lazare and Sadi Carnot. A Scientific and Filial Relationship* (Gillispie and Pisano 2014). We thank all referees for their valuable remarks, which have been of great help. We express gratitude to Dr. Julie Robarts (Melbourne University, Australia) for her observations of English language; eventual remaining English mistakes are up to us. The English proofreading funding was kindly supported by IEMN, Lille University—UMR CNRS 8520, France. Thank you!

Finally, of great importance, we address our acknowledgments to Shahid Rahman and Christi Jongepier–Lue, respectively, Springer Editor-in-Chief book Series and Springer Associate Editor for their fine work and positive reception of our project on the *Essay on Machines in General (1786) Text, Translation and Commentaries. Lazare Carnot’s Mechanics—Volume 1.*

The Authors
November 2019

Remarks For the Reader

This book, entitled *Essay on Machines in General* (*Essai sur les machines en général*, 1786), is the first volume of a three-volume set, a unique and major project on the works of Lazare Carnot, *Lazare Carnot's Mechanics: Text, Translations and Commentaries* (expected 2018–2022). The other two volumes are: *Fundamental Principles of Equilibrium and Motion* (*Principes fondamentaux de l'équilibre et du mouvement*, 1803) and *Geometry of Position* (*Géométrie de position*, 1803), also to be published by Springer.

The research for this major project dates back to the end of the 1990s and was carried out by Raffaele Pisano, then also in collaboration with Charles Coulston Gillispie (1918–2015). This research project is devoted to history and historical epistemology of science, integrating history and epistemology of scientific methods and combining epistemological and historical approaches to clearly identify significant historical hypotheses. Therefore bibliographical references, the relationships between physics–mathematics and physics–geometry, and the role played by science in context are strongly stressed.

These volumes (this one and the others cited above, prior to publication) are critical translations, motivated by an important goal: to valorise and spread the name and works of the scientist, Lazare Carnot, to an Anglophone audience.

In this first volume, the translation was mainly carried out by Jennifer Coopersmith and Murray Peake, and the critical commentaries were mainly produced by Raffaele Pisano. The edition used for our critical English translation of the *Essai sur les machines en général* (1786) is archived at the *Archives et patrimoine historique de l 'Académie des sciences, Paris, France BNF* and displayed by National French Library (BnF) website Gallica. An official permission was asked and obtained. The full notice, adapted from BnF (cfr. Gallica website), is:

> Title: Essai sur les machines en général . Par M . Carnot,… Nouvelle édition
> Author: Carnot, Lazare (1753–1823).
> Publisher: impr. de Defay (Dijon) | Nyon l'aîné (Paris)
> Publication date: 1786
> Format: 107 p.; in-8
> Rights: public domain
> Identifier: ark:/12148/bpt6k65435732
> Source: Bibliothèque nationale de France, département Sciences et techniques, 8-V-11886
> Relationship: http://catalogue.bnf.fr/ark:/12148/cb30197796
> Date of online availability: 2013, 31st July

The aim has been to keep to the character of the original *Essai sur les machines en général* (1786) but also to make it easy to understand for the modern reader.

With regard to the first aim, we do not simplify, modernize, or in any other way correct the ideas within the *Essai sur les machines en general*. With regard to the second aim, language explanation and critical comments are reported in footnotes and endnotes, respectively. We have sometimes broken up very long sentences into shorter sentences or changed the word-order. Also, on occasion, extra words have been added for continuity of phrasing or to add a short in-text explanation or to give an alternative modern term. These additions are always in square brackets and in situ. (It should always be clear from the context whether we have to do with continuity or explanation.)

We have corrected obvious printers' errors and indicated this in a footnote. We also use footnotes to draw attention to corrections applied from the *Errata*. We remark that this *Errata* does not appear in the BnF edition that we used for our critical translation. On the contrary, this *Errata* appears in several digital editions (e.g., see Google books). It appears that some editions added this *Errata*. We also note that Lazare Carnot Mathematical Works (1797) does not include this *Errata*. For completeness we decided to use the *Errata* from one of these digital editions (Google, public domain). We warmly thank Google digital books and its cited source as University of Lausanne (Cfr. Google books website).

The layout is such that the translated pages always keep in step with the original manuscript, the French always having an odd page number and the English an even number. In so far as English has a smaller word-count than French, the translated pages are shorter than the French original. However, in displaying the mathematical relations most clearly, sometimes more spacing has been used than in the original.

The reader should be aware that Lazare Carnot's *Essai sur les machines en général* was written some seventy years before the physical concept of "energy" had been discovered and therefore before the term *energy* had entered the physics lexicon. Other physics terms such as *force, action, percussion, soliciting force, moment of activity, puissance*, and so on, have changed or firmed-up their meaning or fallen into disuse. These terms are left in place but, where necessary, the modern term is given in square brackets straight afterwards, or explained in a footnote or endnote. Two terms deserve special mention. One is *hard*, the other is *weight*. Carnot usually uses the property hard, of a body, to mean that the body is deformable but not elastic (like a lump of putty, the body may change its shape but it does not rebound). The term *plastic* is used to convey this meaning. Carnot sometimes uses the term *weight* to mean, loosely speaking, some mass or body, and other times to mean the force exerted on a mass due to gravity. In this case we write *force-weight* and explain in an endnote. Further remarks and commentaries on terminology may be found in *Lazare and Sadi Carnot. A Filial and Scientific Relationship* (Springer, 2014) by Charles Coulston Gillispie and Raffaele Pisano.

As well as terminology, some symbols have changed their use or fallen into disuse. According to Florian Cajori (1859–1930) in his *A History of Mathematical Notations* (1928–1929), the Lazare Carnot years were a time of transition during which the integral sign was used both for integration and for summation. We have continued to use the integral sign, $\int$, except where there is no infinitesimal quantity (dt, ds, etc.). In these cases, the integral sign, $\int$, is replaced by the summation sign, $\sum$. One other symbol to note is the eighteenth-century sign for the French *livre tournois*, ₶, a French unit of currency, used by Carnot to mean a pound weight. We have translated this as lb and added a footnote. There exists one previous translation into English, the work of 1808, published in instalments by Alexander Tilloch in *The Philosophical Magazine*:

> Carnot L (1808a) Essai sur les machines en général (Part I) In: Philosophical Magazine: comprehending the various branches of science, the liberal and fine arts, agriculture, manufactures, and commerce. Tilloch A (ed). Vol. XXX. Murray J, London, pp. 8–15; pp. 154–158; pp. 207–221; pp. 310–320. Very interesting are the avant-titre page of the book where a portrait from the original of Lazare Carnot is published and a short biography/comment of Lazare Carnot written by the editor (pp. 370–371).
>
> Carnot L (1808b) Essai sur les machines en général (Part II). In: Philosophical Magazine: comprehending the various branches of science, the liberal and fine arts, agriculture, manufactures, and commerce. Tilloch A (ed). Vol. XXXI. Murray J, London, pp. 28–36; pp. 136–144, pp. 220–228; pp. 295–305.
>
> Carnot L (1808c) Essai sur les machines en général (Part III). In: Philosophical Magazine: comprehending the various branches of science, the liberal and fine arts, agriculture, manufactures, and commerce. Tilloch A (ed). Vol. XXXII. Murray J, London, pp. 124–130.

Our present work is an utterly original translation, more modern in style, and benefits from more than a hundred years of advance in science over the earlier translation and remarkable research on the subject (Gillispie 70s–90s; Drago and Pisano 90s–2000s; Gillispie and Pisano 2014; Pisano since 2000s–etc. See also endnotes and references sections below). It also differs from this earlier translation in certain particulars: (1) We have used $\int$, and $\sum$, as appropriate, in place of S, (2) We have translated fils as threads rather than as wires, (3) We have incorporated Carnot's own *Errata*.

This volume has been independently blind-peer refereed.

The Authors
November 2019

About the Authors

Prof. Dr. Raffaele Pisano HDR (Italy, 1970) is a Physicist and Full Professor at the IEMN CNRS-Lille University, Lille, France. His fields of research | teaching are: History of Physics, History of Physics-Mathematics/Theoretical Physics, History of Applied Sciences and Technology, Historical Epistemology of Science, Comparative and Intellectual History, NoS | History of Science and Teaching. Since 2017, he is elected President of the Inter-Divisional Teaching Commission(DLMPST/IUHPST/DHST).

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Murray Peake is a retired Information Technology consultant now living in Brittany.

Introduction

Lazare Carnot, *l'organisateur de la victoire*

This is the first research volume, in translation and with English commentary, of the Springer French editions of three remarkable works of Lazare Nicolas Marguérite Carnot: *Essai sur les machines en general* (Carnot [1783] 1786). The other two pre-print volumes are: *Principes fondamentaux de l'équilibre et du mouvement* (Carnot 1803a) and *Géométrie de position* (Carnot 1803b). For Lazare Carnot's biography, we direct the reader to the main recent authorities' works (Gillispie and Pisano 2014; Dhombres and Dhombres 1997, Gillispie 1971; Gillispie and Youschkevitch 1979, 1982; Dupre 1892). In agreement with Master Charles Coulston Gillispie (1918–2015):

> There is no difficulty in understanding why the scientific community should have ignored Carnot's *Essai sur les machines en général* [...] in the 1780s. His book does not read like the rational mechanics of the eighteenth century. It had long since become normal to compose treatises of mechanics addressed to a professional public in the language of mathematical analysis; though Carnot reasoned no less rigorously than did contemporary mathematical argument, he conducted the discussion verbally, conceived the mathematical expressions he did employ in a geometric or trigonometric rather than algebraic spirit, and usually went on to explain in words what the formulas contained. The genre was apparently of an altogether lower order than that of d'Alembert and Lagrange or Euler and the Bernoulli family. Judging by the style alone, prolix

> and naive, a contemporary reader might easily have supposed the book to be among the many negligible writings that retailed merely elementary mechanics under one pretext or another. Yet, the essay, despite its title, could never have served the purpose of a practical manual for designing or employing actual machinery. (Gillispie and Pisano 2014, pp. 15–16)

The following introductory research presents Lazare Carnot as military man, politician and scientist on the one hand, and his mechanics, particularly his *Essai sur les machines en général* in context, on the other.

1.1 Lazare Nicolas Marguérite Carnot (1753–1823)

Lazare Nicolas Marguérite Carnot (Nolay, France, 1753 May 13th—Magdeburg, Prussia [Germany] 1823 August 2nd) was also called *L'Organisateur de la Victoire* and *Le Grand Carnot*, due to the services he rendered to politics during the French revolution, in the army as a general (i.e., the battle of *Wattignies La Victoire*) and scientist (physics, mathematics, geometry, fortifications and mechanical machines). Despite the dominant Newtonian and Lagrangian mechanics, and taking into account his political background, Carnot's works are of great importance to both the history of physics and the history of mathematics (Chamay 1984–1985; Gillispie and Pisano 2014).[1]

Son of Claude Carnot (1719–1797) *Notaire Royal/avocat au parlement de Bourgogne* and Marguerite Pothier (1726–1788), Lazare Carnot was born in Nolay, a village in the current Côte-d'Or

[1]Some paragraphs of the second part of this introduction are an adaptation of theoretical advancement from Gillispie and Pisano 2014, pp. 16–23; pp. 353–356; pp. 376–380. Necessary parts are quoted from them as a self-citation. We thank *Rights and Permissions Springer Nature* for its kind authorisation. See also: *A Development of the Principle of Virtual Laws and its Conceptual Framework in Mechanics as Fundamental Relationship between Physics and Mathematics* (Pisano 2017) and *Reading Science, Technology and Education: A Tradition Dating back to Science into the History and Historiography* (Pisano, Anakkar et al. 2017) both published by Transversal (see also Pisano and Capecchi 2015). We show appreciation and thankful to these notable publishers—journals. As we already remarked in the *Acknowledgments* section, we also express gratitude to *Gallica–National French Library* (BnF), *Archives et patrimoine historique de l'Académie des sciences, Paris* (France), *Académie François Bourdon, Le Creusot—Archives Lazare Carnot* (France), *Collections archives de la bibliothèque de l'École polytechnique de Palaiseau* (Essonne, France). An infinite and particular gratitude is addressed to *Monsieur Gaetan Carnot, member of Carnot's family*, for his kind permission to use the images.

department in eastern France. He began his studies in humanities and philosophy at the Collège d'Autun. Then, under the *Societas Presbyterorum a Santo Sulpitio (Society of the Priests of Saint Sulpice)* he focused on theology, logic and mathematics. Thanks to these scientific studies and the meeting with Duke Louis–Marie–Victor d'Aumont de Rochebaron (1632–1704), a French Army officer and Marquis de Nolay, he undertook his military and scientific education (Reinhard 1950–1952).

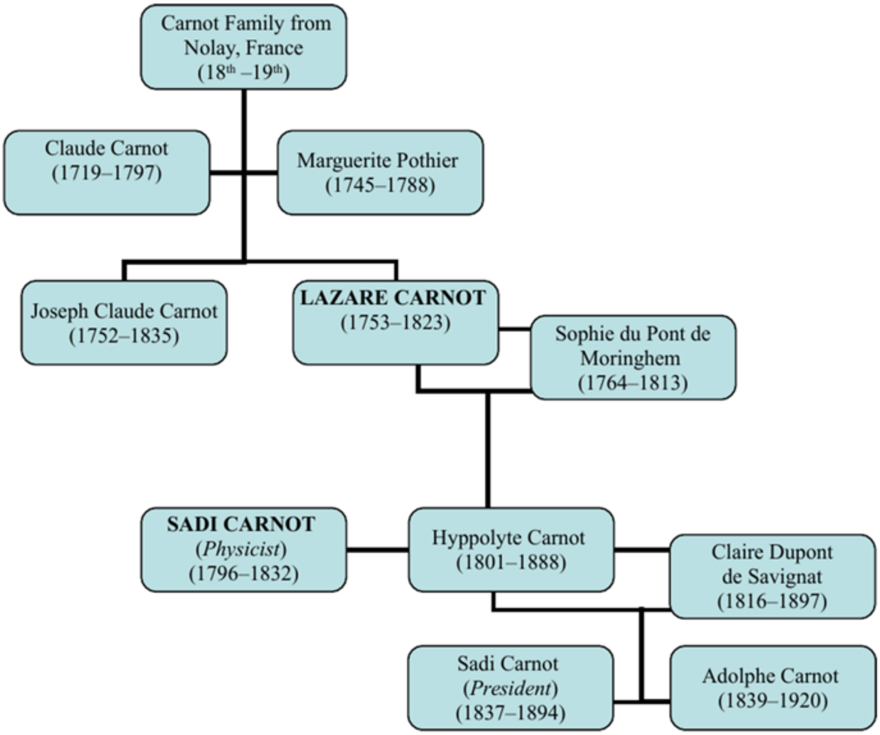
In 1771, Carnot was accepted by *École royale du génie de Mézières* and appointed as a second lieutenant (1773) where he improved his education in mechanics, technical drawing and geography. One of his professors was Gaspard Monge (1746–1818; Monge 1799). Some years before, in 1764, Louis–Alexandre Berthier (1753–1815) was also accepted at the *École royale du génie de Mézières*. By the age of twenty Carnot had graduated from the school as first lieutenant, after also attending Louis Joseph (1736–1818) Prince of Condé's engineer corps. During this period, physics, mathematics, engineering and military fortifications were his main interdisciplinary fields within military strategic (geometric) defence. Typical of Carnot's designs for fortresses in the tradition of Vauban is a loop-holed wall, built as the *channel of fortification*, now called the Carnot wall.

Parallel to science, politics also absorbed Carnot's life: he became one of the first delegates (1791) of the new *Assemblée législative (Legislative Assembly)*. He was also elected (1791) member of the *Comité de l'Instruction Publique (Committee of Public Instruction)*. It was a committee—part of the *Legislative Assembly*—established to reorganize the French education system in this period. Carnot suggested interesting reforms to both the teaching and public education systems that were not implemented. The ferociously combative environment and social instability were not favorable preconditions for new, advanced, ideas.

In Salperwick (North of France) on 1791, May 17th Lazare Carnot married (with a dowry of 30,000 books) Jacqueline Sophie du Pont

(Dupont) also de Lierdt (1764–1813) from Moringhem (North of France). They had two sons: Nicolas Léonard Sadi Carnot (1796–1832), physician/engineer founder of thermodynamics, and Hippolyte Carnot (1801–1888), an important French political leader. The latter was born in Saint–Omer (North of France) not far from Lille.

The following Diagram 1 shows ancestry and descendants of Lazare Carnot from Nolay (France) branch. For our aims it is limited to 1719–1920:



Diagr. 1 Extract of Lazare Carnot Family from Nolay (France): 1719–1920. We thank M. Jean Le Bret, member of Carnot's family who gave us (RP) copies of Lazare Carnot's Ascendant (1719–1489) and Descendant (1753–2020) Genealogies

Fig. 1 «Le Comte Carnot, membre de l'Institut Royal de France (section Sciences physique et mathématiques—Mécanique) (s. d.)». *Plate from the original portrait conserved at Académie François Bourdon, Le Creusot—Archives Lazare Carnot. Very kindly authorized by Monsieur Gaetan Carnot, member of Carnot's family*

At the end of 1770s Carnot was busy completing his *Essai sur les machines en general* for his first (lost) edition of 1783 (Fig. 11). At the beginning of the 1780s he also participated in a competition organised by the *Académie des sciences*. He wrote two unpublished memoirs: *Mémoire sur la théorie des machines pour concourir au prix de 1779 proposé par l'Académie royale des sciences de Paris* (Carnot 1778[9]; Fig. 9; hereafter 1779) and *Mémoire sur la théorie des machines pour concourir au prix que l'Académie royale des sciences de Paris doit adjuger en 1781* (Carnot 1780[1]; Fig. 9; hereafter 1780).

This was also a successful period of important appointments in his military career. He became *capitaine au corps royal du génie* (1783), then in rapid succession *lieutenant–colonel, colonel, général de brigade* and *général de division* (1784; the same year he wrote his famous *Éloge de M. le Maréchal de Vauban*; Cfr. Duthuron 1940).

In 1787 he was an official member of the *Académie de Dijon*.

In 1792, the *Legislative Assembly* (1791, 1st October to 1792, 20th September) concluded its activity. From the 22nd of September 1792 to the 2nd of November 1795 the *Convention national* (*National Convention*) governed France, and Carnot was elected the *43rd President of the National Convention* (1794; see Fig. 3). He demonstrated his military prowess, for example, participating in a military mission (1792, Bayonne) to improve the current defensive systems against Spanish attacks. One year later, a significant political event was his vote in favour of the death sentence of King Louis XVI (1754–1793).

In 1793 Carnot was elected member of the *Comité de salut public (Committee of Public Safety)* a new committee established by *National Convention*. He was also charged as one of the *Ministers of War* (Culp 1914). He achieved several military victories through the implementation of his singular defensive strategies, especially against the prevailing methods of European armies. Among others, these included a unique massive army, that is the organization of military forces capable of fighting a total war, achieving the neutrality of

Prussia, and interrupting communications with Austria and England. Essentially, it consisted of a substantial army divided by units, which rapidly attacked from the flanks rather than head on. Carnot also saw to—in his capacity of *Minister of War*—several emergencies, such as the scarcity of munitions (due to the lack of saltpetre[2] and copper) and other technical needs. He successfully organized the army and *ad hoc* strategies, including the Northern front, battle of Wattignies,1793, 15–16 October, and came to be known at this time as the Organizer of Victory (Fig. 2).

In Arras—North of France, not far from Lille—Lazare Carnot met Maximilien François Marie Isidore de Robespierre (1758–1794). The two officers were both members of the *Société des Rosati d'Arras*, a literary society established in 1778, 12th June. Carnot enrolled in the society in 1786, Robespierre in 1787. The society was essentially inspired by Jean de la Fontaine (1621–1695), Guillaume Amfrye de Chaulieu (1639–1720) and Jean de la Chapelles' (1651–1723) writings. Later on, in the 1790s, especially in the course of Carnot's roles at the *National Convention*, the tension between them became extreme: anti-Prussian Robespierre and anti-English Carnot; and also in regard to Jacobin politics. We should remark that Carnot had shown no opposition to the *la Terreur* until he, and other of his colleagues of the *National Convention*, arrested (1794, 27th July) Robespierre and his twenty-one associates and condemned them to capital punishment by beheading (1794, 28th July).

In 1794, Gaspard Monge, Jacques–Elie Lamblardie (1747–1797) and Lazare Carnot founded the *École centrale des travaux publics*. In 1795 this institution was renamed the *École Polytechnique*, as it is still known.

[2]*Potassium nitrate* (KNO3) is a nitrogen-containing compound. The niter (from nitrogen as a source) exists in nature as a mineral. Generally, it is referred to as saltpeter or saltpetre.

In 1795 Napoléon Bonaparte (1769–1821) was general in-chief of the *Armée d'Italie* (Army of Italy) located on the Italian border. Due to his political disagreement with Paul–François vicomte de Barras' (1755–1829) reactionary ideas, in the same year, Carnot left the *Comité de salut public*. On 1795, April 11th Lazare Carnot became (1795–1797) one of the five directors of the *Directoire (Directory)*. The latter governed France until 1799. He supported Napoléon's initiatives; he was the only one of the *Directory* to do so (Dhombres and Dhombres 1997; Hicks). The life of this *Directory* was not easy, due to the several cultural and political differences between the members.

In 1796 (April 30th) Carnot was elected president of *Directory*; on June 1st, his first son, Nicolas Léonard Sadi, was born (Figs. 4 and 5). His presidency ended on 1796, 29th July.

Sadi Carnot is the father and "inventeur de la Thermodynamique" as noted in the History of *École polytechnique* (Callot 1980, p. 390). In 1824, one year after Lazare Carnot's death, Sadi Carnot's only published work appeared: *Réflexions sur la puissance motrice du feu* (Carnot 1824 [1824], 1978; Girard 1824, Gondinet 1833; Challey 1971; Costabel 1976). He wrote another two (unpublished) manuscripts. The first is *Notes sur les mathématiques, la physique et autres sujets* (Carnot 1878b, pp. 89–102; see also Carnot 1878a; Robelin 1832; Rosenfeld 1941; Rumford 1798). The manuscript is conserved in the archives of the *Académie des science–Archives et patrimoine historique de l'Académie des sciences*, Paris (Gillispie and Pisano 2014). Sadi Carnot wrote crucial details about early ideas on the law of conservation (Gillispie and Pisano 2014)[3]. The second manuscript is *Recherche d'une formule propre à représenter la puissance motrice de la vapeur d'eau* (Carnot 1978, pp. 223–234; Cfr. Clément 1819a, b, 1970). In this work, Sadi Carnot indicated a mathematical expression for motive power applicable to heat machines *en général*.

[3]Recent studies (Pisano's works; Gillispie and Pisano 2014) remarked on some differences between the 1824 edition and the 1878 a manuscript given to the *Académie des sciences* (Carnot 1878a). The manuscripts edited by Gauthier–Villars (Carnot 1878b) are not always reproduced in their entirety. To consult Sadi Carnot's complete manuscripts see Pisano's works; Gillispie and Pisano 2014; see also Carnot 1978, 1986, masterfully edited by Robert Fox.

Fig. 2 «Lazare Carnot à Wattignies: portrait à cheval en uniforme de commissaire aux armées par R. Desvarreux (1909)». *Plate from the original portrait conserved at Académie François Bourdon, Le Creusot—Archives Lazare Carnot. Very kindly authorized by Monsieur Gaetan Carnot, member of Carnot's family*

Fig. 3 «Lazare Carnot annonce à la tribune de la Convention la prise de Condé, nouvelle reçue par le télégramme Chape en 1794, gravure (s. d.)». *Plate from the original portrait conserved at Académie François Bourdon, Le Creusot—Archives Lazare Carnot. Very kindly authorized by Monsieur Gaetan Carnot, member of Carnot's family*

This work is very connected to the filial and scientific relationships between Lazare and Sadi on a common scientific project (Gillispie and Pisano 2014), concerning the study of the efficiency of mechanical and heat machines; the project originally belonged to Lazare (*ibidem*). The manuscript was found in 1966 (Gabbey and Herivel 1966). It was presumably written between November 1819 and March 1827 (Gillispie and Pisano 2014): decisive evidence still is lacking to determine the precise date. Generally speaking, it was written before Carnot's publication of 1824 (Gillispie in Gillispie and Pisano 2014, Chap. 3, ft. 42) and after April 1823 (Fox in Carnot 1986, p. 168).[4]

In 1797 (September 4th) three members of the *Directory*—with military support from Charles Pierre–François Augereau (1757–1816)—staged a *coup d'état* called the *Coup of 18 Fructidor*—Year V. They were Paul–François de Barras, Jean–François Rewbell (1747–1807) and Louis–Marie de la Révellière–Lépeaux (1753–1824). Lazare Carnot was removed and obliged to seek protection in Switzerland (Geneva). In this period, he wrote *Réflexions sur la métaphysique du calcul infinitésimal* (Carnot 1797a), in order to explain and justify the role of the mathematics used in the previous *Essai sur les machines en général* (Carnot [1783] 1786).

In 1799 Carnot came back to France and was appointed (1800 April 2nd) by Napoléon as *Minister of War*. He was in charge until October 8th, which included the events of the *Battle of Marengo* (1800 June 14th). Two years later, he did not support *Napoléon's Consular powers for life and descent heritage*: he voted against this proposal. This also was the period of *Principes fondamentaux de l'équilibre et du mouvement* (Carnot 1803a) and *Géométrie de position* (Carnot 1803b).

Between 1800 and 1806 Lazare Carnot devoted himself to the geometry inspired by earlier mechanical and mathematical works,

[4]Charles Gillispie argued (*Ibidem*) the role played by the concepts of reversibility and incompleteness and completeness of a cycle. Robert Fox (Carnot 1986, pp. 168–169) argued his "tentative inclination to suppose" (Carnot 1986, p. 169) suggesting a date between November 1819 (when Clément lectured as a professor at *Conservatoire des arts et métiers* in Paris) and 8 March 1827, when the latter acknowledged a "distinguished mathematician" (Carnot 1986, p. 167) for information which added to his lecture. However, whether the composition of the unpublished manuscript was elaborated before or after (or during: Pisano) the composition of *Réflexions sur la puissance motrice du feu* is still an outstanding question that requires resolution.

such as those on equilibrium, analysis and geometry. He wrote four masterpieces (Carnot 1800, 1801, 1803a, 1806).

In 1800 Carnot addressed a remarkable letter to Charles Bossut (1730–1814) in which new results and intellectual standpoints in geometry, particularly in trigonometry (Carnot 1800, pp. 401–421; Cfr. Bossut 1800a, b) were presented.

In 1801 he wrote *De la corrélation des figures de géométrie* (Carnot 1801), and his second son, Lazare Hippolyte, was born (Saint–Omer, April 6th). In this book he presented several of Euclid's theorems and various forms of a theorem, later called as *Carnot's theorem*[5], or the *law of cosines* (cfr. Carnot 1801, § 220, pp. 162–164). This remarkable theorem (Lagrange 1813, p. 406) which referred to the triangle—such as the generalisation of Menelaus from Alexandria (ca. 70–140; Chemla 1998, 1990)—was also presented in the *Géométrie de la position* (Carnot 1803b, p. 168; p. 291; pp. 436–437), where it is fully derived and written in the modern forms. As such, the latter is considered to complement the previous *De la corrélation des figures de géométrie* (Carnot 1801). Another important work on the subject was *Mémoire sur la Relation qui existe entre les distances respectives de cinq points quelconques pris dans l'espace; suivi d'un essai sur la théorie des transversales* (Carnot 1806b).

Lazare second's son Hippolyte Carnot had an excellent political career: *Député* (1839–1849; 1850–1851; 1871–1875), *membre du Corps législatif* (1864–1869), *Ministre de l'instruction publique* (1848) and *Sénateur inamovible* (1875–), and wrote several works on politics and on teaching.

[5]As an extension of Pythagoras's theorem for the case of triangles, it concerns the lengths of the sides of a triangle to the cosine of one of its angles. It is—in some manner—also attributed to al–Biruni (ca. 973–1048), al–Kash (ca. 1380–1429) and François Viète (1540–1603); Bonaventura Cavalieri also presented it and its complete proof in *Trigonometria plana, et sphaerica, linearis and Logarithmica* (Cavalieri 1643; see also 1635). It is also possible to find it in the Euclid's *Elements*, Proposition 13, Book II (see also Proposition 12). The *théorème japonais de Carnot* is a theorem presented in *Géométrie de position* (Carnot 1803a, p. 168) related to the proof of the *Japanese theorem for concyclic polygons*. Lagrange appreciated this theorem very much (Lagrange 1813, pp. 406–407). On the contrary, Cauchy was very critical of the theorem: "In the various treatises of mechanics it is taught that live forces are lost every time bodies undergo a sudden change in velocity, and that this loss of live force is the sum of the live forces due to the velocities that are lost. But this proposition, which has been named Carnot's theorem, is evidently inexact as is the demonstration on which it purportedly rests." (Cauchy 1829, p. 116).

Fig. 4 «Lazare Carnot en uniforme de membre du Directoire exécutif (s. d)». *Plate from the original portrait conserved at Académie François Bourdon, Le Creusot—Archives Lazare Carnot. Very kindly authorized by Monsieur Gaetan Carnot, member of Carnot's family*

Fig. 5 «Sadi en uniforme de polytechnicien en 1813, reproduction d'après Boilly (s. d)». *Plate from the original portrait conserved at Académie François Bourdon, Le Creusot—Archives Lazare Carnot. Very kindly authorized by Monsieur Gaetan Carnot, member of Carnot's family*

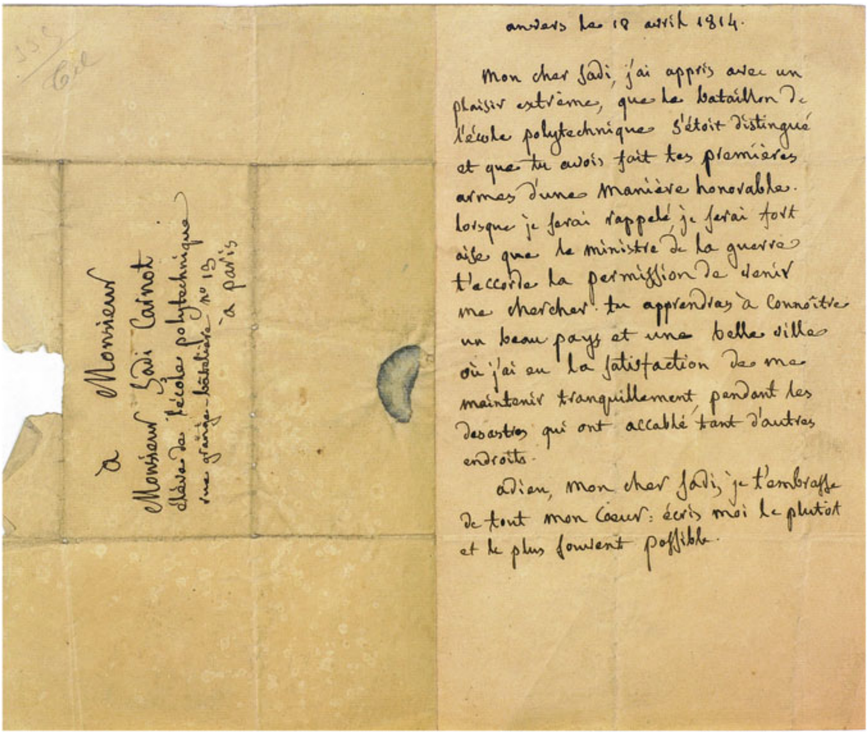


Fig. 6 «Lazare Carnot's letter to his son Sadi Carnot (1814)». *Plate from the original addressed to Sadi Carnot. Dossier Sadi Carnot, VI 2a2 (1812). Very kindly authorized by the Collections archives de la bibliothèque de l'École polytechnique de Palaiseau (Essonne, France)*

Hippolyte was also responsible for the delay in the circulation of his brother Sadi's documents and his biography (Gillispie and Pisano 2014, Chaps. 6 and 9). A collection of Sadi Carnot's manuscripts were given to the *Académie des sciences* on December 16th 1878: 46 years after Sadi's death. One of them was an autograph version of *Réflexions sur la puissance motrice du feu* (Carnot 1878a). It is conserved (4 cahiers, 92 f. r/v) at the *Académie des sciences–Institut de France* in Paris (Gillispie and Pisano 2014). One reason for this delay could be the ambiguity surrounding the cause of Sadi's death (1832, 24th August) in Ivry–sur–Seine. A prevailing opinion suggests a death from disease, which appears to be "choléra" (Arago and Gay–Lussac 1833, LII, p. 211; Henrion 1833, p. 167). But recent research (Cfr. Pisano's studies) strongly suggests that he developed mental illness (Cfr. Birembaut 1974) when the cholera epidemic of 1832 hit Paris. Some of Sadi's friends submitted an obituary (Girard 1824, pp. 411–414; Robelin 1832, pp. 528–530; Gondinet 1833, p. 46). Given the roles played by Lazare in politics, the revolution and the army, this mysterious aspect of Lazare Carnot's family is very important. Continuing the family's prestige, Marie François Sadi Carnot (1837–1894) Hyppolite's son, become the fourth President of the Third French Republic (1887).

In 1804, 2nd December Napoléon crowned himself emperor. Lazare Carnot's republican attitudes and political ideas impeded further public roles under the *Empire Français (First French Empire)*, so he retreated to private life and his studies on military fortifications, especially on the geometry of *bastions systems*. In this period, Lazare Carnot wrote *De la Défense des places fortes*. The book had three editions in French (Carnot 1810, 1811, 1812a) and one edition translated into English, and published in London by a military library (Carnot 1814).

In 1812—and after the fiasco invasion of Russia—Carnot returned to active military service, in particular he worked for the defence of Antwerp during the *War of the Sixth Coalition* (also *Guerre de la libération*). The coalition was composed of Austria, Prussia, Russia, the United Kingdom, Portugal, Sweden, Spain and some German states. The Count of Artois demanded that Napoléon capitulate. Paris was captured around the Spring of 1814. Napoléon was obliged to abdicate in April and exiled to the island of Elba in Italy; but, he escaped (1815, February) and was restored to power in France once again. During the war of *Hundred Days*, Carnot was Minister of the Interior under Napoléon (Fig. 7).

The decisive *Battle of Waterloo* was in 1815, June 18th.

Fig. 7 «Lazare Carnot, portrait dans son bureau en uniforme de Ministre de l'Intérieur en 1815, (s.d.)». *Plate from the original portrait conserved at Académie François Bourdon, Le Creusot—Archives Lazare Carnot. Very kindly authorized by Monsieur Gaetan Carnot, member of Carnot's family*

The British allied army under the command of the Duke of Wellington, a Prussian army under the command of Generalfeldmarschall (field Marshal) Gebhard Leberecht von Blücher (1742–1819) brought an end to Napoléon and occupied Paris (July 7th). Finally, Napoléon was condemned to exile (1815, December) on the island of Saint–Helena (Longwood, South Atlantic west coast of Africa). He was fifty-one years old when he died (1821, May 5th) in exile.

During the Second (Bourbon) Restoration Carnot was accused as the deliberate *killer of a monarc(-hy)* and as a *regicide* during the *Second White Terror*—1815–1816 (Louis XVIII).

In 1816, Carnot was exiled[6] to Poland. First, he lived in Warsaw, then in the Kingdom of Prussia (Germany). He was accepted with honour in both countries. One year before the outstanding publication, *Réflexion sur la puissance motrice du feu* of his oldest son Sadi Carnot (Carnot [1824] 1978, 1986; Pisano 2010; Gillispie and Pisano 2014), Lazare Carnot died in Magdeburg (Fig. 8). He died at seventy years of age and was buried at the Johanneskirche (Sankt–Johannis Kirche).

Only in 1889 were Carnot's remains transferred to the Panthéon on 4th August. He was commemorated by a royal and magnificent ceremony when his grandson, Sadi Carnot[7] was still President of the French Republic.

Carnot produced distinguished writing and masterpieces in several fields such as mechanical machines, mechanics, geometry, mathematics, fortification and military treaties. Therefore, it is interesting to show the extraordinary trivalent role, political, military, and scientific, played by Lazare Carnot in the course of his main responsibilities and works[8]:

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[6]Napoléon Bonaparte and Lazare Carnot only were exiled.

[7]Marie François Sadi Carnot—as above cited—was the fourth President of the Third French Republic. On June 24th, 1894, Sante Geronimo Caserio (1873–1894), an Italian anarchist, assassinated the President Carnot who died after midnight on 25 June. The Board of Pardons decided against all appeals for clemency on August 14. Caserio was executed by guillotine in Lyon at 5 am, August 16, 1894. President Carnot was honoured with an elaborate funeral ceremony in the Panthéon, Paris.

[8]Cfr. Gillispie 1971; Gillispie and Pisano 2014. Particularly on Lazare Carnot as a politician and military officer, including documents and archives, see Jean Dhombres and Nicole Dhombres (Dhombres and Dhombres 1997).

  - 1767. Early humanities studies in Autun, located in the department of Saône–et–Loire, Department of Bourgogne–Franche–Comté (Centre-East of France).
  - 1768. Séminaire d'Autun (military lyceum).
  - 1769. Carnot prepared his application (*concours*) to enrol the *École du Génie*.
  - 1770. Carnot was positively evaluated (*classified 3rd*) for *École royale du Génie de Mézières* (North of France).
  - 1771. Carnot attended the *École royale du génie de Mézières*.
  - 1773. Carnot was twenty-years old when he became *Ingénieur Royal*.
  - 1773. Carnot went to military garrison in Calais (North of France).
  - 1777. Carnot went to military garrison in Cherbourg (North of France).

- 1778[79]. *Mémoire sur la théorie des machines pour concourir au prix de 1779 proposé par l'Académie royale des sciences de Paris* (Carnot 1778; Fig. 9).
- 1780[81]. *Mémoire sur la théorie des machines pour concourir au prix que l'Académie royale des sciences de Paris doit adjuger en 1781* (Carnot 1780; Fig. 9).

  - 1780. Carnot went to military garrison in Béthune (North of France).
  - 1781. Carnot went to military garrison in Arras (North of France).

- 1783. *Essai sur les machines en général. Par un officier du Corps royal du Génie. Defay. Dijon.* It is his masterpiece on mechanical machines.[9]

[9]As we read in the end of the second edition (1786, Fig. 11) a first edition was completed in 1782 and published one year after: "Depuis la première édition de cet ouvrage [*Principes fondamentaux de l'équilibre et du mouvement*] en 1783, sous le nom d'*Essai sur les machines en général* [...]" (Carnot [1783] 1786; Fig. 11). In *Principes fondamentaux de l'équilibre et du mouvement*'s Preface Carnot cited the *Essai* of 1783 and its role with respect to the new book in question (Carnot 1803a, Préface, p. v).

- 1784. *Lettre sur les aérostats*. This manuscript is lost.[10]
- 1784. *Éloge de M. le maréchal de Vauban* (Carnot 1784).
- 1785. *Dissertation sur la théorie de l'infini mathématique* (Carnot 1785).
- 1786. *Essai sur les machines en général*. It is the new edition[11] (Fig. 11; Carnot [1783] 1786).
- 1786. *Observations sur la lettre de M. Choderlos de Laclos à Messieurs de l'Académie françoise, concernant l'Éloge de Monsieur le Maréchal de Vauban* (Carnot 1786).
  - 1786. Carnot enrolled in the Société des Rosati d'Arras.
  - 1787. Robespierre enrolled in the *Société des Rosati d'Arras*.
- 1787. *Lettre de M. Carnot, Capitaine en premier au Corps Royal du Génie, à M. le Marquis de Montalembert. In: Réponse au Mémoire sur la fortification perpendiculaire* (Carnot 1787a).
- 1787. *Le Pouvoir de l'habitude*, read[12] at the *Académie d'Arras*, 1787 25th May (Carnot 1787b).
- 1789. *Mémoire présenté au Conseil de la Guerre* [...] *Est-il avantageux au Roi de France qu'il y ait des Places fortes sur les frontières de ses États*? (Carnot 1789a).
- 1789. *Réclamation adressée à l'Assemblée nationale contre le régime oppressif sous lequel est gouverné le Corps royal du Génie* [...] *28 septembre 1789* (Carnot 1789b).

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[10]On 1784, 17th January Lazare Carnot wrote this *Lettre sur les Aérostats* inspired by the first human flight. On 5th June 1783, the brothers Montgolfier–Joseph–Michel (1740–1810) and Jacques–Étienne (1745-1799) produced their first public official flight of the balloon filled with heated air. The flight was in Annonay (Ardèche, Region of Auvergne-Rhône-Alpes). Cfr.: Meusnier de la Place (1783), Darboux (1887–1896) and Gillispie (1983). Cfr: Dhombres and Dhombres 1997; Gillispie and Pisano 2014.

[11]In this edition, Lazare Carnot is well cited: "M. Carnot, Capitaine au Corps royal du Génie, de l'Académie des Sciences, Arts, et Belles–Lettres de Dijon, Correspondant du Musée de Paris". The 1786 edition—as we remarked at the beginning in this volume—had an early English translation, divided into 3 issues (XXX,XXXI, XXXII) by *Philosophical Magazine* (Carno 1808a, b, c). In part one the reader can find both a portrait from the original of Lazare Carnot (Carnot 1808a, first pages of the volume XXX) and a short biography/comment of Lazare Carnot written by the editor (*Ibidem*,pp. 370–371).

[12]This edition (Arras, 1971) also includes Maximilien Robespierre's speeches "Le droit et l'état des bâtards" read at the *Académie d'Arras*, 1786, 27th April.

- 1791. Carnot was finally *affecté* in Saint–Omer (North of France).
- 1791. Carnot became *Président des Amis de la Constitution d'Aire*
- 1791. Carnot became *Député at the Assemblée législative* and member of the *Comité de l'Instruction Publique*.
- 1791. Carnot became *Député du Pas–de–Calais* (North of France).
- 1792. Carnot became *Commissaire à l'Armée du Rhin*.
- 1792. Carnot proposed a special military strategy to avoid the conquest of the fortresses.
- 1792. The *Convention* elected Carnot *Député*.
- 1792. 10th August. Louis XVI was suspended.
- 1792. 13th August. Louis XVI is officially arrested.
- 1792. 20th September, *Battle of Valmy*. The first major victory by the army of France. It was located between Sainte–Menehould and Valmy (Marne department, N–E France).
- 1792. 21st September. The *Convention* abolished the Absolute Monarchy and declared the First French Republic.
- 1793. 14th January. The *Convention* begins to discuss the terms of the judgment for Louis XVI. Finally, *three motions* and *four questions* were decisive.
- 1793. 15th January. First motion. The *Convention* proposed to vote two main questions of culpability: *conspiration* (673 *pro*, 32 on different claiming, 3 did not answer and 10 abstaining, absents 31; total 718) and *ratification au people* (286 *pro*, 423 *contra*, 12 abstaining, absents 28; total 721). Carnot, respectively, voted *pro* and *contra*.
- 1793. 16th–17th January. Second motion. The *Convention* proposed to vote the third question of *punishment*: 387 *pro* (whom 26 *pro* by Mailhe's amendment), 44 in favour of *mort avec sursis*, 290 *pro* other punishments, 5 abstaining, absents 23; total 726. Carnot voted *death penalty*.

  - 1793. 17th January. The *Convention* sentenced to death Louis XVI (*Louis Capet*).
  - 1793. 19th January. Third motion. The *Convention* proposed to vote the fourth question of *sursis à l'exécution*: 310 *pro*, 380 *contr*a, 2 attached certain conditions to their votes, 10 abstaining, absents 47; total 702). Carnot voted *contra*.
  - 1793. 20th January. The convention officially announced the verdict to the *maison du Temple*.
  - 1793. 20th January. Louis XVI asked various requests to the *Convention*; one of them was a delay of 3 days before executing. The Convention granted all except the additional three days.
  - 1793. 21st January. Louis XVI at age 38 was executed (guillotine) on in Paris at the *Place de la révolution* (since 1795, *Place de la Concorde*). Charles–Henri Sanson (1739–1806), public executioner, inflicted it.
  - 1792. Carnot became *chargé de mission auprès des armées*.

- 1792. *Sur les citadelles, Carnot l'aîné, député du département du Pas-de Calais à ses collègues, 5 janvier, l'an IV de la Liberté* (Carnot 1792).

  - 1793. Carnot is member of *Comité de Salut Public* and *chargé des questions militaires*.
  - 1793. Carnot became *Commissaire de l'Armée du Nord*.
  - 1793. Carnot became *Capitaine du Corps du Génie*.

- 1793. *Rapport fait à la Convention Nationale par ses commissaires Carnot, Garrau et Lamarque*, [*Convention* 1793, 12th January] (Carnot 1793a).
- 1793. *Déclaration des droits du citoyen, proposée par L. Carnot, Député du Pas de–Calais, 10 mars 1793* (Carnot 1793b).

  - 1793. Carnot gave a substantial contribution to the victory of Wattignies.

  - 1794. Carnot became the 43rd *President of the National Convention*. [George Couthon (1755–1794), Maximilien–François–Marie–Isidore de Robespierre (1758–1794) and Louis–Antoine–Léon de Saint–Just (1767–1794) were condemned to death].
  - 1794. Gaspard Monge, Jacques–Elie Lamblardie (1747–1797) and Lazare Carnot founded the *École centrale des travaux publics*.

- 1794. *Rapport et projet de décret sur la suppression du conseil exécutif provisoire et son remplacement par des commissions particulières* [by Carnot on behalf of *Comité de salut public*] (Carnot 1794).

  - 1795. *École centrale des travaux publics* was renamed *École Polytechnique*.
  - 1795. Carnot became *Député au Conseil des Cinq-Cents*.
  - 1795. Carnot became one of the five directors of the *Directoire*.
  - 1795. Carnot appointed Napoléon Bonaparte as general in chief of the *Armée d'Italie*.

- 1795. [Report[13] at the *National Convention*] *Campagne des Français depuis le 8 septembre 1793 répondant au 22 fructidor de l'an Ier de la République jusqu'au 15 pluviôse an III* (Carnot 1795a).
- 1795. *Opinion de Carnot, représentant du peuple, sur l'accusation proposée contre Billaud-Varenne, Collot–d'Herbois, Barère, et Vadier; par la commission des Vingt et un* (Carnot 1785b).

  - 1796. Carnot was elected President of the *Directoire*. He was in charge for a short term (30th April–29th July).
  - 1796. Carnot's first son, Sadi was born in Paris.

- 1796. *Exploits des Français depuis le 22 fructidor an I, jusqu'au 15 pluviôse an III* (Carnot 1796a).

[13] In 1797 a second report (*tableau de Campagnes*) was edited.

- 1796. *Discours prononcé par le Président du Directoire Exécutif à la fête de la Reconnaissance* (Carnot 1796b).
  - 1797. Bonaparte took power. The *Consulat* replaces the *Directoire*.
  - 1797. Carnot, after *Coup d'État de Fructidor* (1797 4th September) went to Geneva (October).
- 1797. *Réflexions*[14] *sur la métaphysique du calcul infinitesimal* (Carnot 1797a).
- 1797. *Œuvres mathématiques*[15] *du Citoyen Carnot* (Carnot 1797b).
- 1797. *Épitre au directeur Carnot, suivie de quelques-unes de ses poésies fugitives, et précédées de notes historiques sur les sociétés de ROSATI.* (Carnot 1797c).
- 1798. *Réponse de L.–N.–M. Carnot, citoyen français, l'un des fondateurs de la République, [...] au rapport fait sur la conjuration du 18 fructidor au Conseil des Cinq–Cents* (Carnot 1798).
  - 1800. Carnot became *Inspecteur général des armées* (February 7th).
  - 1800. Carnot is entitled *Ministre de la Guerre* (April 2nd).
  - 1800. Carnot resigned (October 8th) from his *Ministre de la Guerre*. This was historically considered an act of disapproval to the appointment of Napoléon as *Consul à vie* (October 8th).

[14]This book had a notable success and very quickly was translated into Portuguese (Lisbon, 1798), German (Frankfurt, 1800), English (London, 1800, Philosophical magazine, Vol. VIII, pp. 222–240; 335-352. Id. Vol. IX, 1801, pp. 39-56; Holarke, 1801; Peddie, 1914) and Italian (Pavia, 1803). In 1813 a second edition (Carnot 1813) was published.

[15]This particular edition includes a portrait of Lazare Carnot and two reprints: *Réflexions sur la métaphysique du calcul infinitésimal* (published by Duprat some months before, 1797) and the second edition of *Essai sur les machines en général* (Dijon, 1786).

- 1800. *Lettre*[16] *du citoyen Carnot au citoyen Bossut, concernant quelques vues nouvelles sur la trigonométrie.* In *Cours de mathématiques. Tome II. Géométrie et application de l'algèbre à la géométrie*, pp. 401–421 (Carnot 1800 in Bossut 1800).
- 1801. *De la corrélation des figures de géométrie* (Carnot 1801).
  - 1801. Carnot's second son, Lazare–Hippolyte was born in Saint–Omer. Later, Lazare–Hippolyte's son, Sadi Carnot who will be the President of the *Troisième République* (1887, 3rd December–1894, 24th June).
  - 1802. Carnot became member of the *Tribunat*
- 1803. *Géométrie de position à l'usage ceux qui se destinent à mesurer les terrains* (Carnot 1803b).
- 1803. *Principes fondamentaux de l'équilibre et du mouvement* (Carnot 1803a).
- 1804. *Discours prononcé par le Citoyen Carnot, sur la motion relative au gouvernent héréditaire Séance extraordinaire du 11 floréal an XII* (Carnot 1804).
- 1806. *Mémoire sur la Relation qui existe entre les distances respectives de cinq points quelconques pris dans l'espace; suivi d'un essai sur la théorie des transversales* (Carnot 1806).
- 1810. *De la Défense des places fortes*. (Carnot 1810; see also Carnot 1858a).
- 1811. *De la Défense des places fortes*. 2nd edition (Carnot 1811).
- 1812. *De la Défense des places fortes*. 3rd Edition (Carnot 1812a).
- 1812. *Discours préliminaire de la troisième édition du Traité de la défense des places fortes* (Carnot 1812b).
- 1813. *Réflexions sur la métaphysique du calcul infinitésimal*. 2nd edition[17] (Carnot 1813; see also Carnot 1936).
  - 1814. Carnot took part in the battle of Anvers (Antwerp) and became its governor (January 30th–April 23rd).

[16] The date of the letter is *30 fructidor an VIII* [1800, 22nd September].

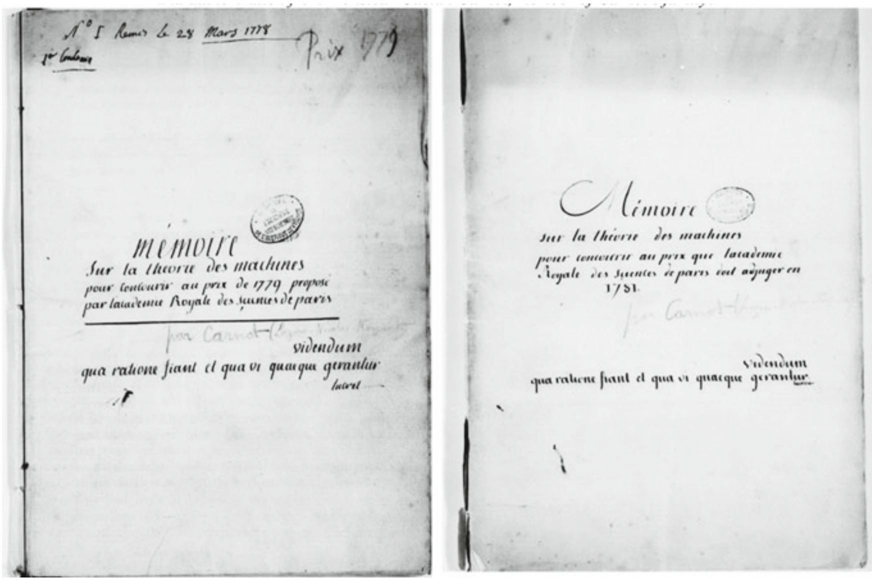
[17] This book had several reeditions (1839, 1860, 1881, 1921, 1970). An early translation (by Browell) was published in Oxford (Carnot 1832).

- 1814. *A Treatise on the Defence of Fortified Places*. Translated from the French (Carnot 1814a).
- 1814. *Mémoire*[18] *adressé au roi, en juillet 1814 par M. Carnot* (Carnot 1814b).
- 1815. *Mémoire adressé au roi, en juillet 1814 par M. Carnot*. 5th edition. (Carnot 1815a).
    - 1815. Carnot was elected *Ministre de l'Intérieur* during the *Cent jours*.
- 1815. *Exposé de la situation de l'Empire* (Carnot 1815b).
    - 1815–1816. Carnot is banished as regicide by Louis XVIII and was exiled to Poland.
- 1818. *Correspondance inédite de Napoléon avec le général Carnot pendant les Cents jours* (Carnot 1818).
- 1819. *Correspondance inédite de Napoléon avec le général Carnot pendant les Cents jours* (Carnot 1819).
- 1820. *Opuscules poétiques du général L.N.M. Carnot* (Carnot 1820; see also Carnot 1894, 1933).
- 1821. *Don Quichotte*[19] *poème héroï–comique en six chants* (Carnot 1821).
    - 1823. Carnot exiled to Magdeburg (Prussia, now Germany) where he died.
- 1823. *Mémoire sur la fortification primitive pour servir de suite au Traité de la défense des places fortes* (Carnot 1823; see also Carnot 1858b).
    - 1889. Carnot's remains were transferred to the Panthéon (Paris) on 4th August.

[18]This book had several there main reeditions (Bruxelles, Londres, Paris).

[19]This book had a reedition in 1891 (Verviers, Paris).

Fig. 8 «Lazare Carnot en exile en Allemagne, d'après un tableau de C. Bochme (1823)» *Plate from the original portrait conserved at Académie François Bourdon, Le Creusot—Archives Lazare Carnot. Very kindly authorized by Monsieur Gaetan Carnot, member of Carnot's family*



1778[9] 1780[1]

Fig. 9 Lazare Carnot's Memoires (1778, 1780)
With kind permission by *Archives et patrimoine historique de l'Académie des sciences, Paris, France*

Lazare Carnot's works were well known and cited by scientists of the period. For example, Joseph–Louis Lagrange (1736–1813) cited him in the *Mécanique Analytique*[20]. Others citations were by: Jean Baptiste Joseph Fourier (1772–1837) on the *Principle of virtual work* (Fourier 1798, pp. 20–60; see also *Id.*, 1888–1890, pp. 475–521, 1807, 1808, 1822, 1829; see also: Richmann 1750; Riemann [1861] 1868); Claude Louis Marie Henri Navier (1785–1836) on *force vive* and *Carnot's theorem* (Navier 1841, pp. 350–351); and (Mach [1896]

[20]Lagrange 1788, II, p. 578. Carnot's quotation of Lagrange's *Théorie des Fonctions Analytiques* (Carnot 1813, p. 47; see also Lagrange 1881, pp. 409–410; 1797, 1806, 1793a, b). For Carnot's theorem included by Lagrange in his 2nd edition, see Lagrange 1813, pp. 406–407. Recently: Dugas 1955; Scott 1970; Gillispie 1971; Drago, Manno, and Mauriello 2001; Drago and Manno 1994; Dhombres and Dhombres 1997; Gillispie and Pisano 2014; Chemla 1990, 1998; Nabonnand 2010, 2011; Pisano's works below.

1986, p. 211). Notwithstanding this recognition, due to his political and military activities, Carnot was always in the line of fire, marking the whole of his life with an unfounded prejudice: expert politician or outsider scientist? Thus, other contemporary works on mechanics, (especially Lagrange's *Mécanique*) received more credit among scientists and later on in the historiography of science (Cfr. Fraser 1983). In fact, rather than diminishing Carnot's importance, the shift in mechanics, which became more and more applied to other sciences (Pisano and Capecchi 2013; Pisano 2017; Gillispie and Pisano 2014; Cardwell 1965, 1967, 1971) was a favorable environment for the introduction of an advanced mathematics into mechanics, in contrast to the application of mechanics to mechanical calculation and geometry problems only. For example, one can see Laplace's *Traité de mécanique céleste* and *Exposition du système du monde* (Laplace [1836] 1984; see also 1805; Lavoisier and Laplace [1780] 1784). His scientific program mainly used central forces and correlated differential equations in order to explain physical phenomena. On the other hand was the role played by mathematics with respect to physical measures, such as in the analytical theory of heat (Pisano and Capecchi 2009a). The application of these—such as propagation and velocity applied to heat—and other subjects produced new research disciplines as shown in the following flow chart (Fig. 10):

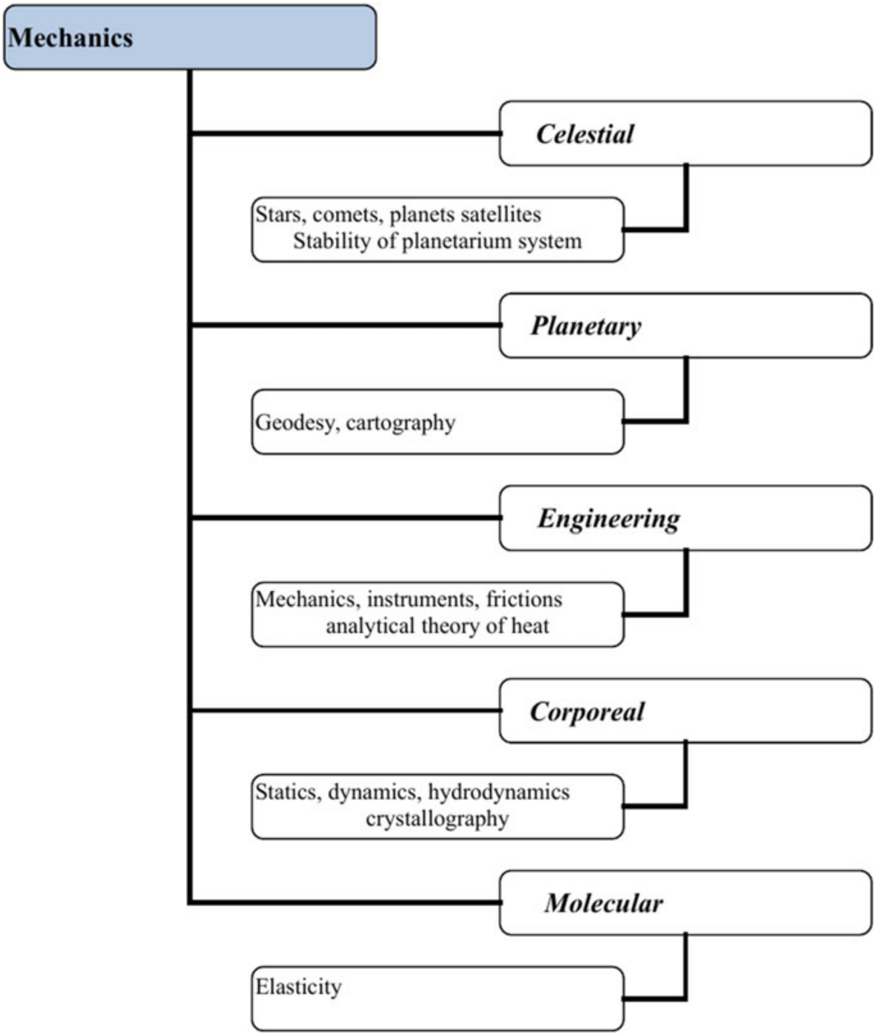


Fig. 10 Applied Mechanics and the birth of new research subjects[21]

[21]Gillispie and Pisano 2014; Cfr. Drago, Manno, and Mauriello 2001; Drago and Manno 1994.

When considering analytical theories, such as those by Fourier[22] first, and later by Gabriel Lamé (1795–1870), in both cases progress was made through the development of advanced mathematics (differential calculus by partial derivatives, integral calculus, series) to describe each field of phenomena. The change of focus from a mechanical nature (mathematical space and time, velocity, differential calculus) to mechanical aspects of the nature—applied to other phenomena—contributed to the birth of new disciplines such as the applied sciences (Cfr. Bouvier). This was the case also for the *Mécanique celeste*: from ancient Astronomy (that is observation, modelling, and trigonometry) to applied mechanics to the astronomy of the 19th century. This achievement was underwritten by the fact that astronomy is the most important scientific system of knowledge, but it is also science's most conservative, and difficult, field to approach.

Lazare Carnot's mechanics/mechanical machines developed, as a typical mathematical operation (its fundamental equations. Gillispie and Pisano Chaps. 2–3 and 11), the sum of all the parts that compose the physical system. That is, by combining a sequence of elements using addition, the result is their sum. In other words, a sum may be expressed (under certain conditions) as a definite integral:

$$\sum_a^b f = \int_a^b \mathrm{fdx}$$

Therefore, Carnot used the ancient mathematical operators, sum and integral[23], with respect to the advanced ones, infinitesimal and differential calculus (see below translations and endnotes).

Carnot's research was a kind of point of convergence amidst emerging engineering, early theoretical studies on machines, the classical tradition and new insights from mechanics. His *Essai sur les machines en général* (Carnot 1783; 1786) can be considered one of the most early relevant theoretical treatises on machines alongside

[22]See also Briot 1869; Chasles 1839, 1852, 1875.

[23]Thus, the area under the curve is physically recognised as the *Work*.

those of Petit, Poisson and Poncelet (Cfr. Arago and Dupin 1827).[24] Through the application of mechanics to machines—that is, by including the role played by machineries—Carnot brought the subject to a high theoretical level by studying the general properties of mechanical machines, for example the independence of efficiency from working substance, to arrive at a theory of *machines en général*.

He followed—in a certain manner—the school of thought of Jean Baptiste Le Rond d'Alembert (1717–1783) who was one the last scholars of *vis viva*—in the tradition of René Descartes (1596–1650), Christiaan Huygens (1629–1695) and Gottfried Wilhelm von Leibniz (1646–1716; Leibniz 1849–1863, 2009). This tradition assisted his research on bodies in collisions—applied to machines, as well. In fact, these phenomena were not assumed: neither in Newton's science (Newton [1686–1687] 1803, 1687, 1714, 1730, 1803, 1999, Cohen and Smith; Guicciardini 1999; Bussotti and Pisano 2014) nor, generally speaking, in the Newtonian deterministic paradigm.

1.2 Notes on the Context before the *Essai*

In the 15th–16th centuries, so-called *men of war* accomplished—without having a certain and structured scientific theoretical background—a reliable competence in the manufacture of cannons and other weapons. The impact of this on Science needs to be historiographically[25] clarified: "[...] before we appreciate their [the men of

[24]See also: Poisson 1823a, b, 1829, 1833, Chatzis 2009; Poncelet 1823a; Poncelet 1827–1829, 1829, 1845, 1874.

[25]On specific criticism/works in history and historiography of science (physics and mathematics) from *Scientia de ponderibus*, machines, Mechanics to 19th century, see (selected): Winter 2007; Wallis 1668, 1693; Tartaglia [1554] 1959, 1546, 1565; Pisano and Capecchi 2014; Brown 1967–1968; Capecchi and Pisano 2007, 2008a, b, 2010a, b, Clagett; Clagett and Moody, Clagett and Murdoch; Crombie 1959, 1963, 1994; Commandino 1565, del Monte [1577] 1581; Huygens 1673; Duhem 1905–1906, 1977; Galilei [1599] 1634, 1890–1909, 2002; Jouguet 1924; Galluzzi 1979, galluzzi and Torrini 1975–1984; Jammer 1957, 1961; Kuhn 1955, 1959, 1960, [1961] 1980, 1961, 1962, 1963, 1970, 1974; Liouville 1836; Mach [1883] 1996; Renn 2000; Pisano 2007, 2009a, b, c, d, 2010, 2011, 2012, 2015, 2016, 2017; Pisano and Gatto, pre-print; Pisano and Bussotti 2012, 2014a, 2015a, b, c, d, 2016a, b, 2017a, b, c, 2020a, b; Pisano and Capecchi 2008a, b, 2012, 2013, 2015; Pisano, Capecchi and Lukešová 2013; Pisano and Gaudiello 2009; Popper 1962; Prigogine 1980; Singer [1954–1958] 1993; Skolem [1920] 1967; Smith and Wise 1989; Scott 1959, 1971; Taton 1964a, 1951; Tackray 1970; Truesdell 1968a, b, 1970, 1980; Truesdell and Bharatha 1977; Westman 1980.

war's] hesitations and grasp the nature of their ignorance and their failures." (Gille 1966, p. 240). For, *how did the relationship between science, applied sciences and machineries work*?

During the 17th and 18th centuries, remarkable practical and early theoretical studies of heat engines developed, due to technological and social industry developed especially in France and in England. The main subject was the search for a source (*working substance*) of unlimited power by means of, for example, the conversion of heat into work: broadly, a general law of conservation of energy. Heat machines were the field of applicability of these studies (Cfr.: Parent [1745] 1706; Gustav 1869; Hirn 1862a, b, 1863, 1864a, b, 1867, 1868, 1887; Payen 1967, 1968).

In 1778 Lazare Carnot produced stimulating studies on machines and the *agent* (*working substance*; Carnot 1778, 1780, §§ 149–160). This allowed him to systematically approach his more structured work on mechanical machines in general (Carnot [1783] 1786).

Thanks to James Prescott Joule's (1818–1889) experiences (Fox 1969; Joule 1844, 1845, 1847, 1965, pp. 277–281, 1847, pp. 173–176) and successive theoretical studies—mainly by Rudolf Clausius (1822–1888) and by William Thomson (1824–1907; Thomson 1848–1849a, b, 1851a, b, 1852, [1890] 1943; see also Clausius 1850, 1865, 1864, 1868–1869)—a mathematical formulation (first principle) of the conversion of heat into work and consequent conceptualisation of energy (an integral) was produced. The crucial role was played by new physical quantities—out of the Mechanical paradigm—such as (Volume) Heat, Temperature and Work. Finally, after Sadi Carnot's work of 1824, thermodynamics consolidated its scientific theoretical entity by means of two principles, historically inverted: the second principle of thermodynamics (Carnot 1824) and the first principle of thermodynamics (Thomson/Clausius from 1848 to 1850s; Pisano's works. Cfr.: Reech 1851, 1852, 1853, 1854, 1869; Tait 1868, 1887; Steward 1866; Callen 1974; Callendar 1910, Hoyer 1974, 1975, 1976; Klein 1969; Mendoza 1959, 1963). In modern terms, generally speaking, the first principle reads:

$$\Delta E = \delta W \pm \delta Q \quad (1)$$

In addition, Thomson (more analytically than Sadi Carnot) also dealt with the second principle of thermodynamics (Thomson 1848–1849, pp. 541–574; 1882–1911, pp. 113–155; 1852, pp. 248–255; see also Smith and Wise 1989, Chaps. 9–11; Pisano, Anakkar, Pellegrino and Nagels 2019).

At that time, Lazare Carnot's mechanics was an attractive alternative to Newtonian mechanics (Gillispie and Pisano 2014) both from the standpoint of content and methodology. Unfortunately, without limit operators it could not reproduce the same power of calculus of the Newtonian apparatus. Particularly, a strong relation to Leibniz's ideas, *theoretical physics must explain facts with facts* made the above cited attraction more evident. For example, one can understand the studies on colliding bodies in relation to those of Leibniz in the *Dynamica de Potentia et Legibus Naturae Corporeae* (Leibniz 1849–1863, II, sectio III, Propositions 1–18, pp. 488–507); *the same can be said* for the early concept of potential energy (*Ivi*, II, sectio I, p. 435). Lazare Carnot's method allowed him to introduce progress in the concept of potential energy in his theory of motion applied to machines (Carnot 1803a, pp. 36–38) and limited him to a necessary mathematics only in order to explain physical phenomena without the use of absolute space and time, that had been typical of previous predictive mechanics (Cfr. Koetsier 2007).

Carnot used the same founding method in all his scientific and military works. Gillispie (Gillispie 1971) *in primis* noted this unitary founding method existed in Carnot's various scientific works. Recently Carnot's method has been fully explained (Gillispie and Pisano 2014). He carried on Leibniz's work in order to find his *characteristica universalis*: a systematic reasoning for all the fields of knowledge (Carnot 1803b, p. 1). Leibniz succeeded in part with infinitesimal analysis and Carnot presented his geometric method as a Leibnizian one[26]. From the point of view of geometric calculation, given a problem on a geometric figure, its elements are made to vary by insensible degrees, always maintaining the same type of figure, in order to find the solution formula for each generated. For that, Carnot uses *correlation tables* (see also Poncelet's works in the references section below).

In 1789, Antoine–Laurent Lavoisier (1793–1794; *Traité élémentaire de Chimie* [1789] 1937, 1862–1893) as well as other chemists of his time, searched in a revolutionary fashion for the basic principles of this new theory. These new principles were different from Newtonian Mechanics (Bussotti and Pisano 2014; Pisano and Bussotti 2016; 2020). Lavoisier's conceptualization excluded a chemical system based on only four elements. He addressed new quantities such as *chaleur, calorique and lumière* (Lavoisier 1789, I, pp. 12–17; see also Betancourt 1792, Berthollet 1803, 1809; Combes 1863, 1867). On that, Pierre Simon de Laplace (1749–1827) also carried out remarkable research in collaboration with Lavoisier (Lavoisier and Laplace 1784). Therefore, two main paths of research were produced: a study on the properties of gases (Newtonian kinetic model of gases) and study on the efficiency of heat machines, later also called thermodynamics. From 1772, Lavoisier worked on the foundations of his new forthcoming theory later revealed in his *Traité élémentaire de Chimie* of 1789. This is referred to in the historiography[27] as the *chemical revolution* (Guerlac 1963), originally based on the phenomenon of combustion (Dagognet 1969). The second half

[26]On the Leibnizian background in Lazare Carnot, one can also see the famous correspondence in 1677 (*Ivi*, VI, pp. 81–106) between Leibniz and Honoratus Fabrius (1607–1688). For a first panoramic view on Leibniz and his dynamics, see Pierre Costabel's (1912–1989) works (Costabel 1960). For the most complete (works and letters) series of Leibniz's mathematical writings, see Eberhard Knobloch's VIII edition for "Berlin–Brandenburgische Akademie der Wissenschaften Leibniz–Edition, Reihe VIII" (Leibniz 2009), Bussotti 2015. Particularly on Leibniz, in the occasion of his anniversary, see also *Leibniz and the Dialogue between Sciences, Philosophy and Engineering*, 1646-2016 (Pisano, Fichant, Bussotti and Oliveira 2017; Bussotti and Pisano 2017; see also Bussotti 2003, 2015).

[27]Historical studies often proposed such an emergence between the 16th and the 18th century evoking Paracelsus (1493–1541, born Philippus Aureolus Theophrastus Bombastus von Hohenheim), Robert Boyle (1627–1691), Nicolas Lémery (1645–1715), Georg Ernst Stahl (1659–1734) or even Michail Vasil'evič Lomonosov (1711–1765).

of the 18th century was also marked by chemical studies on air and gases, which included early results on affinities (Thackray 1970). Newtonian rational mechanics as *calculated physics*[28] was finally documented as being *Newtonian* (Bussotti and Pisano 2014a) by two main historiographic approaches (Kragh 1987): (1) physical phenomena should be incorporated into a mathematical apparatus related to the final law of universal gravitation, (2) by *Hypotheses non fingo* (Bussotti and Pisano 2014ab; Pisano, Franckowiak and Anakkar; Pisano and Franckowiak). But the scholars busy with chemistry were aware that their science was considered weak because its lack of mathematisation (i.e., Black 1803). Therefore, they focused their studies on measurements and not on any pre-determined *a priori* system. In 1777, Lavoisier tried to include mathematics in chemistry (Lavoisier 1862; Cfr. Scerri 2013; Scheele 1781, 1785), by representing chemical reactions as *ad hoc* equations. Later he worked using air and water as compounds, gases and oxygen–acidity (Cfr. Perkins 1820, 1821). Particularly, Lavoisier studied fifty-five substances that could not be reduced to a final substance and proposed two new elements: *light and caloric*. He also defined *chaleur, calorique and lumière* (Lavoisier [1789] 1937 pp. 12–17; 1862–1893; see also Bailyn 1985; Cfr. Landriani 1785). As above cited, he worked with Laplace and produced an ice-calorimeter apparatus to measure the specific heat of different bodies.

In 1816–1819, Pierre Louis Dulong (1785–1838) and Alexis Thérèse Petit (1791–1820) presented cases where the relationship between the specific heat and temperature is relevant (Dulong and Petit 1816; Dulong 1829). The laws of gases were an object of interest and studied by theoretical scholars, too (physicists and chemists). In particular, the adiabatic law (Poisson 1823, pp. 5–16; Laplace 1822) had various formulations. When Siméon Denis Poisson (1781–1840) expressed the right form of the equation, most scientists did not consider the question solved.

[28]"Il est clair que la révolution qui placerait la *Chimie* dans le rang qu'elle mérite, qui la mettrait au moins à côté de la Physique calculée ; que cette révolution, dis-je, ne peut être opérée que par un chimiste habile, enthousiaste, and hardi, qui, se trouvant dans une position favorable, and profitant habilement de quelques circonstances heureuses, saurait réveiller l'attention des savants, d'abord par une ostentation bruyante, par un ton décidé and affirmatif, and ensuite par des raisons, si ses premières armes avaient entamé le préjugé" (Venel, 1751–1772 [1753], III pp. 409–410; original author's italic and capital letters).

Sadi Carnot wrote two unpublished works (Carnot 1878a, b) and *Réflexions sur la Puissance Motrice du Feu* (Carnot 1824, hereafter *Réflexions*) and invented thermodynamics and its application to heat machines (Gillispie and Pisano 2014; Cfr. Callot 1980). In his unpublished work, *Notes sur les mathématiques, la physique et autres sujets* (Carnot s. d.), he used the hypothesis on *puissance motrice* of the conversion of heat into work[29] (Carnot 1878, pp. 134–135). In the *Recherche d'une formule propre à représenter la puissance motrice de la vapeur d'eau* (between November 1819 and March 1827) he provided a cycle by three phases only. The *Réflexions* (Gillispie and Pisano 2014, pp. 176–183) and his *Recherche d'une formule propre à représenter la puissance motrice de la Vapeur d'Eau* (Carnot 1978, pp. 223–225) were a new conceptualisation of the relationship between physics and mathematics within a new theory which seemed to echo engineering practices. It was also quite different from the analytical approach adopted by Jean Baptiste Joseph Fourier (1768–1830) some years before, e.g., for heat propagation in solids (1807; Cfr. Pisano and Capecchi 2009a).

Several decades after, Gabriel Lamé (*Ivi*) wrote *Leçons sur la théorie de la chaleur* (Lamé 1861b; see also 1861a, 1836, 1852). He was not interested in thermodynamics as a unique science; he devoted himself to analytical heat transmission by means of differential equations and volume integral in order to calculate energy; a new branch of physics, *physics mathematics* was thereby defined (Pisano 2013 and other Pisano's works on the subject below). In this period, heat engines were studied and produced. In particular, steam engines were used, that required water to be pumped (Cfr.: Edmunds 1902; Einstein [1949] 1970; Tredgold 1838). In *Réflexions* Sadi Carnot calculated[30] efficiency as a quantity independent of any *working substance* (Carnot 1824, pp. 73–79, ft. 1; Carnot 1978 p. 38). Sadi introduced crucial concepts like *state of a system, reversible*

[29]In *Notes sur les mathématiques, la physique et autre sujets* Sadi Carnot introduced his "thèse générale" on energy: "[...] la puissance motrice est en quantité invariable dans la nature, qu'elle n'est jamais à proprement parler ni produite, ni détruite" (Carnot 1878a, folio 7r; Picard 1927, p 81, line 14; see also Robert Fox in Carnot 1986, p. 191; Gillispie and Pisano 2014, Chap. 11).

[30]This calculus presents some inaccurate passages. A full analysis is available (Gillispie and Pisano 2014, Chap. 9).

processes, cycle, phases, impossibility of perpetual motion and a fundamental theorem showed by an *ad absurdum* proof, untypically for physics at that time. The impossibility of perpetual motion addressed the state of a system, reversible processes and cycle (four phases). At the *École Polytechnique*, Benoît Paul Émile Clapeyron (1799–1864) a friend of Sadi's, mathematically and analytically rewrote the *Réflexions* in his *Mémoire sur la puissance motrice de la chaleur* (Clapeyron 1834, pp. 153–190) adding a cycle[31] which came to be incorrectly known as "Carnot's cycle" (see also Clausius 1850, pp. 368–397; pp. 500–524; p. 379).

1.3 The *Essai sur les machines en général* (1786)

In the *Essai sur les machines en général* (Carnot [1783] 1786) mechanics refers to mechanical machines. Particularly Lazare Carnot reasoned independently from working substance bodies and particular mechanisms. In his words:

> [...] this Essai only concerns machines in general; each of them has their own particular properties […].[32]

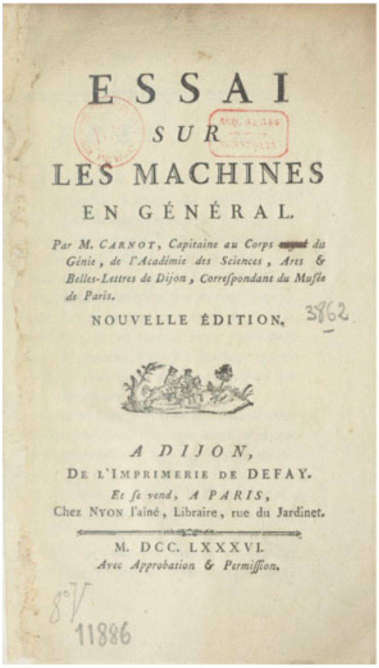
> [...] we compare these different efforts regarding the working substances that produce them, because the nature of the working substances cannot change the forces they must exert to fulfill the different objects for which the Machines are intended.[33]

Carnot is usually considered to be the first author to claim that the empirical nature of mechanics was both theoretical and mechanical (Gillispie and Pisano 2014). He expressed his view of mechanics in the introductory parts of *Essai sur les machines en général* (Carnot 1786) and *Principes fondamentaux de l'équilibre et du mouvement* (Carnot 1803a).

[31]This PV diagram (also Clapeyron diagram) has no metric (such as, e.g., with respect to Descartes diagram).

[32]Carnot 1786, p. x, line 14.

[33]Carnot 1786, p. 62, line 2.



ESSAI
SUR
LES MACHINES
EN GÉNÉRAL.

Par M. CARNOT, Capitaine au Corps du Génie, de l'Académie des Sciences, Arts & Belles-Lettres de Dijon, Correſpondant du Muſée de Paris.

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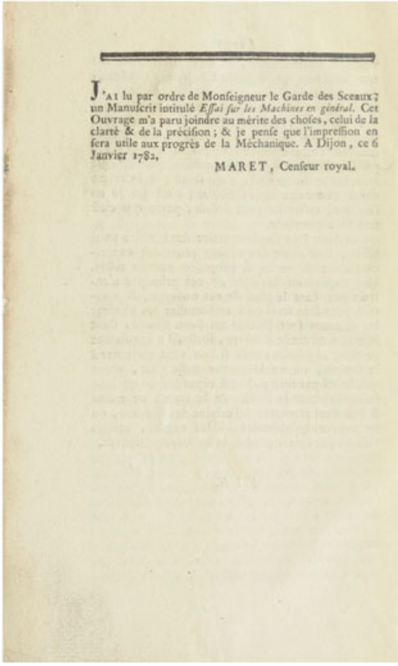

J'AI lu par ordre de Monſeigneur le Garde des Sceaux; un Manuſcrit intitulé *Eſſai ſur les Machines en général.* Cet Ouvrage m'a paru joindre au mérite des choſes, celui de la clarté & de la préciſion; & je penſe que l'impreſſion en ſera utile aux progrès de la Méchanique. A Dijon, ce 6 Janvier 1782.

MARET, Cenſeur royal.

b

Fig. 11 Lazare Carnot's *Essai sur les machines en général* (1786), first (a) and last page (b) where the manuscript of 1782 is mentioned. With Kind permission by *Gallica–Bibliothèque Nationale de France*

A machine was thought of as an intermediary body in order to communicate/produce movement between two or more principal bodies who do not act directly on one another. On that, a remarkable criticism of mechanics and machines was provided by Carnot in his *Preface*:

> Preface. Although the theory here presented is applicable to all issues concerning the communication of motions, *Essay on machines in general* was given as the title of this pamphlet; first of all, because it is mainly the Machines that are considered as the most important argument of mechanics; secondly, because no particular machine is dealt with but we only deal with properties which are common to all of them.
>
> This theory is based upon three main definitions; the first looks at some motions that I call *geometric*, because they can be only determined by the principles of geometry, and are absolutely independent of the rules of dynamics; I did not think that we would omit without creating obscurity in the statement of the main proportions, as, in particular, I let you see in the case of the principle of *Descartes*. By my second definition, I try to fix the meaning of the terms *impelling forces* and

resisting force: it seems to me that without knowing a precise definition between these two different forces, we cannot clearly compare the causes and effects of the machines, without a well characterized distinction between these forces; and upon this distinction it seems to me that something vague and indeterminate was always left. Finally, my third definition is that by which I give the name of *moment of activity*[34] of a force referring to a quantity which includes both a real force or an activity in motion that every instant employed by that force, that is to say, the time during which it acts. In any case, an agreement should be that this quantity, under whatever name one wishes to designate, to meet it in the analysis of Machines in motion is frequent. Using these definitions, I arrive at very simple propositions; I deduced them using the same fundamental equation. [...]. This equation is the most simple, generally extends to every conceivable case of equilibrium and motion, both this motion suddenly changes, that it change by insensible degrees; it also applies to all bodies, both hard [plastic[35]] that they have any degree of elasticity; [...]. I easily obtain from this equation a general principle of equilibrium and motion for Machines properly so called; [...]. Everyone claims [a principle] that, for Machines in motion, what is gained in force is lost in time or speed; but after reading the best books of mechanics, where we should find proof and explanation of this principle, can we capture its importance and its true meaning? Since for most Readers its generality has irresistible evidence which must characterize mathematical truths. If Readers should find this guarantee striking, do not they see themselves as meccanici, educated by these works, [and] immediately abandon their chimerical projects? Do not they believe or at least suppose, in spite of everything one can tell them, that some magic is present in the Machines? The counter examples proposed are limited to simple Machines; they considered them not to be of such great effect; but none show them that it must be valid in every imaginable case; the case of two forces is considered only, and, for other cases, an analogy[36] seems to be

[34]In this context, *puissance* means/is translated by a general term as *force*. The *moment of activity* corresponds to *work*. For the latter, Lazare Carnot refers to the related quantity time and not to space. In other cases, like for other scholars in this period, Carnot used the term “force” (in general) to indicate (in modern terms) *inertia*, *variation of quantity of motion in time, motive* or *acceleration*.

[35]Lazare Carnot adopted the lost quantity of motion. This part of his mechanics and machines theory included the irrelevance of the geometrical form of a machine that produced a kind of conceptual equivalence between *hard bodies* and *plastic bodies* (see the endnotes in the anthological part below). Lazare Carnot’s second fundamental law of hard bodies “corps durs” (Carnot 1786, p. 22; see also 1830a) or *perfectly hard* “parfaitement durs et sans resort” (see also d’Alembert [1743] 1758, Lemme XI, pp. 144–145) can also mean bodies without their (natural) elasticity (Carnot 1786, pp. 22–23; see also Carnot 1803a, pp. 8–10). Carnot considered elastic bodies as a kind of limit case of hard bodies. Thus, he considered elastic bodies and the composition of infinitely hard bodies, each separated from the next by elastic springs (Carnot 1786, p. 23; see also “corps *solides*” in Carnot 1803a, p. 8). Generally speaking, Carnot tried to create a simplified model of *corps durs* (see below).

[36]It should be noted that the analogy did not go any further.

> sufficient enough. [...]. The way to eradicate this error, is without doubt, [1] to fight its source, showing that, not only in all known Machines, but in all possible Machines, *loss in time or speed is always what is gained in force*; it is an inevitable law; and [2] to explain what this law clearly means; but in order to do that, one must move towards the greatest possible level of generality, and not study any Machine in particular, not adopt any analogy; in the end, it is necessary to propose a general proof, immediately and geometrically deduced by the first axioms of mechanics: that is exactly what I tried to do in this Essay.[37] X. The Science of Machines in general is reduced to the following question: By *knowing the virtual motion of any system of bodies (that is to say, that it would take each of these bodies, if it was free) to find the real motion which will be the next instant* [after the collision], *since there is mutual action of bodies, thus considering them as they are in nature, that is, having inertia common to all parts of matter.*[38]

The search for a universal working substance (*machine en général*) for mechanical machines—that included subsequent studies on equilibrium and movement (Carnot 1803a, b)—was one of Lazare Carnot's major projects from the 1760s. On the subject, he wrote two *Memoires* (Carnot 1778, 1780) and then the *Essai* (1783).

In the following, Lazare Carnot's main assumptions (in the *Essai*) underlying this mechanical project are listed (Gillispie and Pisano 2014 Chap. 11):

The cause of the motion of mechanical machines.[39]
What is the best way of utilizing the *greatest possible effect* produced by a mechanical machine in motion?[40]
Lacking a complete theory of *impelling forces* and *resisting forces* in mechanics.[41]
Searching for a general theory of machines and principle of equilibrium and motion.[42]

[37]Carnot 1786, pp. iij–ix, line 1. (Author's italic).

[38]Carnot 1786 § X, p. 21, line 7. (Author's italic).

[39]Carnot 1786, p. vj, pp. 13–14; see also Carnot 1780, § 103; Gillispie 1971, Appendix C, § 103, pp. 301–302.

[40]Carnot 1786, pp. ix–x, pp 89–94; see also Carnot 1780, §§ 149–160; Gillispie 1971, Appendix C, §§ 149–160, pp. 327–340; Carnot 1803a, p. xxj, p 149, pp. 247–250.

[41]Carnot 1786, pp. iv–v; Carnot 1780 § 129; Gillispie 1971 § 129, p. 316.

[42]Carnot 1786, pp. iv–v, pp. 11–12; see also Carnot 1778 §§ 27–79; Carnot 1780, § 102, §§ 133–141; Gillispie 1971, Appendix C, § 102, pp. 301–303, §§ 133–141, pp. 317–321.

Communication of motion and work.[43]
Reducing the problems of mechanics to a practice—calculation and geometry.[44]
The machine and its use.[45]
Absorbed motion and lost motion in a mechanical machine.[46]
Moment d'activité.[47]
The advantage.[48]
Finding the actual ("réel") motion after interaction among bodies.[49]
The aim of running and general mechanical machines.[50]
To obtain the *maximum effect produced*, no useless motions and interruptions have occurred.[51]
The operative conditions to *establish the maximum effect produced for a hydraulic engine*.[52]
Searching for actual ("réelle") motion in mechanical machines.[53]
For moving machines, what is lost in time or speed is always what is gained in force. (*Golden rule*).[54]
The impossibility of a perpetual motion.[55]

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[43] Carnot 1786, p. iij–iv, p. 44; Carnot 1803a, pp. xiij–xvj; see also Carnot 1780, footnote " * ", § 148; Gillispie 1971, Appendix C, footnote " * ", § 148, p. 309, pp. 326–327.

[44] Carnot 1786, p. 12; see also Carnot 1780, § 113 and footnote " * "; Gillispie 1971, Appendix C, § 113 and footnote " * " , pp. 308–309.

[45] Carnot 1786, p. 19, pp. 60–62; see also Carnot 1780, § 108; Gillispie 1971, Appendix C, § 108, p. 303.

[46] Carnot 1786, pp. 19–20; see also Carnot 1780, §§ 108–109; Gillispie 1971, Appendix C, §§ 108–109, pp. 303–304.

[47] Carnot 1786, p. 88; see also Carnot 1780, §§ 129–132, § 149; Gillispie 1971, Appendix C, §§ 129–132, pp. 316–317, § 149, p. 327.

[48] Carnot 1786, p. 85; see also Carnot L 1780, § 151; Gillispie 1971, Appendix C, § 151, p. 328.

[49] Carnot 1786 pp. 21–24; see also Carnot S 1878a folio 2r Ia, pp. 34–35; Picard p. 73.

[50] Carnot 1786, pp. 88–91; see also Carnot 1780, § 102, §§ 152–153; Gillispie 1971, Appendix C, § 102, pp. 301–303, §§ 152–153, pp. 328–332.

[51] Carnot 1786, pp. 89–91, pp. 93–99. He searched for the maximum work. He also proposed additional arguments in his famous Corollary on the equality "Q = q" (Ivi, Corollary V, §XLI, pp. 75–76, pp. 83–84; see also Carnot 1780, § 149; Gillispie 1971, Appendix C, § 149, pp. 327–328). The work plays an important role in Lazare Carnot's mechanics of running machines. On concept of work see Carnot 1786, pp. 65–66; pp. 83–85; pp. 96–97.

[52] Carnot 1786, pp. 89–94; see also Carnot 1780, §§ 149–152, §§ 155–157; Gillispie 1971, Appendix C, §§ 149–152, pp. 327–330, §§ 155–157, pp. 332–334.

[53] Carnot 1786, pp. 44–46; see also Carnot 1778 §§ 80–85; Carnot 1780, § 106; Gillispie 1971, Appendix C, § 106, p. 302.

[54] Carnot 1786, pp. iv–viii; see also Carnot 1780, § 153; Gillispie 1971, Appendix C, § 153, p. 330.

[55] Carnot 1786, p. ix, pp. 94–96; "[...] le mouvement perpétuel est une chose absolument impossible [...]" (Ivi, p. 94, line 18); see also Carnot 1780, § 146, § 157; Gillispie 1971, Appendix C, § 146, pp. 323–324, § 157, pp. 333–337; Carnot 1803a, p. xxi, pp. 256–257.

Ad absurdum reasonings and proofs.[56]
Geometric motions.[57]
Abstraction to study a machine.[58]
Science of (mechanical) machines.[59]
The *effect produced* is always limited.[60]
Working substances.[61]
Considering geometric motion independently of any dynamics rules.[62]
Reasonings by synthetic method.[63]
Produced work and consumed work.[64]
The argument—hydraulic engine.[65]
The role played by friction.[66]
Percussions or brusque change. Impact between bodies and loss of *moment-of-activity*.[67]
The calculation of the *effect produced* for any machine; ……….. the initial conditions should be restored at the end of the process.[68]

Lazare Carnot had special consideration for force, space and *void*. He assumed a bivalent position on the concept of the *void*, after Descartes and d'Alembert. For example, in his consequent book on equilibrium and movement (1803a), he wrote:

[56]Carnot 1786, pp. 28–36, p. 107; see also Carnot 1780, §§ 113–114; Gillispie 1971, Appendix C, §§ 113–114, pp. 308–310.

[57]Carnot 1786, pp. 28–34, pp. 41–45; see also Carnot 1780, § 113; Gillispie 1971, Appendix C, § 113, pp. 308–309.

[58]Carnot 1786, pp. 19–20, pp. 60–63; see also Carnot 1780, § 108, §§ 116–118; Gillispie 1971, Appendix C, § 108, p. 303, § 116–118, pp. 312–313; Carnot 1803a, pp. 256–257.

[59]Carnot 1786, p. 21; see also Carnot 1780, § 107, §§ 109–111; Gillispie 1971, Appendix C, § 107, §§ 109–111, pp. 303, pp. 304–306.

[60]Carnot 1786, pp. vij–ix, pp. 86–87; see also Carnot 1780, §§ 151–152; Gillispie 1971, Appendix C, §§ 151–152, pp. 328–329.

[61]Carnot 1786, pp. 86–87, pp. 89–93; see also Carnot 1780, §§ 155–156; Gillispie 1971, Appendix C, §§ 155–156, pp. 332–333. Carnot 1878a, folio 5r, pp. 40–43; Picard pp. 77–78.

[62]Carnot 1786, p. iii; see also Carnot 1780, footnote "*"; Gillispie 1971, Appendix C, footnote "*", p. 309. Cfr. d'Arcy 1752, 1754, 1763.

[63]Carnot 1786, pp. 33–35; p 85; Carnot 1813, pp. 12–21, p 189, p. 200, pp. 242–243, pp. 217–253.

[64]Carnot 1786, p. 66, p. 85; see also Carnot 1780, §§ 129–132, §§ 153–154; Gillispie 1971, Appendix C, §§ 129–132, pp. 316–317, §§ 153–154, pp. 330–332.

[65]Carnot 1786, pp. ix–x, pp. 88–81. Carnot 1803a, pp. xxi, p. 149, pp. 247–250.

[66]Carnot 1786, pp. 43–44; pp. 60–63, pp. 94–95; see also Carnot L 1778 §§ 1–26; Carnot 1780, §§ 1–100, § 160; Gillispie 1971, Appendix C, § 160, pp. 337–340.

[67]Carnot 1786, pp. (pp. 45–48 and) pp. 91–95; see also Carnot 1780, §§ 146–147, § 152, § 157; Gillispie 1971, Appendix C, §§ 146–147, pp. 323–325, § 152, pp. 328–329, § 157, p. 333. Please also note: "[...] qu'on appelle force ou puissance, dont la recherche est l'objet de la théorie des Machines proprement dites" (Carnot 1786, p. 62, line 29).

[68]Carnot 1803a, pp. 259–261.

Following this idea ["to avoid metaphysical notion of force" and to use "the theory of communications of motions"[69]] we will soon see, as I previously mentioned, the necessity of turning to the experiment, and that is what I did, without neglecting to support myself with reasonings that can confirm it in the most plausible way, using or generalizing the results per induction. At times I even used the name of the force in the vague sense of which I spoke above […].[70]

[...] *Primitive ideas concerning the matter, the space, the time, the rest, the motion, etc.* 7. The first rule to establish in such delicate research on the laws of nature is to only admit notions so clear that they can comprise the bounds of our logic. We must therefore reject the definitions of *matter, time, space, rest*, and *motion* as expressions that are impossible to express with more clear terms, and the ideas that these expressions produce in us primitive ideas outside of which it is impossible to construct. But once these expressions are admitted, we will easily see that which is a body, speed, a motive force, etc. 8. The body is a given part of matter. 9. The apparent space that a body occupies is called its volume; the actual space that this same body occupies, or its real quantity of matter, is called its *mass*. When the body is such that equal parts of its volume always correspond to equal parts of its mass, we say that it has a uniform *density*, or that it is equally *dense* in all of its parts; and the relationship from mass to volume, or the quotient of one times the other, is called the *density* of this body. But if unequal masses correspond to equal volumes, we say that the density is variable and for each particle of matter, we call *density* the volume of this particle divided by its mass, or rather, the last reason of these two quantities. The empty parts or gaps lodged between the parts of the matter, and that make the volume or apparent space greater than the actual space are called *pores*.[71]

[On the concept of force in the theory]. [...] in my opinion, no rigorous proof of the parallelogram of forces is possible: the mere existence of the *force* in the announcement of the proposition is able to make this demonstration impossible for the nature of things in itself. "It is extremely difficult", as Euler said, "to reason on primary principles of our knowledge [...]". This obscurity disappears in the second way [theory of motion] to conceive the mechanics, but another inconvenience appears; that is, the fundamental principles that in the first way [theory of forces where cause produces motion] are established such as axioms in favor of the metaphysical expression [...] that is to say, [...] force, are, in this second case [theory of motion], nothing less than self-evident propositions, and in order to establish them, we need to include the recourse to the experience.[72]

[69] Carnot 1803a, p. XVI, line 5.

[70] Carnot 1803a, p. XVI, line 10.

[71] Carnot 1803a, pp. 6–7, line 1 (Author's italic).

[72] Carnot 1803a, pp. xiij–xiv, line 17.

Collision theory moves through the conservation of momentum and energy. In both *Essai* (1786) and *Principes fondamentaux de l'équilibre et du mouvement* (1803) Lazare Carnot laid out his laws of conservation mainly by means of *insensible degrees* (Carnot 1803a, § 293, pp. 261–262). They can be considered a sequence of infinitesimally small percussions.

The *Essai* adopted a very simple level of mathematics combined with a simplified geometrical model of machines/mechanics and advanced studies such as machines in motion. The particular apparatus of machineries within machines (equilibrium and in motion) were systematically deduced from his *Hypothèses* (laws) of mechanics.

Carnot rigorously used geometry and trigonometric (vector calculus) rather than the advanced mathematics of his contemporaries. Combining the *Geometric motion, Moment of activity–concept of Work* with the *moment of quantity of motion*, he found the invariants of the *communication of motio*n and established the maximum effect produced by mechanical machines, e.g., the continuity in the transmission of power. He was convinced that an innovative general theory of machines—an applied science of machines—was a crucial necessity for the economic development of a society. His works came to be of interest to technicians by the 1780s as well. But the principles and related theories then in currency were insufficient to perform motion and equilibrium for all machines. This was one of the main challenges of his *machines en général*:

> Given the virtual motion of any system of bodies (i.e., that which each of the bodies would describe if it were free), find the real motion it will assume in the next instant in consequence of the mutual interaction of the bodies considered as they exist in nature, i.e., endowed with the inertia that is common to all the parts of matter. (Carnot [1783] 1786, § X, p. 21).

Since the argument was verbally expressed, the *Essai* was also beyond the comprehension of readers without formal education. The *Essai* seemed naturally to address its contents to certain trained engineers, which—Carnot included—represented the forthcoming generation of engineers. Thus, the latter, *a priori*, could be very positively sensible to the content of the *Essai*. However, the methodological novelty of *Essai* joined with an evident non-advanced use of mathematics created some difficulties to the readers and to Carnot's scientific career. In other words, to fit Carnot's mechanics/machines in the context of France at beginning of 19th century, one should consider Carnot such as a real pioneer in a conventional scientific environment. As cited above, from the pages of the *Essai* the readers encounter the dissimilarity between *hard bodies* and *elastic bodies* within an early collision theory with respect to a theory of elasticity in the 19th century. On that, Gillispie suggested that

> Carnot's work came into that development somewhat past its midpoint and inherited as assumptions the positions adopted by Maupertuis [Maupertuis 1746; Scott 1959, pp. 199–210]. Perfectly hard bodies were held to be indeformable and perfectly elastic bodies to contain forces capable of restoring their initial shape and volume after compression or shock of impact. (Gillispie and Pisano 2014, p. 17).

John Wallis (1616–1703) had already dealt with inelastic bodies in collision (Wallis 1668). In the processes of modelling of other bodies, for example the passage from liquid to gas and related known mechanical properties like incompressibility and deformability, Carnot was faced with a supplementary process of modelling based on the definition of fundamental quantities, which finally led him to the physics of *Work, Power* and *Efficiency*. In this period the challenge was to resolve the debate as to which of two quantities, *momentum (mv)* and *live force* (mv^2), could be considered as *conserved* in interchanges of motion. This was a serious problem for the development of principle of live force. In fact, the collision theory at that time conserved its quantity for cases of perfectly elastic bodies, while for cases of hard bodies, mv^2 was supposed to be conserved only for movements by insensible degrees; it was not conserved in cases of impact or collision, general speaking.

In *Essai*, Carnot's physical *Work* could be considered to be the quantity to calculate the activity of a machine. He was possibly one of the first to introduce the concept of work[73] *(moment of activity)* in modern terms as *F•ds* (Carnot 1786, pp. 65–66; pp. 83–85; pp. 96–97). By taking into account his geometric motion, it is possible that Carnot moved his conceptualisation of the *Work* quantity towards such a convertibility, within the equivalence in the law of fall of mgh to $\frac{1}{2}mv^2$. This is historically plausible because it is in line with his engineering outlook: thinking of a) machines as massive bodies in motion and b) *live force conservation and hard body interactions*. The later studies dealt with hydrodynamics (Bernoulli 1738; see also 1733 [1748] 1750. Cfr. Borda 1770a, b; Bossut 1775) a parallel subject of research where the principle of live force assumed a crucial role in the solution of engineering problems. The hydraulic applications crossed (without being special cases) Carnot's reasoning in almost all his scientific writings[74]. In these cases, the mechanical work produced only depends on the two levels of potential (Fig. 12). At that time—though not physically correct—it might have plausible to associate motion (for example of liquids) to the *principle of continuity*. In fact, by taking into account the common incompressibility of liquids and hard bodies, one could realise that the conservation of live force might be the principle for these types of interactions. In *Essai*, Carnot moved beyond the dichotomy between continuous and discontinuous change of motion in hard-body interaction. He focused on how the *principle of live force* might be applied similarly in continuous–discontinuous interchange of motion, for elastic or inelastic

[73]In 1830–36s Gaspard–Gustave de Coriolis (1830–1837) named quantity work to one-half the live force, so that still later kinetic energy became defined as half the product of mass times velocity squared (Coriolis 1830–1836, 1829, III, V, pp. 33–34; see also 1844). On that, one can also see Navier's notes on Bélidor (Navier 1819; see also, 1818; 1826, 1841, 1856; 1864; Navier and Girard 1829) and Prony (Prony 1796, 1790, 1837; Prony, Girard and Navier 1829; Gillispie and Pisano 2014, Chap. 4; Gillispie 1971, Appendix B, § 27, ft c, pp. 272–273).

[74]The analogy between a mechanical/heat engine and an hydraulic engine belonged to, respectively, both of two Carnot, father (e.g., Carnot 1786, pp. ix–x, pp. 88–81; Carnot 1803a, pp. xxi, p. 149, pp. 247–250) and son (e.g., Carnot 1978, 16–17). Cfr. Gillispie and Pisano 2014, Chap. 11.

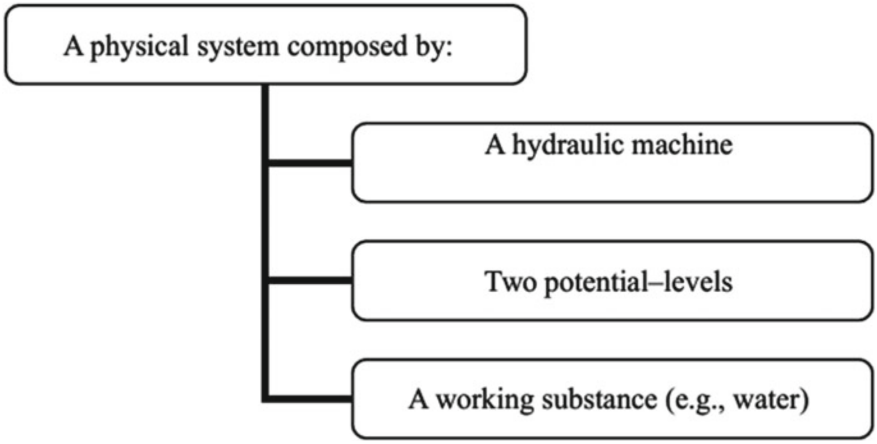

Fig. 12 A Simplified Model of a Carnot Hydraulic Machine

bodies. In this sense, as he often argued, the engineers inaptitude for metaphysics arises, and their tendency to always and only distinguish two main approaches to science, the so-called *experiential* and *rational*. The first one concerns description and comprehension of nature by means of basic magnitudes and notions such as body (mass), time, distance, power, equilibrium ($v = 0$). The second approach, at that time made (for certain scientists) Mechanics purely rational: hypotheses, modelling and correlated calculated quantities (e.g., derivatives etc.) such as velocity, acceleration, generally speaking, motion ($v \neq 0$). etc. In the 19th century a new discipline *physics mathematics* (different from *mathematical physics*. Cfr. Pisano 2013a, b) was born. For example, one can see this in analytical theories by Fourier, Lamé, rather than Maxwell's electromagnetism. A new methodological approach arose where some quantities were measured physically (i.e., distance and time) and others were calculated mathematically (i.e., velocity, acceleration, dynamics forces, work, energy). Thus scientists worked with quantities that were—at the same time—physical and mathematical. What about the measurement[75]? In effect for these scientific theories, *physics mathematics*, the measurement was not a priority (Cfr. Lamé 1836, 1861b). One could only calculate the *lim* including the instantaneous velocity, because some quantities in this *physics mathematics* were only considered mathematically. For example one could only determine the derivative of velocity in time because the latter was considered as a function of two variables (distance and time). On contrary, if one lost this mathematical aspect of the theory (i.e., distance and time as variables of the derivative function) then it was not possible to obtain quantities like velocity and acceleration. Therefore, in order to calculate the partial derivatives it was necessary that the velocity was a function of two mathematical variables, distance (x) and time (t): $v = f(x, t)$. How did they proceed?

An idea was to formulate *definitions* entirely free of ambiguity. This is exactly what he would not and need not attempt in his *Essai* (Carnot 1786, pp. 105–106). It was his reserve with respect to this early positivist approach that made the *Essai* difficult to read, even for skilled scholars of the 18th and 19th century. The problems with both terminology and these strong heuristic aspects of the theory made the content of the book ambiguous for his contemporaries, who could dismiss it as a non-rational scientific book. Considering the political and scientific roles played by Carnot, his influence was evident. We can suppose these difficulties were felt at that time, which might explain the addition of a ad hoc glossary to the *Principes de l'équilibre et du mouvement* (Carnot 1803a, § 18, pp. 10–13; see also xi–xij). It is also important to note Carnot's own usage with respect to translations: vitesse as velocity, force (usually) as a quantity of motion (momentum in moving bodies; Cfr. Galluzzi 1979). He did not need to stress Newton's second law. The latter could be

[75]Cfr. Pisano 2010, 2011, 2012, 2013a ,b ,c, 2016, 2017; Pisano and Bussotti 2015b, c, d, 2016a, b, 2017a, b; Gillispie and Pisano 2014, and Pisano's works below. On electricity and electromagnetic theory see Ampère (Ampère 1806, 1822a, b, 1826, 1827; Pisano 2013a).

understood, alluding to machines at the end of 18th century, as *force motrice* (also *motive force*).

Lazare Carnot suggested two main principles (Carnot 1786, pp. 12, pp. 14–15):

1. A General law of equilibrium in weight-driven machines the condition being that the center of gravity of the physical system be at the lowest point possible (Cfr. Pisano 2017).
2. A so-called by Carnot "fameuse loi d'équilibre de *Descartes*"[76] (*Descartes*' famous law of equilibrium). In Carnot's words: "The second principle on which we propose to make a few observations is the famous equilibrium law of Descartes; it comes to this, two forces in equilibrium are always in the reciprocal ratio of their velocities, estimated in the direction of the forces, such that when one supposes that one of the two [forces] gains infinitesimally on the other, [it is] in a manner such as to give birth to a tiny motion." (Carnot 1786, p. 15).

Principle (2) reads that two forces in equilibrium are in inverse ratio to their *vitesses* at the instant when one prevailed (in modern terms) infinitesimally over the other, thus initiating a *small motio*n (*petit movement*: Carnot 1786, p. 15)[77]. In agreement with Gillispie:

> Carnot preferred the center of gravity principle. [...]. It was always possible to reduce the operation of other forces to that of gravity by replacing their agency in principle with that of a weight acting over a pulley. This imaginary transformation of the system may at first annoy the modern reader as a somewhat juvenile evasion of the difficulty, but it would be better to see it as the engineers way of reducing an abstract problem to his own terms. Anyone who has been to engineering school will have resorted to similar devices, although it would be more germane to recall for a moment the analytic role of the experience and manipulation of weight in the mechanics of an Archimedes a Stevin or a Galileo Even Lagrange was not above it. There was a further, more serious, objection. (Gillispie and Pisano 2014, pp. 20–21; Cfr. Gillispie 1980).

[76]In effect it was self-quotation strictly personalised by Carnot. Recently on Descartes: Bussotti and Pisano 2013; Schuster 2013a, b, 2000; Schuster and Sutton 2000; see also Descartes 1897–1913).

[77]This scientific aptitude was transferred (also for advancement "[...] for insensible degrees [...]" Carnot 1786, p. 92; see also 1813a, p. 20) to his son Sadi on heat machines (Carnot 1824, p. 18; Carnot 1978, 1872, 1943, 1953; see also 1996, 1921; Desormes and Clément 1819; Despretz 1821, 1823; Jacquier 1867; Jamin 1868, 1886).

In the history of science (i.e., physics and mathematics), the statement in accordance with the equilibrium of a physical system related to a center of gravity, which should be at the lowest possible point, also involving *maxima* and *minima* (Carnot 1786, p. 14) has been the object of several difficulties[78] and formulations from antiquity (Aristotle, Archimedes to Galileo, Torricelli, Riccati until the end of 19th century). At the beginning, Lazare Carnot applied it to the machineries (Cfr. Reuleaux 1876; Hachette 1811) of an ideal machine —not in motion—by an arbitrary geometrical form and characterized by mass, only. In this configuration, the sum of the resistances of the fixed supports estimated vertically must equal the weight of the system. Subsequently, if an infinitesimal motion (*petit movement*) is provided, then a portion of the mass should have gone into producing motion, the other portion is invested in fixed supports.

In the following, a concise reconstruction of Lazare and Sadi Carnot's main reasonings in the *Essai* (Fig. 13) is presented:

[78]For example, one can see Galileo in his *Mecaniche* (Galileo 1890–1909, II, pp. 155–191) and in *Discorsi intorno alle cose che stanno in su l'acqua* (Galileo 1890–1909, IV, pp. 3–141). The latter attributed the law of *virtual velocity* to Aristotle (1936 [1955], 847a 10–15, 847b 10, pp. 329–332; see also Baldi 1621) also adding that the idea of the *principle of virtual work* was born thanks to the observation of the motion of points, which rotate along a circumference. Galileo also dealt with the law of the *virtual displacement* (Galileo 1890–1909 II, pp. 240–242, IV, pp. 68–69; VIII, pp. 310–331, pp. 329–330). We should wait for 1644, when Evangelista Torricelli (1608–1647), in his *Opera geometrica* (Torricelli 1644) claimed a rational criterion for equilibrium, playing a fundamental role in mechanics and in the history of mechanics (Capecchi and Pisano 2007, 2010a, b, Pisano and Capecchi 2013; Pisano, Dhombres, Radelet–de Grave, Bussotti *370th Anniversary of Torricelli's Opera Geometrica (1644): Statics, Mathematical and Geometrical Conceptual Streams, forthcoming)*. It can surely be considered the origin of the modern statement of the *principle of virtual work*: "Two heavy bodies linked together cannot move by themselves unless their common centre of gravity does not descend" (Torricelli 1644, *Liber primus de motu gravium naturaliter descendentium*, p. 99). With regard to Torricelli's principle, one can also consider John Wallis's assumptions (Wallis 1693), and Pierre Varignon's (1654–1722; Varignon 1725) essential and rigorous formulation as a scientific production which aimed at founding all statics upon an easily geometric principle: the composition of forces. In this sense, it is also alternative to the *principle of virtual work*. Let us remark that in his letter to *Johann* Bernoulli (1667–1748), Varignon also dealt with concept of *virtual velocities*, as components of *virtual infinitesimal displacements* towards the direction of the forces (Bernoulli Johann 1742a, b, II, 1727; see also Bernoulli Jakob 1703; see Radelet–de Grave 1996, 2007, 2009 and her other remarkable works on Bernoulli family). After Bernoulli, the most significant contribution to the development of the *principle of virtual work* is probably thanks to Vincenzo Riccati (1707–1775) who tried to establish it upon simple principles easily accepted by his contemporaries, introducing *Principles of actions* in his *Dialogo di Vincenzo Riccati della Compagnia di Gesù dove ne' congressi di più giornate delle forze vive e dell'azioni delle forze morte si tien discorso* (Riccati 1749) and in *De' principi della meccanica* (Riccati 1772). On the subject, see recently Pisano 2017. Below the principle of virtual works in relation to Lazare Carnot is presented.

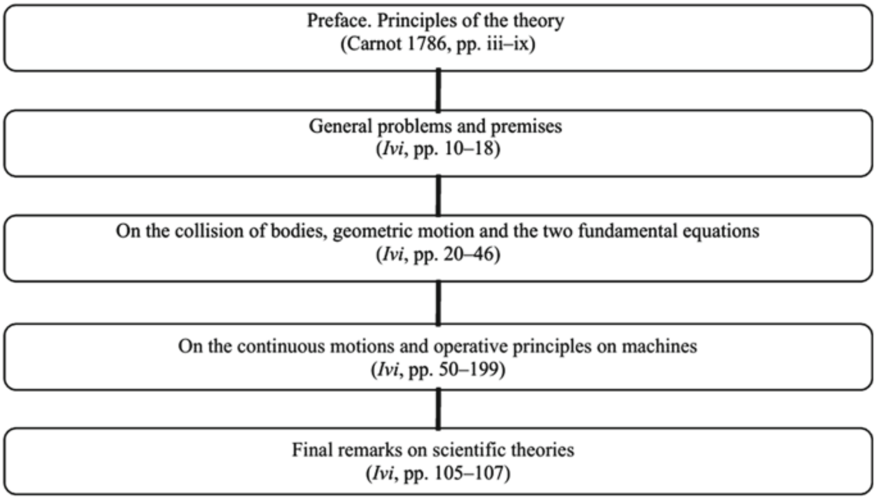

Fig. 13 Lazare Carnot's Main Reasonings[79]

Taking into account Lazare Carnot's reasoning on machines (Fig. 13) and principle (2) discussed, one can conclude that if the center of gravity did not descend, then a state of equilibrium should be provided:

> *To assure that several weights* [masses] *applied to whatever Machine should be in mutual equilibrium, it suffices to prove that if the Machine was left on its own, the centre of gravity of the system would not descent.* (Carnot 1786, II, p. 14; Author's italic).

It is historically and epistemologically interesting to observe how Carnot made progress in his explanations between scientific content and methodology. As above cited, he remarked to the reader the lack of ideal principles and mathematical idealisations in his *Essai*. In other words, if a proof was mentioned, then the reader might expect a proof as structured modelling, that is an example of a certain experience. In effect, his reasoning and above claim "Pour s'assurer [to assure] [...]" (*Ibidem*) essentially consisted in the exclusion of idealized thought experiences because the latter cannot include the real of objects in equilibrium and/or in motion; that is, they cannot be

[79]Adapted from Gillispie and Pisano 2014, p. 351; Pisano 2010; Cfr. Drago and Manno 1994.

scientifically processed as physical and/or mathematical quantities. This is a heritage of a certain logic (mainly classical or non-classical) used in science (Gillispie and Pisano 2014, Chaps. 7, 8, 11; Pisano 2010a; Pisano and Gaudiello 2009a, b; Capecchi and Pisano 2007). The idea was that a statement was correct if its contrary led to physically absurd situations. In the history of science, we can see many of these cases. For example, Stevin's law (Stevin [1605] 1608; Dijksterhuis 1955) of the inclined plane which excluded a perpetual motion; Sadi Carnot's theorem (Carnot 1978, p. 38; Pisano in Gillispie and Pisano 2014, Chaps. 7 and 10) of the efficiency of an heat machine proved by an *ad absurdum proof*[80], the system composed of body (mass and its motion) described by infinitesimal analysis. Particularly, the latter idealizes the system mass–infinitesimal point in order to apply this kind of mathematics to physics; namely, rational mechanics which later became part of physics–mathematics of the 19th century (Cfr. Pisano's works on the subject).

On that, Gillispie noted that:

> As for the so-called principle of Descartes, Carnot found in it disqualifying flaws. It was less general than the center-of-gravity principle thus transformed, from which it could be deduced by conversion of its forces into weights acting over pulleys. It applied only to systems in which no more than two forces were at work. More seriously, it envisaged only the relative amounts of the forces in equilibrium, whereas in requiring their vertical projections the center-of-gravity principle specified also the direction of those forces. (It is just in these passages that one may begin to appreciate how Carnot's attempts to analyse the manner in which forces transmitted by shafts, cords, and pulleys would constrain and move points within systems composed of rigid members, clumsy though these constructs seem, they nevertheless belong to the pre-history of vector analysis in exhibiting awareness that the quantity of a force comprises direction as well as intensity). (Gillispie and Pisano 2014, pp. 21–22).

The statement of the principle of Descartes (Schuster 2013) did not require opposite forces in direction in order to be in equilibrium (Bussotti and Pisano 2013). Taking into account the whole content of the *Essai*, Carnot specified the geometrical difference—so important for mechanical machines—between "forces sollicitantes" (*impelling forces*) and forces résistantes (*resisting forces*) (Carnot 1786, pp. iv–v).

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[80]Cfr. Pisano's works in the references section below.

This was an argument of intense interest to Carnot that he had already argued in his *Memoires* (Carnot 1780, § 129).

Lazare Carnot was possibly one of the first to clarify (physically) the vector definition of *impelling forces* and *resisting forces*. He avoided using them as metaphysical causes of variation of motion. He considered the (work) moment of activity "*q*", operated by *resisting forces*, as the *effect produced* by *impelling forces*. Thus, he considered the (work) moment of activity "*Q*", consumed by *impelling forces* at a given *t*—time (Carnot 1786, §§ LII–LIII, pp. 83–84, §§ LXIII–LXIV, pp. 95–99; see also Gillispie and Pisano 2014, Chaps. 2, 3, and 11). In effect, Carnot's criticism of Descartes' principle moved towards the *principle of virtual velocities* as discussed below (Cfr. Pisano 2017; Gillispie and Pisano Chaps. 10 and 11; Coopersmith 2015, 2017); although Carnot did not name it.

The main problem was *how* and *when* infinitesimal movements occurred for one machine in motion; and for two machines in two states infinitely close to each other. This raised the question: *what was the physical and geometrical configuration of the system?* Finally, his *geometric motions* would help him to deal with the problem. In fact, the latter were different from virtual velocities because they were finite quantities (Cfr. Poinsot 1838, 1975). Considered more closely, they were possible or actual displacements, and the internal consumed work of the system was zero. But these considerations were applicable to an equilibrium state only. It is also remarkable to see how Carnot referred to d'Alembert's principle (Carnot 1786, pp. 16–26; see also Carnot 1778, 1780; Carnot 1803a pp. 41–43; pp. 24–26; Hankins 1972, p. 203) in order to find a manner to justify the extension of the equilibrium principles to motion; historiographically speaking, *how is possible to pass from statics to kinematics and then to dynamics*? Carnot, in practice, did not use both principles. His reasoning moved towards the *conservation of live force*.

In the history of science, an undue use of the term axiomatization/axiomatic concerning theories is diffused. Usually, in mathematics and mathematical physics, the term axiomatization of a scientific theory represents a formulation of a scientific system of statements (e.g., axioms/primitive terms) in order to build a consistent, coherent corpus of statements (e.g., propositions) which may be logically and deductively derived from these statements. The proof of any statement (i.e., theorems) should be taken into account and traceable back to these axioms. Of course, the latter is a difficult condition to be universally claimed: i.e., see the case study of Archimedean's *On the equilibrium of planes* (Capecchi and Pisano 2007, 2010b, Pisano 2009b, Pisano and Capecchi 2008, 2009a, b, 2010a, b), and non-Euclidean geometry. Therefore, the use of axioms (in the history of science) as self-evident statements in a theory does not mean that this theory system is axiomatically built (Pisano 2008). In fact, three fundamental properties should be formally respected: 1) an axiomatic system is said to be consistent if it lacks contradiction, i.e., the ability to derive both a statement and its denial from the system's axioms; 2) in an axiomatic system, an axiom is called independent if it is not a theorem that can be derived from other axioms in the system; a system will be called independent if each of its underlying axioms is independent. Although independence is not a necessary requirement for a system, consistency is; 3) An axiomatic system will be called complete if for every statement, either itself or its negation is derivable. For example, Euclid of Alexandria authored the earliest extant axiomatic geometry and number theory presentation that can be formally considered: *an axiomatic system, a model theory, and mathematical proofs within a formal system.* All of that evidently is lacking in Lazare Carnot. In Carnot the axioms only mean a tentative step towards ordering a new theory (on machines) by means of primitive statements and eventually derived proportion; and also to order a scientific reasoning extrapolated from a previous known theory (for example Newton's law or axiom of motion). This aspect belongs to several periods of the history of science.[81] For, an interesting epistemological aspect of the content of the *Essai*, generally speaking, is Carnot's theoretical conceptualisation of equilibrium—and—motion as the adaptation of two axiomatic statements of mechanics at that time:

[81]See below Pisano's works on the relationship between physics and mathematics into history; see also Schuster 2013; Bussotti and Pisano 2013.

1. The first one concerned the physical static situation of equality in opposite senses of action and reaction (for all bodies), as the third *Axiom or Law of motion* by Newton (Newton 1803, p. 19; Bussotti and Pisano 214a, b; Pisano and Bussotti 2016a, b, c); even if Carnot did not directly refer to the Newtonian third law.
2. The second one concerned hard bodies only and in motion. Bodies in their interactions (impact or pressure) had their relative velocity in the next instant equal to zero.

Following Lazare Carnot's reasoning, he discussed two more concerns:

> That the intensity of the impact or action [that is exerted] between two bodies that meet does not depend on their absolute motion but only on their relative motion.
> That the force or quantity of motion which each exerts on the other, due to the impact, is always directed perpendicularly to their common surface at the point of contact.
> XII. Of these two fundamental laws, the *first* is generally applied to all bodies in nature, as well as to the two subordinate laws which we are going to see, while the *second* is only for hard [plastic] bodies. However, since bodies that are not hard [plastic] have various degrees of elasticity, one usually falls back again to the hard [plastic]-body laws and uses these as bodies of comparison. In other words, one regards elastic bodies as being composed of an infinity of little hard bodies separated by little compressible rods and attributes to this all the elastic properties of these bodies. Thus one does not think that, properly speaking, in nature, bodies are [ever] animated by different motive forces. We will follow this method, as being the simpler; thus we will reduce the question to researching the laws observed by hard bodies, and we will then make some applications to the case where the bodies have different degrees of elasticity. (Carnot 1786, XI, p. 23; Author's italic)

Taking into account these principles and their corollaries, Carnot suggested that in a system of hard bodies in motion, the net result of mutual internal interactions of the system was zero. For example, generalizing them to a system of hard bodies, these arguments could be supposed equivalent to an expression of the *conservation of live force*. But, it should be noted that here Carnot met a situation of non-applicability (*en général*); it was the case of hard bodies. The successive introduction of his class of geometric motions helped him to overcome this problem of generalisation. In fact, he worked with displacements in geometry in order to avoid using physics (dynamics). This is why Carnot did not account for inelastic collisions where instead a loss of live force (a kind of non-conservative case) might have occurred. In addition, adopting *geometric motions* (e.g., *Ivi*, pp. 68–69) he had at his disposal indeterminate arbitrary values useful to solve particular cases. These arguments consisted of a derivation of conservation of moment of momentum (i.e., torque) from conservation of energy (work). In order to discuss a machine *en général* Carnot needed to ignore internal vincula and interactions (bodies forces). Thus, he made use of the *moment de la quantité de mouvement* (conservation of moment of momentum as a fundamental principle in mechanics *en général*.

> XXI. Let us imagine an arbitrary system of bodies in motion [...] $\sum muVcosy$ will be named the "moment of the quantity of motion of the system" with respect to the geometric motion that we gave to the system. In this way the moment of the quantity of motion of a system of bodies, with respect to any geometric motion whatever, is the sum of the products of the quantities of motion of the bodies which compose it, each multiplied by the geometric speed of the bodies, measured in the direction of this quantity of motion. So, using the same denominations, $\sum muWcosx$ is the moment of the quantity of motion before the collision; and $\sum muVcosY$ is the moment of the quantity of motion after the collision; and $\sum muUcosz$ is the moment of the quantity of motion lost in the collision : (all the moments being related to the same geometric motion). Thus from the fundamental equation (F) [$\sum muUcosz = 0$; (*Ivi*, p. 40)] one can conclude that in the collision of hard [plastic] bodies, whether these bodies are all mobile, or whether some are fixed, or—what comes to the same thing—whether the collision is immediate, or done by means of any Machine without springs, the moment of the quantity of motion lost by the general system is equal to zero. (Carnot 1786, XXI, pp. 41–42).

The *Principle of Least Action* is applied (Cfr. Coopersmith 2017; see also 2015). It becomes a kind of *ad hoc* formulation of *conservation of live force* devoted to manifest the power of machines rather than a movement. This is a crucial aspect of his approach to mechanical science, perhaps mixed with his political and military experiences as a statesman. Carnot applied his principles of mechanics and geometric formulation to running machines. It is here that he formulated an early idea of Work that he called *Moment of Activity*. It was a manner to calculate input–output in a running machine.

> XXXII. If a force P moves with speed u, and the angle formed by the directions of u and P is z, the quantity $P\cos z u dt$ in which dt expresses the element of time, will be named moment of activity consumed by the force P during dt. That is to say, the moment of activity consumed by a force, P, in an infinitely short time, is the product of this force (in the direction of the speed) and the distance undertaken (from the point of application) in this infinitely short time.
> I will denominate by *moment of activity*, consumed by this force, in a given time, the sum of moments of activity, consumed by it at each instant, such that $\sum P\cos z u dt$ [modern terms: force per distance] is the *moment of activity*, consumed in some indeterminate time. (Carnot 1786, XXXII, p. 65).

By means of his *moment of momentum* and for machines in smooth motions, Carnot expressed a more general rule concerning the equality between the *Work* done by the impelling forces and the *Work* done by the resisting forces (Cfr. Carnot 1786, pp. 60–61; XXXIII, p. 66). In his *memoire* (Carnot 1780) he had already written:

> 109. The science of machines in general and all mechanics is thus reduced to the following question. Knowing the virtual motion of a system of bodies that is the one that it would be taken by each body if it was free to find the real motion that it will have the next moment because of the interplay of bodies assuming that each of them is endowed with inertia as common to all the parts of the matter. And since this problem is simpler if we would find, among the bodies, some that are deprived of this inertia it is clear that we cannot have a general theory of machines without having solved this problem in its full scope. That is what we will try to do.[82]

The *Essai* ends with designs for machineries (i.e., hydraulic case studies; Carnot 1786, VI, p. 80) and a remarkable concluding *Scholium* (Carnot 1786, pp. 81–104). It aimed to make clear Carnot's scientific aptitude in both mechanics and machines theory. Thus, specific arguments concerning the mechanical advantages of machines, experiences and rational approaches to the science of mechanics (Cfr. Cauchy 1828, 1829) concluded this masterpiece on *Machines en général*:

> The two fundamental laws from which I started (XI) are therefore purely experimental truths and I have proposed them as such. A detailed explanation of these principles did not enter into the plan of this work, and would perhaps have only served to obscure things. The sciences are like a beautiful river,

[82]Carnot L 1780, §§ 108–109; see also Gillispie 1971, Appendix C, §§ 108–109, pp. 303–305; see also Carnot 1786, §§ XXIX–XXX, pp. 60–61.

> whose course is easy to follow once it has acquired a certain regularity. But if one wants to go back to the source, one cannot find it anywhere, because it is everywhere; it is spread in some way over the whole surface of the earth. Similarly, if one wants to go back to the origin of the sciences, one will find only obscurity, vague ideas, vicious circles; and one is lost in primitive ideas. (Carnot 1786, p. 107).

His son Sadi followed in his footsteps, part of a filial and scientific relationship based on opposition to a (Newtonian) rational approach to science and to perpetual motion (Carnot 1978, pp. 21–22, ft. 1) in his *Réflexions sur la puissance motrice du feu* (1824, Pisano's works; Gillispie and Pisano 2014, Chaps. 6–11).

Both Lazare and Sadi Carnots' sciences[83] (Gillispie and Pisano 2014) avoided the use of Newtonian's scientific apparatus for a new way to approach the science of machines. For mechanical machines, Lazare considered the *production of mechanical work* occurs with the transference of motion from one body to another. For heat machines, Sadi considered the *production of heat work* occurred by transmission of heat between two thermostats by a small difference in temperature. According to Lazare Carnot, Sadi included the *production of the movement of heat* in order to establish a principle *en général* and to produce a new physical situation of the conversion of heat to Work.

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[83] A recent historical, epistemological investigation found that *Double Negative Sentences* (DNSs) belonged to non-classical logic, the scientific content of which does not correspond (once one deletes the double negation) to their positive sentences. They were found in Sadi Carnot's book of 1824 (Gillispie and Pisano 2014, pp. 412–419). They introduced the scientific–logical style and background of Sadi and his father's scientific filiation on mechanical and heat machines *en général*. For example, if a DNS (A) is not equivalent to its corresponding affirmative sentence (A), than (generally speaking) it loses its scientific argument and follows that it belongs to non-classical logic; i.e., see the law of double negation: A $\neq$ A. In logic "A" reads "non non A"; Gillispie and Pisano 2014, Chap. 7; see also Pisano's works; Batens and Meheus; Beth, Bevir; Destouches 1948, 1951, 1959, 1966). This inquiring was used, e.g., for analysing the analogy between mechanical and heat machines should be noted. If in thermodynamics Q is analogous to f, since neither are state functions f must be substituted by potential $\Delta V = f\Delta s$, while Q must be substituted by entropy, which however has a different formula $\Delta S = \Delta Q/t$. (Thomson 1851b, I, pp 175–183; see also Thomson 1851a; Clausius 1850, pp. 368–397; pp. 500–524). Moreover, it should be also noted that in the second case, it is not a special physical distance but it is temperature range, $\Delta t \neq 0$. Sadi Carnot wrote this at the beginning of the discursive part of *Réflexions sur la puissance motrice du feu*, fully of DNSs, and repeats it several times as well as at the end of the demonstration of his celebrated theorem (Carnot 1978, p. 38): *work can be obtained every time there is a difference in temperature between which heat passes*. Thus, it is possible to note a common way of conceiving work in comparison with special and heat motions (Pisano 2010; Gillispie and Pisano 2014).

1.4 Paralleling *Essai* and *Principes*, 1786–1803

By following the Newtonian paradigm until Laplace[84] (Fox 1974; Dhombres and Alvarez 2012) one can see a development of an advanced use of mathematics (i.e., differential equations) to certain fields of science. This became a kind of alert for Carnot as he declared in the end of *Essai* and in successive *Principes fondamentaux de l'équilibre et du mouvement* (1803a; here after Principes). Paralleling these two books (1786 and 1803a), Carnot wrote:

[*Essai*]. *Reflections on the fundamental laws of equilibrium and motion*. Among the Philosophers who occupy themselves with researching the laws of motion, some are in Mechanics, an experimental science,[85] the others, purely rational. That is to say, they first compare the phenomena of nature, decompose them, so to speak, in order to understand what they have in common and so reduce them to a small number of principal facts—which then serve to explain the others and to predict what will happen in any circumstance; the others start from hypotheses, then reason from their suppositions, succeed in discovering the laws that the bodies would follow in their motions if their hypotheses conformed to nature, then, comparing their results with phenomena and finding that they agree, conclude that their hypotheses are accurate; that is to say, the bodies in effect do follow the laws which they had at first only assumed. The first of these two classes of Philosophers therefore follow in their researches the primitive ideas that nature has imposed on us and the experiences that she continually offers us. The others start from definitions and hypotheses: for the first, [they assign] the names of bodies, forces, equilibrium, motion, that correspond to primitive ideas; they neither can nor must define them; the others, on the contrary, having to draw everything from their own depths, are obliged to define these terms with precision, and explain clearly all their suppositions. But if this [second] method appears more elegant, it is also more difficult than the other one; as there is nothing more perplexing in most of the rational sciences, and above all in this one—than to first pose exact definitions in which no ambiguity remains—it would throw me into metaphysical difficulties, well beyond my powers, to want to explore all those proposed up till now. I will content myself with [examining] the first and most simple [definition]. (Carnot 1786, pp. 104–106; Author's italic).

[*Principes*]. Following this idea ["to avoid metaphysical notion of force" and to use "the theory of communications of motions" (Carnot 1803a, p. XVI, line 5)] we will soon see, as I previously mentioned, the necessity of turning to the experiment, and that is what I did, without neglecting to support myself with reasonings that can confirm it in the most plausible way, using or generalizing the results per induction. At times I even used the name of the force in the vague sense of which I spoke above […].

(Carnot 1803a, p. XVI, line 10).[86]

[84]In the 1816 Laplace pointed out that the speed of sound in air depended on the heat capacity ratio and corrected Newton's surprising error (Biot 1858, pp. 1–9, 1802, pp. 173–182; see also 1802, 1816).

[85]Cfr. Desaguliers 1751, 1734.

[86][...] Primitive ideas concerning the matter, the space, the time, the rest, the motion, etc. 7. The first rule to establish in such delicate research on the laws of nature is to only admit notions so clear that they can

Lazare Carnot adopted *velocity* and *quantity of motion* as the equations of motion (Gillispie and Pisano 2014, Chaps. 2–4, 11). In this kind of mechanics, the basic concepts, time and space are finite and delimited (e.g., in machines); differently from Newtonian tradition, they are not absolute and infinite. Lazare Carnot's mechanics was limited to algebraic and trigonometric equations. In fact, his equations of the invariants of motion are to be solved by velocities only.

The following Table 1 presents the basic notions both in *Essai* and in *Principes*:

Table 1 Carnot's Physical–Mathematical–Geometrical Approach

Main concepts	Lazare Carnot (1786; 1803a; see also 1813)
Space and time	No absolute and infinite Newtonian space and time
Physics–mathematics relationship	Only physical
Geometry–physics relationship	Independence from position in space
Derivate quantities	Velocity, Quantity of motion, Work
Mathematical approach	No local and infinitesimal variables
Main mathematical–geometrical technique	Geometric motion

comprise the bounds of our logic and scientific language (Cfr. Condillac 1821). We must therefore reject the definitions of matter, time, space, rest, and motion as expressions that are impossible to express with more clear terms, and the ideas that these expressions produce in us primitive ideas outside of which it is impossible to construct. But once these expressions are admitted, we will easily see that which is a body, speed, a motive force, etc. 8. The body is a given part of matter. 9. The apparent space that a body occupies is called its volume; the actual space that this same body occupies, or its real quantity of matter, is called its mass. When the body is such that equal parts of its volume always correspond to equal parts of its mass, we say that it has a uniform density, or that it is equally dense in all of its parts; and the relationship from mass to volume, or the quotient of one times the other, is called the density of this body. But if unequal masses correspond to equal volumes, we say that the density is variable and for each particle of matter, we call density the volume of this particle divided by its mass, or rather, the last reason of these two quantities. The empty parts or gaps lodged between the parts of the matter, and that make the volume or apparent space greater than the actual space are called pores. (Carnot 1803a, pp. 6–7, line 1. [Author's italic]). [On the concept of force in the theory]. [...] in my opinion, no rigorous proof of the parallelogram of forces is possible: the mere existence of the force in the announcement of the proposition is able to make this demonstration impossible for the nature of things in itself. "It is extremely difficult", as Euler said, "to reason on primary principles of our knowledge [...]". This obscurity disappears in the second way [theory of motion] to conceive the mechanics, but another inconvenience appears; that is, the fundamental principles that in the first way [theory of forces where cause produces motion] are established such as axioms in favor of the metaphysical expression [...] that is to say [...] force, are, in this second case [theory of motion], nothing less than self-evident propositions, and in order to establish them, we need to include the recourse to the experience (Carnot 1803a, pp. xiij–xiv, line 17; Cfr. Pisano and Gillispie 2014).

In the following, Carnot's main hypotheses—related to discussion on *Principle of virtual laws*—are presented (Table 2).

Table 2 Some of Carnot's hypotheses (Carnot 1803a)

1	Once at rest a body cannot move by itself and once put in motion it cannot change either its velocity nor its direction by itself (Carnot 1803a, p. 49).
3	When many forces, either passive or active, equilibrate themselves, each of these forces is always equal and opposite to the resultant of all the others (*Ivi*, p. 49).
5	The action that two bodies contiguous bodies exert on each other by impact, pressure or tension, does not depend in any way on their absolute velocity, but only by their relative velocity (*Ivi*, p. 49).
7	When bodies that impact are perfectly hard or perfectly soft, they proceed always together after the impact; that is according to the straight line of their mutual action [...] (*Ivi*, p. 50).

Carnot declared (Carnot 1786, pp. 104–107) his preferences: both analytical and empirical ones. In the introduction of *Principes* he reaffirmed his empirical ideas (Carnot 1803a, p. 2) also expressing a particular final conceptualisation of his mechanics as either empirical or fully rational (Carnot 1803a, pp. 3–5). In other forums, he referred to his *seven hypotheses* (Carnot 1803a, pp. 46–47; see also Pisano and Capecchi 2013) in order to replace the three Newtonian laws.

1.4.1 The Embryonic and Genesis of the Magnitude Work

Both in *Essai* and *Principes* Lazare Carnot's approach of conceptualisation of force as a physical quantity (Carnot 1803a, p. xj) was quite evident: sometimes following common sense, and at others even meaning *Work*. As noted above, the term that he used to indicate work was *moment of activity* (Carnot 1786, XXXII, pp. 65–66).

The total moment of activity, at a finite time, is given (in an integral form) by the following expression (Carnot 1786, p. 66):

$$P \int udtcosz$$

For, he formulated his own conservation of work by means of smooth movements, such as a corollary:

> *Corollary V. Specific law concerning Machines whose motion changes by insensible degrees.*
> *XLI. In a Machine whose motion changes by insensible degrees, the moment of activity, consumed in a given time by the soliciting forces, is equal to the moment of activity exerted in the same time by the resisting forces* (Carnot 1786, pp. 75–76; Author's italic).

The *production of work* was produced by mechanical machines (Pisano and Bussotti 2015a, b). Thus, taking into account f_i forces linked to δs_i and displacements of bodies, *the production of a mechanical work* was produced by the transferring of motion from one body to another one. This reasoning is specified both in the *Essai* (e.g., Carnot 1786, p. iij, pp. 65–66; see also pp. 96–97) and in *Principes*. In the latter Carnot wrote:

> [...] we come back specifically to the second way [theory of motion] of looking at the problem, that is to say, that mechanics are nothing else than the theory of the laws of the communications of the motions. (Carnot 1803a, p. xiij).
> [...] The first method [theory of forces where cause produces motion] offers much more ease; so it is, as I mentioned here above, almost generally followed. Nevertheless, I adopted the second [theory of motion] as I already did in the first edition; because I wanted to avoid the notion of metaphysics of forces, to leave undistinguished the cause and the effect, in short, to bring everything to the only theory of communication of motions. (Carnot 1803b, pp. xv–xvj).

According to Carnot, an action of every force (weight) can be reproduced. Then the *communication of motion* (Carnot 1803a, pp. xiij–xvj) and the *Work* are able to produce a new physical situation—related to machines *en général*—based on the impossibility of a perpetual motion and the independence of *efficiency* from the working substance. Below a sequence of Carnot's main reasoning on the subject is presented. Table 3 summarizes the development of the conceptualisation of the Work—with respect to vincula—in Carnot's two mentioned works:

- Everyone says that in Machines in motion, one loses in time or speed what one gains in force.
- (Carnot 1786, p. vj; see also *Ivi*, p. viii).
- The reflections that I propose on this law lead me to say a word about perpetual motion, and I show not only that every machine left to itself must stop but I even assign the moment when this must happen. (Carnot 1786, p. ix).
- However, I repeat, the object of this Essay is [the theory of] Machines in general. Each machine has its own particular properties but here we are concerned only with those properties that are common to all. (Carnot 1786, p. x).
- […] and one compares these different efforts without regard to the agents that produce them as the nature of these agents can change nothing about the forces which they are obliged to exert in order to fulfill the various purposes that the Machines are [designed] for. (Carnot 1786, p. 62).
- LVII. What is then the true purpose of Machines in motion? We have already said, it is to procure the ability to vary at will the terms of the quantity *Q* (the *momentum* of activity) that must be exerted by the moving forces. (Carnot 1786, pp. 88–89; Authors' italic).
- LXII. One may conclude from what we are coming to say on the subject of friction and other passive forces, that perpetual motion is absolutely impossible, as it employs—for the production of perpetual motion—only bodies that are not solicited by any motive force, or even by [the descent of] heavy bodies […]; (Carnot 1786, p. 94).
- It is therefore evident that one must absolutely despair of producing what one calls perpetual motion, if it is true that all the motive forces which exist in nature […]. (Carnot 1786, p. 95).

Tab. 3 The way of conceiving vincula and the production of work

Lazare Carnot (1780; 1786)
• The work as a *product* of a mechanical machine; *vincula* bodies. • Mechanical vincula: M≫m • (Principle of virtual work). Systems of bodies, non-infinitesimal points,but global and with *vincula*. • More than one body having infinite mass cannot be a machine: no work from *vincula*, only. • It is impossible to link (in a direct way) different potential systems to produce work freely (impossibility of perpetual motion).

Particularly, in the following a selection of early Carnot's reasoning on the subject:

> 108. When a body acts on another one it is always directly or through some intermediary body. This intermediate body is in general what we call a machine. The motion that is lost at every moment in each of the bodies applied to this machine is partly absorbed by the machine itself and partly revised by the other bodies of the system but as it may happen that the subject of the matter is only to find the interplay of the bodies applied to the intermediate bodies without the need to know the effect on the intermediate bodies, we have imagined, in order to simplify the question, to ignore the mass of this body, however keeping all the other properties of matter. Hence the science of machines has become a sort of isolated branch of mechanics in which it is to be considered the mutual interplay of different parts of a system of bodies among which there are some that, lacking the inertia as common to all the parts of the matter as it exists in nature, withheld the names of machines. This abstraction might simplify in special cases where circumstances indicating those bodies for whom it was proper to neglect the mass to make it easier for the objective, but we easily know that the theory of machines in general has become much more complicated than before because then this theory was confined in the theory of motion of bodies as they are offered to us by nature, but now it is necessary to consider at the same time two kinds of bodies, one kind as actually existing, the other partially deprived of its natural properties. Now it is clear that the first problem is a special case, since it is more complicated than the other so that by similar hypotheses, we easily find the laws of the equilibrium and of motion in each particular machine such that the lever, the winch, the screw, resulting in a blend of knowledge whose binding can be hardly perceived and only by a kind of analogy; this must necessarily happen as we will resort to the particular figure of each machine to show the property which is common to it and to all the others. Since these properties are the ones we have mainly seen in this first section, it is clear that we will be able to find them only by putting aside the particular forms. So let us start by simplifying the state of the issue by ceasing to consider the system bodies of different natures; finally giving back to machines their inertia it will be easy afterwards to neglect the mass in the

> result, we will hold the possibility to consider it or not, and therefore the solution of the problem will be general and easier at the same time. (Carnot 1780, § 108).

1.4.2 From Work to Principle of Virtual Laws

Bodies with infinite mass (the constraints only) were not considered by Carnot because they do not produce *Work* (Carnot 1786, pp. 58–59). In fact, to produce work, other intermediary mechanisms are necessary. While in the previous sections, the discussion dealt with Carnot's engineering/geometrical approach to mechanics–machines, in the following the reader finds scientific arguments about Carnot's physical and mathematical roots devoted to the concept of *Work* and *Principle of Virtual Laws*.

The *Principle of virtual laws* (also *law, works, work, displacement, displacements*, depending on specific cases) in *Mechanics* throughout history encountered several intellectual difficulties, both for fundamental and applied research. For example, see *rational mechanics*, which did not allow indeterminable situations. In the *mechanics of rigid bodies*, the application of the *principle of virtual law* solves certain problems related to *vincula*. The latter *vincula* are related to *constraint reactions*, such as *auxiliary unknowns*, which are then eliminated by substitution during the solution of the single static problems; idem by *friction constraints* (Cfr. Coulomb 1784, 1785, 1799) one can formulate certain *constitutive laws*. On the contrary, in *continuum mechanics*, it is very important.

The *principle of virtual works* states that the *total virtual work* performed by all the forces acting on a system in static equilibrium is zero for a set of infinitesimal *virtual displacements* from equilibrium. The infinitesimal displacements are virtual because they need not be obtained by a displacement that actually occurs in the physical system. The virtual work is the work performed by the virtual displacements, which can be arbitrary, and are consistent with the constraints of the system.

Its common mathematical expression is:

$$\delta W = \sum_i F_i^{(a)} \delta s_i = 0$$

The theory of mechanical machines may be based on the *principle of virtual work*, and thought of as a consequence of the principle of the impossibility of perpetual motion, as applied to machines and constraints: *it is impossible that the reactions of the constraints on the actions of the bodies, which make up the machine, produce positive work*. In other words, it is impossible for forces of bodies of constraints to produce work:

$$\sum_i R_i \delta s_i \leq 0$$

The generalization of Lazare Carnot's *Principle of virtual work* is historically very important because it precedes Lagrange's approach[87] on the subject. Carnot began by stating his principles, which he referred to also as laws, to underline their empirical content. In his *Principes* Carnot wrote:

> Any motion that, when imparted to a system of bodies, has no effect on the intensity of the actions that they exert or can exert on each other in the course of any other motions imparted to them, will be named geometric. (Carnot 1803a, § 136, p. 108).

In this regard Gillispie observed that Lazare Carnot

> […] did achieve a greater clarity, most notably in the passages defining geometric motion [...]. Neither in the 1780 memoir nor in the *Essai sur les*

[87]In 1762, in *Application de la méthode exposée dans la mémoire précédente à la solution des différents problèmes de dynamique* (Lagrange 1762 ; see also: 1764, 1788, 1870–1873, 1892), Lagrange published the results of his research on the *Principle of Least action*. He perfected its formulation and substantiated it with a convincing proof. Nevertheless, previously Lagrange had conceived of a more general principle than that of least action. In a letter to Euler on 24th November 1759, Lagrange wrote about having composed elements of differential calculus and mechanics and developing the true metaphysics of its principle. Euler's *Correspondence with Joseph Louis de Lagrange (Opera Omnia*. Series IV, Vol I, retrieved via:http://eulerarchive.maa.org/correspondence/correspondents/Lagrange.html *Idem* letter was previously edited in Lagrange 1892, Tome XIV, pp. 170–174; see also: Euler 1738 [1730–1731], 1774 [1773], 1736, 1749, 1751 [1750] 1752, 1757, 1775. Cfr: Pisano 2017; Pisano and Capecchi 2013). Generally speaking, one can see: Johann Bernoulli, *Theoremata selecta pro conservatione virium vivarum demonstranda et esperimenta confirmanda* (Bernoulli Johann 1727), *Remarques sur le principe de la conservation des forces vives pris dans un sens général* (Bernoulli D 1750 [1748]). D'Alembert's *Traité de dynamique* (D'Alembert 1758 [1743]; see also Hankins 1970; D'Alembert 1751–1780; 1767) and Lagrange's *Recherches sur la libration de la Lune* (Lagrange 1764; see also: Truesdell 1968a, b, 1970, 1976, 1980 ; see also Fossombroni 1794).

> *machines en général* had Carnot adapted his concept of geometric motions from the principle of virtual velocities. In the *Principes fondamentaux de l'équilibre et du mouvement*, however, he went on to recognize the analogy between such motions and that principle in the use Lagrange made of the latter. (Gillispie and Pisano 2014, p. 72).

On the subject, in *Essai* we read:

> First law. *The reaction is always equal and opposite to the action.*
> Second law. *When two hard bodies act on each other by collision or pressure—that is to say, they act on each other in virtue of their impenetrability—their relative speed, immediately after the interaction, is always null.* (Carnot 1786, pp. 21–22. Author's italic)

The first law states that all bodies change their state of rest or motion always due to the action of another body. All bodies resist this change of state. Carnot often used the term "force d'inertie"[88]. On this subtle concept Carnot distanced himself from Euler's formulation (Pisano 2012) who considered it to be a theorem (theorem 7; see also theorem 6) by means of activity (force) and passivity (inertia); typically of applied mechanics. In detail, Euler's theorem founding principles of dynamics strictly follow those assumed in *Mechanica sive motus scientia analytice exposita* (1736) where he tried to reformulate Newtonian mechanics in a more systematic form.

> PROPOSITION 7. THEOREM. 56. A body remains in a state of absolute rest, unless it is disturbed to move by some external cause.[89]

Thus, the *Axiom or Law of motion à la Newton* is now presented as theorems. This methodological difference assumed a great importance for the development of mechanics and successive applied mechanics. It means that in certain cases, the principle of inertia is not a principle but a mere theorem; so it can be proved within a certain mathematics. Euler also assumed that laws of mechanics can be deduced without any recourse to experiments. On can also see D'Alembert (1743) who called it "loi" (law) and not a theorem:

> II. Law. 6. A body once put in motion by whichever cause, must persevere uniformly and in straight line, unless a new cause, different from that has caused the motion, will act on it. (D'Alembert (1743 [1758], p. 4).

[88]Carnot 1786, pp. 21–22; see also p. 20, p. 60, p. 61, pp. 62–63, pp. 68–69, p. 77, p. 78, p. 81, p. 86, p. 97.

[89]Euler 1736, p. 21; see also Euler [1730–1731] 1738; [1773] 1774; 1749; [1750] 1752; 1757; 1775.

Coming back to Carnot, his inertia force is a result of two combined motions. In his words:

> A body that one forces to change its state of rest or of motion resists (XI) the agent which produces the change, and it is this resistance which one calls the force of inertia. To evaluate this force it is necessary then to decompose the actual motion of the body into two parts, one of which is [the motion] that it will have the instant afterwards. [because] The other [part of the motion] will evidently be that which must be destroyed in order to force the body's change of state, that is to say, the resistance with which it opposes this change' or its force of inertia', from which it is easy to conclude, that the force of inertia of a body is the resultant of its actual motion & the motion equal and opposite to that which it must have the following instant. (Carnot 1786, p. 61).

The second law in *Essai* concerns hard (or completely soft) bodies. Carnot thought that the content of this law allowed moving the attention to the hard bodies only. Thus, elastic bodies can be re-conducted to hard bodies. But this assumption was clearly a forced justification. By applying his principles to a system of free hard bodies, Carnot formulated a first principle of mechanics as in the following:

$$\sum mVUcosZ = 0$$

It is the "first fundamental equation of mechanics (E)" (Carnot 1786, p. 27): m is the mass of the corpuscles of the system, V the true velocity after the impact, U the lost velocity (such that $W = V + U$ is the velocity the mass would have before the impact) and Z the angle between V and U. At this point Carnot introduced the concept of *geometric motion*.

> XVI. […] *if a system of bodies moves from a given position, with an arbitrary motion, but such that it was possible also to make another absolutely equal and directly opposite motion, then each of these motions will be called a geometric motion* […]. (Carnot 1786, p. 28. Author's italic. See also Ivi, pp. 29–34, pp. 41–45).

In *Principes* the definition was formulated slightly different:

> DEFINITIONS. 136. *Any motion will be called geometric if, when it is impressed upon a system of bodies, it has no effect on the intensity of the actions that they do or can exert on each other when any other motion is impressed upon them.* (Carnot 1803a, p. 108. Author's italic. See also: Gillispie 1971, p. 43).

The first one is purely geometric: *geometric motions* are reversible and congruent with constraints. The second one, by means of "action" alluded to force or Work, as well. In *Essai*, geometric motions could be also infinitesimal. This moves directly to *Principes*, too (e.g., see Carnot 1803a, theorem IX, p. 130). In effect, even if Carnot was not interested in infinitesimal cases, if finite or infinitesimal nature of geometrical motion, in the end it made no difference for Carnot's formulations. In fact, he used a quantity u associated to a single geometric motion. He called it *geometric velocity*, which, in modern terms is not so far from *virtual velocity* or *virtual displacement*. Geometric motion was one of his main contributions to mechanics:

> The theory of *geometric motions* is very important; it is as I have already noted like a mean science between ordinary geometry and mechanics [...] This science has never been treated in details, it is completely to create, and deserves both for its beauty and utility any care by Savan[t]s. (Carnot 1803a, p. 116. Author's italic).

By means of geometric motion, he wrote his first fundamental equation of motion when the true velocity Vafter the impact substituted the geometric velocity u, as in the following:

$$\sum muUcosz = 0 \quad (z \text{ is the angle between } u \text{ and } U)$$

Carnot called this equation the "second fundamental equation of mechanics (F)" (Carnot 1786, p. 32); by changing u among all possible geometric motions, one can obtain all equations needed to find the lost motions U of all masses implicated. In this way, the following problem is solved: given the initial velocities V of a system of masses to find the final velocities W after the impact. Given U, the final velocities are:

$$W = U + V.$$

By taking into account the discussion on hard bodies as above cited, this result could be uncorroborated (Cfr. Popper 1959), simply because hard bodies are ideal bodies, so far from the practical use of mechanics within machines theory. Carnot tried to solve this correlated problem by moving towards real cases. Instead, to consider the motion lost in the impact, he used the motion lost by imperceptible degrees and identifies mU (the lost motion) with a force F. The latter could be read with the modern meaning, as well. In the following, Carnot's reasoning on the subject:

> Whether it acts on the Machine in pulling it by a cord or in pushing it by a rod, the tension of this cord or the pressure of this rod expresses equally the effort which it exerts on the Machine, and the quantity of motion that it itself loses by the reaction which it experiences. If therefore one calls this force F then this quantity F will be the same thing as that which is expressed by mU in our equations (I). (Carnot 1786, p. 63; author's capital letters and italic style).

We can then write the second fundamental equation "F" expressed by the following form:

$$\sum Fucosz = 0$$

In *Essai*, Carnot could have alluded to the principle of virtual work most by means of the application of his *geometric motion* (i.e., *virtual velocities*; Carnot 1786, § XXXIV, pp. 68–70). As above cited, Carnot's theory of *geometric motions* coincided with velocities and not with displacements. Thus, Carnot avoided a formulation of the *principle of virtual work* by infinitesimal displacements. The latter could have produced some scientific embarrassment with respect to his projects (Carnot 1813). Furthermore, by applying the principle of virtual velocity to mechanical machines, the (forces) weights that balance each other are reciprocal to their virtual velocities. Incidentally, the two conceptually different approaches/formulations can be mathematical equivalents. Particularly, Lazare Carnot in applying the principle of virtual work also discussed the role played by forces applied to the bodies. In his words:

Fundamental Theorem
General Principle of equilibrium and of motion in Machines.

> XXXIV. *Whatever the state of rest or of motion found in any system of forces applied to a Machine, if we suddenly give it* [the system] *an arbitrary geometric motion, without changing these forces*, [then] *the sum of the products of each force and each* [initial] *speed* [when taken from the point of application, and in the direction of the force] *will be equal to zero*.
> That is to say, therefore, that in naming by, F, each of these forces (1), u the velocity at the first point of application, and z the angle between the directions of F and u, and if one imparts a geometric motion to the Machine, then it must be proved that for the whole system, $[\sum]Fucosz = 0$. But, this equation is precisely the equation (AA) [$\sum FucosZ = 0$ (Carnot 1786 p. 63] found (XXX) [*Ivi*, pp. 60–63] which is nothing else at root but the same [second] fundamental equation (F) [$\sum muUcosz = 0$ (*Ivi*, p. 32] presented in a different form.
> It is easy to perceive that this general principle is only, properly speaking, that *Descartes* to which one gives a sufficient generalisation, for it to include not only all the conditions between two forces, but as well all those of equilibrium and motion in a system composed of any number of forces. Also, the first consequence of this theorem will be the principle of *Descartes*, rendered complete by the conditions that we have seen were omitted by him (V).
> (Carnot 1786, § XXXIV, pp. 68–70 and footnote "(I)". Author's italic and Capital letters).

By working on these subjects, Carnot aimed to obtain a mathematical expression of its invariant with respect to all possible kinds of working substances. Consequently, to find invariants with regard to the efficiency (and reversibility) of a mechanical machine was necessary. In fact, the *principle of virtual work* establishes the condition of *equilibrium* of the forces that act on the bodies to produce work. On that, as above cited, he added an important corollary:

> *Corollary II. General principle of equilibrium in weight Machines.*
> *XXXVI. When several weights applied to any Machine are mutually in equilibrium, and one imparts any geometric motion to this Machine, then the speed of the centre of gravity of the system, measured in a vertical sense, will be zero in the first instant.* (Carnot 1786, p. 71. Author's italic; see also Carnot 1803a).

Coming back to his *first* equation "E" (Carnot 1786, p. 27) and *second* one "F" (Ivi, p. 32) and taking into account $\sum Fucosz = 0$, one can generalize to multi-body systems and write:

$$\sum mVUcos(<\vec{U},\vec{V}>) = 0 \tag{E}$$

$$\sum muUcos(<\vec{U},\vec{u}>) = 0 \tag{F}$$

m = mass of the body
W = velocity before interaction
V = velocity after interaction
U = W–V
u = *arbitrariness geometric motion*

In short:

- The mass of the parts of a machine.
- Global magnitudes, abstracting from the mass of the mechanism.
- Kinematics first, then dynamics, and statics is a special case of dynamics.
- A theory of machines concerns a theory of the *communication of motions*.
- A machine is a connected system of (hard) bodies.
- The connections between the bodies constrain the *communication* of motion of the bodies.
- The theory of interaction collisions by means of insensible degrees (e.g., see Carnot 1803a, § 293, pp. 261–262) as the result of a sequence of infinitesimally small percussions.

Taking into account the paralleling between *Essai* (1786) and *Principes* (1803a) about the *principle of virtual work*, we can write down the *law of conservation* for *hard* (*plastic*) *bodies* in the following way:

$$\sum_i m_i\vec{U}_i = 0$$

m_i = mass of i–th body [for isolate system]
U_i = velocity lost (by that body) during the collision
W_i = velocity before interaction
V_i = velocity after interaction
$\vec{U}_i = \vec{W}_i - \vec{V}_i$

Therefore, using the hypotheses of *parfaitement élastiques* bodies (i.e., in *Principes*, Carnot 1803a, p. 105), we can write:

$$\sum_i m_i \vec{U}_i \vec{V}_i = 0$$

V_i = velocity after interaction is the same for all of them
Generalization for all bodies using n—elasticity index.

Lazare Carnot proposed a tentative of generalization from plastic bodies to all bodies both in *Essai* (e.g., Carnot 1786 pp. 15–22) then in *Principes* (Carnot 1803a, pp. 103–106, pp. 131–146). But in these two books, the generalizations are differently presented. Precisely, the first has an inverse procedure with respect to second one.

At this stage, the *law of conservation of kinetic energy* for soft bodies can be formulated as in the following:

$$\sum_i m_i \vec{W}_i^2 = \sum_i m_i V_i^2 = 0$$

By introducing *geometric motions* (Carnot 1786) and in regard with the above *law of conservation*, we have:

$$\sum_i m_i \vec{U}_i \cdot \vec{u}_i = 0$$

m_i = mass of the i-th body
U_i = velocity lost (by that body) during the collision
u_i = velocity called *"mouvement géométrique"*

u is the velocity of any *geometric motion*. However, mathematically it is an indeterminate variable. Its specification produces an equation applicable to the physical system considered. For example, given

$$\vec{u}_i = const$$

we have an extension of the *principle of virtual velocity* to the collision of several bodies and we can write:

$$\sum_i m_i \vec{U}_i \cdot \vec{u}_i = 0 \rightarrow \vec{u} \sum_i m_i \vec{U}_i = 0.$$

Instead, given

$$u\!-\!arbitrariness$$

then, the equation becomes

$$\sum_i m_i \vec{U}_i = 0 \;(where\; \vec{U}_i = \vec{W}_i - \vec{V}_i)$$

Therefore we have the following situation:

- A theory of interacting bodies by means of collisions.
- A collision is a basic phenomenon. Continuously accelerated motion is a limiting case of a system driven by a series of pulses.
- Newton's second law is replaced by Carnot's second fundamental equation (F) for a system of n bodies.
- Due to the arbitrariness of u_i, it can be a constant, as well. Thus, geometric uniform motions applied to all bodies.
- In addition, by considering another an *ad hoc* geometric motion

$$\vec{u}_i = \vec{\omega} \times \vec{r}_i$$

(e.g., the rotation of the system with angular velocity around a fixed axis) and by using the properties of the triple product and the arbitrariness of "$\vec{\omega}$", we have the following Laws of conservation as invariants of motion:

$\sum_i m_i \vec{W}_i = \sum_i m_i \vec{V}_i$	*Law of conservation of the total quantity of motion.*
$\sum_i m_i \vec{r}_i \times \vec{W}_i = \sum_i m_i \vec{r}_i \times \vec{V}_i$	*Law of conservation of the total angular momentum.*

Lazare Carnot's argument on *geometric motion* (e.g., Carnot 1786, pp. 28–45) essentially expresses non-mechanical interactions as *invertible motions*. In other words: a motion assigned to a physical system of interacting bodies is geometric if the opposite motion is also possible. The result is a *possible motion*, but it is not always *invertible* (e.g., the motion of a sliding ring on a rotating rod). Therefore, one should add a hypothesis of *invertibility* in order to define a *geometric motion* (Pisano and Gillispie 2014, pp. 375–392). A *geometric motion* (via integral) produces an *invertible motion*. Particularly, given *vincula* independent of time, a *geometric displacement* is equivalent to a *virtual invertible displacement* (but not vice versa). On the contrary, only a *possible displacement (invertible)* produces (via derivative) a *geometric motion*. Therefore, a *geometric motion* is a kind of uniform motion when an equivalence of the state of rest and the state of uniform motion is considered.

1.5 Concluding Remarks

Carnot renewed the physics–mathematics relationship of applied mechanics in this period. He was a pioneer of both vector calculus applied to mechanics and of an alternative to Cartesian and Newtonian space. He solved his equations by using velocity only (*communication of motion*) reducing the mathematical problems to algebraic equations. In fact, he adopted a mathematics without infinitesimal entities (Carnot 1813). His mechanical theory (mechanics applied to machines) can be conceived as a generalisation from his second fundamental equation to virtual works by means of his geometric motions. In this sense they could also be considered similar (but not exactly) equivalent to modern virtual movements within collision theory (Cfr. Carnot 1786, §§ VIII–IX, §§ XI–XII, §§ XVI–XVII, §§ XXVIII–XXIX, §§ XXXI–XXXII, §§ XLVIII–

LXIV). Therefore, since collisions cannot be understood through infinitesimal analysis[90] the fundamental equation cannot be no longer $F = ma$, but rather a generalisation of Carnot's second fundamental equation, that is the *principle of virtual work*, applied to bodies in motion and collision (Cfr. Carnot 1803a, p. x).

A new approach to mechanical problems—with respect to Newtonian one—was invented by *l'organisateur de la victoire*.

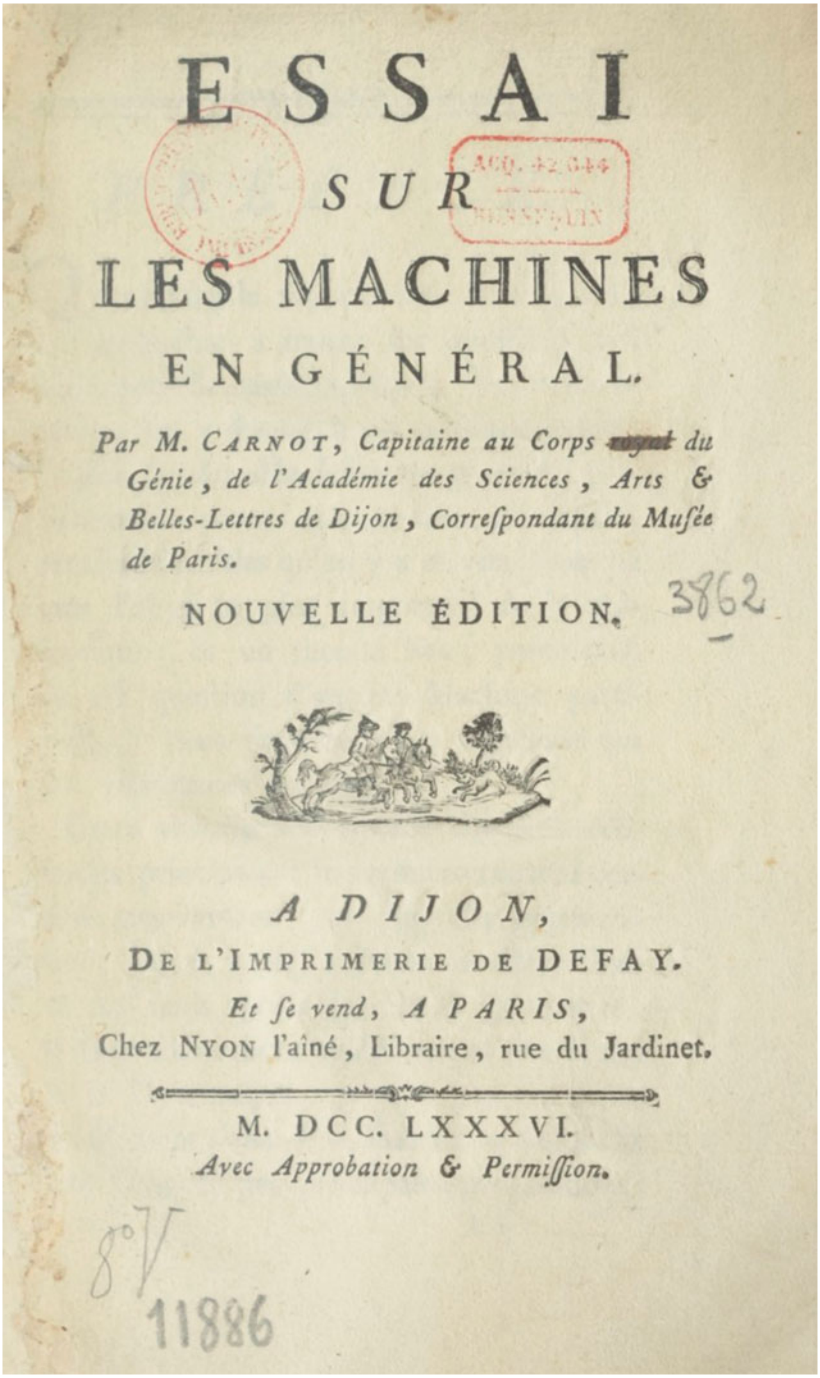
Raffaele Pisano
Lille (France), November 2019

[90]In fact, the time interval is zero and the forces are discontinuous and unlimited.

Contents

Chapter 1
A Critical Translation

R. Pisano et al., *Essay* on Machines in General *(1786)*, Logic, Epistemology,
and the Unity of Science 47, https://doi.org/10.1007/978-3-030-44385-6_1



ESSAI
SUR
LES MACHINES
EN GÉNÉRAL.

Par M. CARNOT, Capitaine au Corps ~~royal~~ du Génie, de l'Académie des Sciences, Arts & Belles-Lettres de Dijon, Correſpondant du Muſée de Paris.

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PRÉFACE.

Quoique la théorie dont il s'agit ici, ſoit applicable à toutes les queſtions qui concernent la communication des mouvements, on a donné à cet opuſcule le titre *d'Eſſai ſur les Machines en général*; premiérement, parce que ce ſont principalement les Machines qu'on y a en vue, comme étant l'objet le plus important de la méchanique; & en ſecond lieu, parce qu'il n'y eſt queſtion d'aucune Machine particuliere, mais ſeulement des propriétés qui ſont communes à toutes.

Cette théorie eſt fondée ſur trois définitions principales; la premiere regarde certains mouvements que j'appelle *géometriques*, parce qu'ils peuvent ſe déterminer par les ſeuls principes de la géometrie, & ſont abſolument indépendants des regles de la Dynamique; je n'ai pas cru qu'on pût aiſément s'en paſſer, ſans laiſſer du louche dans l'énoncé des principales propoſitions,

PREFACE.

Although the theory which we are discussing here is applicable to all questions which concern the transfer of motion, this little work has been given the title *Essay on Machines in General*, first, because it is principally Machines we have seen as being the most important object of mechanics; & in the second place, because it is not a question of any particular kind of Machine, but only of the properties which are common to all.

This theory is founded on three principal definitions; the first regards certain motions which I call *geometric* because they can be determined by the principles of geometry alone, & are absolutely independent of the laws of Dynamics; I did not believe we could easily do without this [first definition] without leaving ambiguities in the enunciation of the principal propositions,

A ij

comme je le fais voir en particulier pour le principe de *Deſcartes.*

Par la ſeconde de mes définitions, je tâche de fixer la ſignification des termes *force ſollicitante* & *force réſiſtante :* on ne peut, ce me ſemble, comparer clairement les cauſes avec les effets dans les Machines, ſans une diſtinction bien caractériſée entre ces différentes forces ; & c'eſt cette diſtinction ſur laquelle il me paroît qu'on a toujours laiſſé quelque choſe de vague & d'indéterminé.

Enfin, ma troiſieme définition, eſt celle par laquelle je donne le nom de *moment d'activité* d'une puiſſance, à une quantité dans laquelle il s'agit d'une puiſſance qui eſt réellement en activité ou en mouvement, & où l'on tient compte auſſi de chacun des inſtants employés par cette force, c'eſt-à-dire, du temps pendant lequel elle agit. Quoi qu'il en ſoit, on ne peut diſconvenir que cette quantité, ſous quelque dénomination qu'on veuille la déſigner, ne ſe rencontre continuellement dans l'analyſe des Machines en mouvement.

as I show especially for the principle of *Descartes*.

In the second of my definitions, I try to fix the meanings of the terms *soliciting force* & *resisting force*. One cannot, it seems to me, clearly compare causes and effects [in machines] without a clear distinction between these different forces [‘soliciting’ and ‘resisting’], & it is this distinction which it appears to me has always been left vague & indeterminate.

Finally, my third definition, is that in which I name the *moment of activity* of a force, a quantity that consists of a force[1] that is really acting or really in motion, & where one takes account also of each of the instants employed by the force - that is to say, the time during which the force acts. Whatever it may be and whatever denomination one assigns to it, one cannot disagree that this quantity is met with continually in the analysis of Machines in motion.

A l'aide de ces définitions, je parviens à des propoſitions qui ſont très-ſimples; je les déduis toutes d'une même équation fondamentale, qui, renfermant une certaine quantité indéterminée à laquelle on peut attribuer différentes valeurs arbitraires, donnera ſucceſſivement, dans chaque cas particulier, toutes les équations déterminées dont on a beſoin pour la ſolution du problême.

Cette équation qui eſt de la plus grande ſimplicité, s'étend généralement à tous les cas imaginables d'équilibre & de mouvement, ſoit que ce mouvement change bruſquement, ou varie par degrés inſenſibles; elle s'applique même à tous les corps, ſoit durs, ſoit doués d'un degré quelconque d'élaſticité; &, ſi je ne me trompe, elle ſuffit ſeule & indépendamment de tout autre principe méchanique, pour réſoudre tous les cas particuliers qui peuvent ſe rencontrer.

Je tire facilement de cette équation un principe général d'équilibre & de mouvement dans les Machines proprement dites, With the aid of these definitions, I arrive at propositions which are very simple; I deduce them all from one fundamental equation, which, containing a certain indeterminate quantity to which one can attribute different arbitrary values, will give successively, in each particular case, all the equations we need for the solution of the problem.

This equation, which is of the greatest simplicity, extends generally, to all imaginable cases of equilibrium & motion, whether that motion changes abruptly or varies by insensible degrees. It applies to all bodies, whether hard[*][2] or of whatever degree of elasticity, & if I am not mistaken, it is sufficient, & independent of all other principles of mechanics, to resolve all the particular cases which may be encountered.

From this equation I easily derive a general principle of equilibrium & motion of Machines proper,

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[*]From now on, by ‘hard’ understand ‘plastic’, that is, deformable but non-elastic.

& de celui-ci dérivent naturellement d'autres principes plus ou moins généraux, dont pluſieurs ſont déjà connus & très-célebres, mais qui ont été juſqu'ici (du moins pour la plupart) ou peu exactement, ou vaguement expliqués, plutôt que rigoureuſement démontrés.

Sans ſortir des principes généraux, j'ai réuni dans un ſcholie, & le plus clairement qu'il m'a été poſſible, les remarques les plus utiles à la pratique, & qui m'ont paru mériter par leur importance un développement particulier; tout le monde répéte que dans les Machines en mouvement on perd toujours en temps ou en vîteſſe ce qu'on gagne en force; mais après la lecture des meilleurs éléments de méchanique, qui ſemblent être la vraie place où doivent ſe trouver la preuve & l'explication de ce principe, ſon étendue & même ſa vraie ſignification ſont-elles faciles à ſaiſir? Sa généralité a-t-elle, pour la plupart des Lecteurs, cette évidence irréſiſtible qui doit caractériſer les vérités mathématiques? S'ils éprouvoient cette conviction frappante, ne

& from it, derive naturally other more or less general principles, many of which are already known & very famous, but which have been until now (at least for the most part) less exact[ly]*, or vaguely explained, rather than rigorously demonstrated.

Without departing from general principles, I have, in a scholium, gathered together & expressed as clearly as I possibly could, all the remarks most useful in practice, & which, by their importance, have appeared to me to merit individual development. Everyone says that in Machines in motion, one loses in time or speed what one gains in force. But, after reading the best treatises on mechanics, which seems to be the proper place to find the proof & explanation of this principle, is the extent & even the true significance of it ['what one loses in time or speed one gains in force'] easy to grasp? Does its generality, for the majority of readers, have that irresistible nature which should characterise mathematical truths? If they [the readers] could only experience this striking conviction,

* See *Errata*.

verroit-on pas des Méchaniciens inſtruits de ces ouvrages, renoncer inceſſamment à leurs projets chimériques? Ne ceſſeroient-ils pas de croire ou de ſoupçonner du moins, malgré tout ce qu'on leur dit, qu'il y a dans les Machines quelque choſe de magique? Les preuves qu'on leur donne du contraire ne s'étendent qu'aux Machines ſimples; auſſi ne croient-ils pas celles-ci capables d'un grand effet; mais on ne leur fait pas voir qu'il doit en être de même dans tous les cas imaginables; on ne parle que de celui où il y a ſeulement deux forces dans le ſyſtême, & l'on ſe contente d'une analogie: voilà pourquoi ces Méchaniciens eſperent toujours que leur ſagacité leur fera découvrir quelque reſſource inconnue, quelque Machine qui ne ſoit pas compriſe dans les regles ordinaires; ils ſe croient d'autant plus ſurs de la rencontrer, qu'ils s'éloignent davantage de tout ce qui paroît avoir de la relation avec les Machines uſitées, parce qu'ils s'imaginent que la théorie établie pour celles-ci, ne peut s'étendre à des conſtructions qui leur ſemwould we not witness an endless stream of practitioners, instructed in these works, abandoning their chimerical projects? Would they not cease to believe, or at least suspect, despite everything said to them, that there is a magical quality in the Machines? The proofs that one gives them to the contrary extend only to simple Machines. Also, do they not believe these [simple machines] are capable of a great effect [can do much work[3]]. But we do not make them see that it [these truths] must apply in every imaginable case; one speaks only of cases where there are just two forces in the system, & we are content with an analogy. This is the reason why these practitioners still have hope that their sagacity will enable them to discover some unknown resource, some Machine not already included in the established rules. They believe they are the more sure to encounter it [some unknown resource] the further they move away from everything connected with machines-in-practice - and this is because they imagine that the established theory may [perhaps] not apply in

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blent n'y avoir aucun rapport ; c'eſt en vain qu'on leur dit que toute Machine ſe réduit au levier : cette aſſertion eſt trop vague & trop tirée, pour qu'on s'y rende ſans un examen profond ; ils ne peuvent ſe perſuader que des Machines qui paroiſſent n'avoir rien de commun avec celles qu'on nomme ſimples, ſoient ſujettes à la même loi, ni qu'on puiſſe prononcer ſur l'inutilité d'un ſecret dont ils n'ont fait confidence à perſonne : de là vient que les idées les plus bizarres, les plus éloignées de la ſimplicité ſi avantageuſe aux Machines, ſont celles qui leur fourniſſent le plus d'eſpoir.

Le moyen de déraciner cette erreur, eſt ſans doute de l'attaquer dans ſa ſource même, en montrant que non-ſeulement dans toutes les Machines connues, mais encore dans toutes les Machines poſſibles, c'eſt une loi inévitable, qu'*on perd toujours en temps ou en vîteſſe ce qu'on gagne en force ;* & d'expliquer clairement ce que ſignifie cette loi ; mais il faut, pour cela, s'élever à la plus grande généralité poſſible, ne s'arrêter à aucune Machine particuliere, these apparently unrelated constructions. It is in vain that one says to them that all Machines [ultimately] reduce to the lever; this assertion is too vague, too much of a stretch; that one could gain understanding without a deep examination. They [the artisans and mathematical practitioners[4]] cannot be persuaded that Machines that appear to have nothing in common with those denominated 'simple', are [nevertheless] subject to the same laws. Neither [do they understand] that we can dismiss as worthless [knowledge deriving from] a secret not shared. From this arises the outcome that only the most bizarre ideas, the ones furthest from the simplicity so advantageous to Machines, are the very ideas that give them the most hope.

The way of rooting out this error is without doubt to attack its common source by showing that not only in all known Machines, but again in all possible Machines, we have the inevitable law: *what one loses in time or speed one gains in force*. We must [then] explain clearly what this law means but, for this, it must be raised to the greatest possible level of generality, not stopping with a particular Machine, ne s'appuyer ſur aucune analogie ; il faut enfin une démonſtration générale, déduite immédiatement & géométriquement des premiers axiomes de la méchanique : c'eſt ce qu'on a tâché de faire dans cet Eſſai ; on a beaucoup inſiſté ſur ce point fondamental, & je ne ſais ſi l'on aura réuſſi à le mettre dans un aſſez grand jour ; mais en attaquant l'erreur, on s'eſt efforcé d'y ſubſtituer la vérité ; on a montré quel eſt le véritable but des Machines : s'il n'eſt pas raiſonnable d'en attendre des prodiges hors de toute vraiſemblance, on verra qu'il leur reſte encore aſſez d'objets d'utilité, pour exercer la plus brillante imagination.

Les réflexions que je propoſe ſur cette loi, me conduiſent à dire un mot du mouvement perpétuel, & je fais voir non-ſeulement que toute Machine abandonnée à elle-même doit s'arrêter, mais j'aſſigne l'inſtant même où cela doit arriver.

On trouvera encore parmi ces réflexions une des plus intéreſſantes propriétés des Machines, qui, je crois, n'a pas encore été remarquée ; c'eſt que pour leur faire pro-and not relying on any analogy.

When it comes down to it, a general demonstration is necessary, and one that has been deduced geometrically & straight from the first principles of mechanics. This is what I have tried to do in this Essay. I have much insisted on this fundamental point, but I do not know if I have managed to bring it sufficiently to general public attention. However, in attacking an error one is forced to replace it by the truth. We have shown what is the true goal of Machines: while [we submit that] it is not rational to wait for marvels outside of all likelihood, we will [nevertheless] find that there are enough useful objectives to exercise the most brilliant minds?

The reflections that I propose on this law lead me to say a word about perpetual motion, & I show not only that every Machine left to itself must stop but I even assign the moment when this must happen.

Furthermore, one will find among these reflections one of the most interesting properties of Machines (one which I believe has not been remarked upon previously). It is that in order to produce x

duire le plus grand effet poſſible, il faut néceſſairement qu'il n'arrive aucune percuſſion, c'eſt-à-dire que le mouvement doit toujours changer par degrés inſenſibles; ce qui donne lieu, entre autres choſes, à quelques remarques ſur les Machines hydrauliques.

Enfin, je termine cet Ecrit par quelques réflexions ſur les loix fondamentales de la communication des mouvements, qui, ſi elles ne ſont pas du goût de tout le monde, ſont du moins aſſez courtes pour ne fatiguer perſonne.

Mais, je le répéte, cet Eſſai n'a pour objet que les Machines en général; chacune d'elles à ſes propriétés particulieres: il ne s'agit ici que de celles qui ſont communes à toutes; ces propriétés, quoique aſſez nombreuſes, ſont en quelque ſorte toutes compriſes dans une même loi fort ſimple: c'eſt cette loi qu'on s'eſt propoſé de rechercher, de démontrer & de développer, en enviſageant toujours les Machines ſous le point de vue le plus général & le plus direct.

the greatest possible ‘effect’ [quantity of work], it is necessary that there is no percussion, that is to say, the motion must always happen by insensible degrees. This leads naturally, amongst other things, to some remarks on hydraulic Machines.

Finally, I finish this work with some reflections on the fundamental laws of the communication of motion which, even if they are not to everyone’s taste, are short enough not to tire people.

However, I repeat, the object of this Essay is [the theory of] Machines in general. Each Machine has its own particular properties but here we are concerned only with those properties that are common to all. These properties, although numerous, are in some way included within one very simple law. It is this law which it is proposed to research, demonstrate & develop, envisaging Machines always from the most general & most direct[5] point of view.

ESSAI
SUR LES MACHINES
EN GÉNÉRAL.

INTRODUCTION.

I. NOUS ne manquons pas d'excellents Traités ſur les Machines ; les propriétés particulieres à celles dont l'uſage eſt fréquent, à celles ſur-tout qu'on eſt convenu d'appeller ſimples, ont été recherchées & approfondies avec toute la ſagacité poſſible ; mais il me ſemble qu'on ne s'eſt pas encore beaucoup attaché, à développer celles de ces propriétés qui ſont communes à toutes les Machines, & qui, par cette raiſon, ne conviennent pas plus aux cordes qu'au levier, à la vis, ou à toute autre Machine ſoit ſimple ſoit compoſée.

Ce n'eſt pas cependant que les Géometres aient négligé de s'élever aux principes généraux d'équilibre & de mouvement ; mais ce n'eſt pour ainſi dire qu'en paſſant qu'ils ont parlé de leur application à la théorie des Machines pro-

ESSAY

ON MACHINES

IN GENERAL.

INTRODUCTION.

I. We are not lacking excellent Treatises on Machines. The properties of the most commonly-used Machines and, above all, those called simple[6], have been researched & studied in depth and with all possible sagacity. However, it seems to me that still hardly any attempt has been made to develop those laws which are common to all Machines & which, for that reason, are just as suitable for strings as for the lever, the screw, or any other Machine, simple or composite.

It is not, however, that Geometers have neglected to reach up to such general principles of equilibrium & of motion. Rather - if I may say so in passing - they have not spoken of their application to the theory of Machines prement dites, & peut-être auſſi n'y a-t-il encore aucun de ces principes qui joigne à une démonſtration rigoureuſe une aſſez grande généralité, pour pouvoir ſuffire ſeul & indépendamment de tout autre, à la ſolution des différentes queſtions qu'on peut propoſer tant ſur l'équilibre que ſur le mouvement des Machines, c'eſt-à-dire, pour réduire toutes les queſtions à une affaire de calcul & de géométrie; ce qui eſt le véritable objet de la méchanique.

II. Parmi les principes plus ou moins généraux qui ont été juſqu'ici propoſés, nous en rappellerons ſeulement deux très-célébres & ſur leſquels nous aurons quelques obſervations à faire.

Le premier eſt celui qui aſſigne pour loi générale de l'équilibre dans les Machines à poids, que le centre de gravité du ſyſtême eſt alors au point le plus bas poſſible; mais quoique cet ancien principe ſoit fort ſimple & fort général, il ne paroît pas qu'on lui ait donné toute l'attention qu'il mérite: c'eſt ſans doute, 1°. parce qu'il eſt ſujet à quelques exceptions, comme tous ceux où il s'agit de *maximum* & de *minimum*; 2°. parce qu'il n'a rapport qu'à une eſpece particuliere de force, qui eſt la peſanteur; 3°. enfin parce qu'il paroît difficile d'en donner une démonſtration générale & rigoureuſe. Mais, 1°. nous allons faire voir qu'en changeant un peu l'énoncé de ce principe, on en peut faire une propoſition très-exacte, très-géométrique & vraie ſans exception; 2°. quoiqu'il n'ait rapport qu'à la peſanteur, cependant il eſt facile de l'appliquer à tous les cas imaginables; il n'y a pour cela qu'à ſubſtituer un poids à la place de chacune des puiſſances qui ſont d'un genre difféproper; & perhaps also they have never adjoined their principles to a rigorous demonstration of enough generality to be sufficient on its own (independent of all others) to the solution of all the different questions one could ever pose, either on the equilibrium or the motion of Machines. That is, [none of these previous Geometers has managed] to reduce all questions to a matter of [pure] calculation & geometry. This is the true object of mechanics.

II. Among the principles, of greater or lesser generality, which have been proposed up until now, we recall only two very famous ones, & on these we will have some observations[7] to make.

The first [example] is that which adduces a general law of equilibrium in weight-lifting Machines that the centre of gravity of the system is at the lowest possible point[8]. However, much as this ancient principle is very simple & very general, it appears to me that it has not been given all the attention it merits. This is without doubt 1st, because it is subject to some exceptions, such as all those cases that have to do with *maximum & minimum* [principles]; 2nd, because it relates only to a particular type of force, the weight[9]; and finally 3rd, because it appears difficult to give a general & rigorous demonstration[10]. But 1st, we are going to show that [just] in changing the statement of this principle a little bit, we are able to generate a proposition that will be very exact, very geometric & true without exception; 2nd, although the principle relates only to the weight, one can, however, easily apply it to all imaginable cases; to achieve this it is only necessary to substitute a weight in place of each force which is of a diffrent ; ce qui eſt très-facile, par le moyen d'un fil paſſant ſur une poulie de renvoi ; de ſorte qu'alors il ne reſte plus à ce principe que le défaut d'être indirect ; 3°. enfin, quoiqu'on ne puiſſe le démontrer rigoureuſement ſans remonter juſqu'aux premiers principes de la méchanique, il eſt cependant facile d'en rendre aſſez bien raiſon, pour qu'il ne fût pas poſſible d'en douter, quand même on n'en auroit pas d'autres preuves, comme nous allons le faire voir en attendant la démonſtration exacte que nous tâcherons d'en donner dans la ſuite de cet Eſſai.

Imaginons donc une Machine à laquelle il n'y ait d'autres forces appliquées que des poids, je la ſuppoſe d'ailleurs d'une forme arbitraire, mais qu'on ne lui ait imprimé aucun mouvement : cela poſé, quelle que ſoit la diſpoſition des corps du ſyſtême, il eſt clair que s'il y a équilibre, la ſomme des réſiſtances des points fixes ou obſtacles quelconques, eſtimées dans le ſens vertical contraire à la peſanteur, ſera égale au poids total du ſyſtême ; mais s'il naît un mouvement, une partie de la peſanteur ſera employée à le produire, & ce n'eſt qu'avec le ſurplus, que les points fixes pourront ſe trouver chargés ; donc dans ce cas la ſomme des réſiſtances verticales des points fixes, ſera moindre au premier inſtant que le poids total du ſyſtême : donc de ces deux forces combinées (la peſanteur du ſyſtême & la charge verticale des points fixes) il en réſultera une ſeule force égale à leur différence, & qui pouſſera le ſyſtême de haut en bas comme s'il étoit libre : donc le centre de gravité deſcendra néceſſairement avec une vîteſſe égale à cette différence diviſée par la maſſe totale du ſyſtême : donc ſi le centre de rent type [i.e., from a force-weight]; this can very easily be done by means of a thread passing over a return pulley. The only remaining fault [with the principle] is that it applies indirectly - to weights rather than to forces of whatever type; 3rd, finally, although we cannot demonstrate it [our most general law] without reconstructing mechanics from first principles, it is nevertheless easy to justify the argument by an appeal to good sense. It is therefore impossible to doubt the veracity of this sound argument even without other proofs; as we are going to show later when we try to give an exact demonstration in this Essay.

Let us imagine then a Machine where the only forces applied are weights - in other regards the form of the Machine is arbitrary - but to which we have not imparted any motion. That given, whatever the disposition of bodies in the system, it is clear that if there is to be equilibrium, the sum of the resistances at fixed points or other constraints, estimated in a vertical sense against the weight[11], will be equal to the total weight of the system. But if a motion begins, then this will be at the expense of a certain portion of the weight, & the load at the fixed points will then be due to the residue. In this case, the sum of the vertical resistances at the fixed points will be less – in the first instant – than the total weight of the system. Then these two forces combined (the weight of the system and the vertical load at the fixed points) will result in a single force equal to their difference, & will push the system from high to low as if it were free [from constraints]. Therefore the centre of gravity of the system will necessarily descend at a speed[12] equal to this difference divided by the total mass of the system. So therefore, if the centre of

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gravité du ſyſtême ne deſcend pas, il y aura néceſſairement équilibre. Donc en général,

Pour s'aſſurer que pluſieurs poids appliqués à une Machine quelconque doivent ſe faire mutuellement équilibre, il ſuffit de prouver que ſi l'on abandonne cette Machine à elle-même, le centre de gravité du ſyſtême ne deſcendra pas.

III. La conſéquence immédiate de ce principe vrai ſans exception, eſt que ſi le centre de gravité du ſyſtême eſt au point le plus bas poſſible, il y aura néceſſairement équilibre; car, ſuivant cette propoſition, il ſuffit, pour le prouver, de faire voir que le centre de gravité ne deſcendra pas; or, comment deſcendroit-il, puiſque par hypotheſe il eſt au point le plus bas poſſible?

IV. Pour donner encore une application de ce principe, je ſuppoſe qu'il s'agiſſe de trouver la loi générale d'équilibre entre deux poids A & B appliqués à une Machine quelconque; je dis donc qu'alors, en conſéquence du principe précédent, il y aura équilibre entre ces deux poids A & B, ſi, en ſuppoſant que l'un des deux vienne à l'emporter, & la Machine à prendre un petit mouvement, il arrivoit que l'un de ces corps montât pendant que l'autre deſcendroit, & qu'en même temps ces poids fuſſent en raiſon réciproque de leurs vîteſſes eſtimées dans le ſens vertical: en effet, ſi l'on ſuppoſe qu'alors A deſcendît avec la vîteſſe verticale V, tandis que la vîteſſe de B, auſſi eſtimée dans le ſens vertical, ſeroit u, on aura, par hypotheſe, $A : B :: u : V$, ou $AV = Bu$, donc $\frac{AV - Bu}{A + B} = 0$. Cela poſé, puiſque les corps ſont ſuppoſés ſe mouvoir, l'un de haut en bas, gravity of the system does not descend, it will necessarily be in equilibrium. Therefore in general,

To assure that several weights [masses] *applied to whatever Machine should be in mutual equilibrium*[13], *it suffices to prove that if the Machine was left on its own, the centre of gravity of the system would not descend*[14].

III. The immediate consequence of this principle, true without exception, is that if the centre of gravity of the system is at the lowest possible point then it will necessarily be in equilibrium. [This is self-evident] because (using the very proposition we seek to prove) it is sufficient to demonstrate that the centre of gravity does not descend. However, how can it descend since by hypothesis it is already at the lowest point possible?

IV. To give another application of this principle, let us suppose that we want to find the general law of equilibrium between two weights, A & B, applied to an arbitrary Machine. I state that, as a consequence of the preceding principle, there will be equilibrium between the two weights, A & B, as follows: suppose that one of the two bodies imparts, & the Machine receives, a tiny motion. [If there is equilibrum] it will follow - for vertical displacements only - that when one body rises while the other descends, then their speeds will be in a reciprocal ratio to their masses.

In effect, if one supposes that A descends with a vertical speed V, while the speed of B, also estimated in a vertical direction [but ascending] is u, then one will have, by hypothesis, $A : B :: u : V$, or $AV = Bu$, and therefore

$$\frac{AV - Bu}{A + B} = 0.$$

This said, since the bodies are supposed to move, one from high to low,

& l'autre de bas en haut, il eſt évident que le premier membre de cette équation eſt la vîteſſe verticale du centre de gravité du ſyſtême; donc ce centre de gravité ne deſcendra pas; donc, par la propoſition précédente, il doit y avoir équilibre.

V. Le ſecond principe ſur lequel nous nous ſommes propoſés de faire quelques obſervations, eſt la fameuſe loi d'équilibre de *Deſcartes*; elle revient à ce que deux puiſſances en équilibre ſont toujours, en raiſon réciproque de leur vîteſſe, eſtimées dans le ſens de ces forces, lorſqu'on ſuppoſe que l'une des deux vient à l'emporter infiniment peu ſur l'autre, de maniere qu'il en naiſſe un petit mouvement.

Mais quoique cette propoſition ſoit très-belle & qu'on la regarde ordinairement comme le principe fondamental de l'équilibre dans les Machines, elle eſt cependant infiniment moins générale que celle qui a été citée en premier lieu, car elle s'applique uniquement au cas où il y a ſeulement deux puiſſances dans le ſyſtême, & d'ailleurs elle ſe déduit très-facilement de ce qui vient d'être dit au ſujet des deux poids *A* & *B*, puiſqu'on ramene viſiblement l'un de ces cas à l'autre, en ſubſtituant, par des poulies de renvoi, des poids à la place des forces dont on cherche le rapport.

De plus, il eſt à remarquer que ce principe n'exprime pas les conditions de l'équilibre entre deux puiſſances, auſſi complétement que celui qui a été cité en premier lieu; car il ne donne que le rapport des quantités de force qui ſe font équilibre, au lieu que celui-ci donne auſſi en quelque ſorte le rapport de leurs directions; par exemple, dans le cas d'équilibre

& the other from low to high, then it is evident that the first term in this equation is the vertical speed of the centre of gravity of the system. Therefore the centre of gravity will not descend and thus, by the preceding proposition, it must be in equilibrium.

V. The second principle on which we propose to make a few observations is the famous equilibrium law of *Descartes*; it comes to this, two forces in equilibrium are always in the reciprocal ratio of their speeds, estimated in the direction of the forces, such that when one supposes that one of the two [forces] gains infinitesimally on the other, [it is] in a manner such as to give birth to a tiny motion.[15*]

But although this proposition is very beautiful, & one regards this ordinarily as the fundamental principle of the equilibrium of Machines, it is, however, infinitely less general than that which has been stated in the first place [*Carnot*'s general Principle], because it [*Descartes*'s Principle] applies only in the case where there are just two forces in the system, &, in any case, this principle [of *Descartes*] can be deduced very easily from what is going to be said on the subject of two weights, A & B, since we manifestly reduce one of these cases [*Descartes*'s principle] to the other [our most general principle], by assuming [the analogy of] return pulleys, and substituting weights for the forces whose ratio one seeks.

Moreover, it is to be remarked that this principle [of *Descartes*] does not express the condition of equilibrium between two forces as completely as that which we stated in the first place - and this [lack] is because it only determines the relative strengths of the forces, whereas our Principle gives in addition some idea of the ratio of their [magnitudes along different] directions. For example, in the case of equilibrium

[*] Carnot's explanation is not easy to follow: the reader is encouraged to look up an explanation of the 'principle of reciprocal velocities'.

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entre deux poids, le principe de *Deſcartes* apprend ſeulement que les poids doivent être en raiſon réciproque de leurs vîteſſes verticales; mais il n'indique pas, comme le premier, que l'un de ces corps doit néceſſairement monter pendant que l'autre deſcendra; pour qu'un treuil, par exemple, à la roue & au cylindre duquel ſont ſuſpendus des poids par des cordes, demeure en équilibre, il ne ſuffit pas que le poids appliqué à la roue ſoit à celui du cylindre, comme le rayon du cylindre eſt au rayon de la roue; il faut encore que ces poids tendent à faire tourner la Machine en ſens contraire l'un de l'autre, c'eſt-à-dire qu'ils ſoient placés de différents côtés, par rapport à l'axe, ſinon leurs efforts étant conſpirants, mettront la Machine en mouvement : il eſt donc évident que ce qui rend le principe de *Deſcartes* incomplet, c'eſt qu'en déterminant le rapport des puiſſances, quant à leurs valeurs ou intenſités, il n'exprime pas que ces puiſſances doivent faire des efforts oppoſés, ni en quoi conſiſte cette oppoſition d'efforts : il eſt clair en effet que pour l'équilibre il faut que l'une des forces réſiſte tandis que l'autre ſollicite; or, c'eſt ce qui n'arrive pas dans le treuil qui vient d'être allégué pour exemple; mais qu'eſt-ce en général qui diſtingue les forces ſollicitantes des forces réſiſtantes? C'eſt, ce me ſemble, ce qui n'a pas encore été déterminé : on verra dans cet Eſſai que la différence caractériſtique de ces forces conſiſte dans l'angle qu'elles forment avec les directions de leurs vîteſſes, de ſorte que les unes font toujours avec leurs vîteſſes des angles aigus, tandis que les autres font des angles obtus avec les leurs.

Enfin,

between two weights, the principle of *Descartes* says only that the weights must be in the reciprocal ratio of their vertical speeds but does not indicate - as our Principle does - that one of the bodies must necessarily rise while the other descends. Thus, [consider a] winch, for example. From the wheel & drum of the winch are suspended weights by ropes. In order for [the system] to remain in equilibrium, it is not enough that the weight applied to the wheel is to the weight applied to the drum as the radius of the drum is to the radius of the wheel. It is also necessary that the weights tend to turn the Machine in the opposite directions to each other - that is to say, they must be on different sides of the [rotational] axis. Otherwise, the effect [the work that each weight does] will be additive, and will [therefore] set the Machine in motion. Therefore it is evident that what makes the principle of *Descartes* incomplete is that, while it does determine the relative force-strengths, it does not determine that these forces must be in opposite directions, or give any indication of why the forces must oppose each other. [However,] in order for [the condition of] equilibrium, it must be that one of the forces resists while the other one solicits - but this is never going to happen in the example of the winch. What is it in general which distinguishes soliciting forces from reaction forces? This, it seems to me, has not been determined before. We will see, in this Essay, that the characteristic difference in these forces [the soliciting and reaction forces] consists in the angle which they make with the direction of their speeds. One kind always makes an acute angle with its speed, while the other [kind always] makes an obtuse angle with its speed.

Finally,

Enfin, un défaut qu'il me paroît qu'on peut encore reprocher au principe de *Deſcartes*, ainſi qu'à tous ceux où il s'agit du petit mouvement qui naîtroit dans le ſyſtême, ſi l'équilibre venoit à être troublé, c'eſt qu'ils n'indiquent pas la maniere de déterminer ce petit mouvement; or, s'il faut pour cela avoir recours à quelque nouveau principe méchanique, le premier n'eſt donc pas ſuffiſant; & ſi on peut le déterminer par pure géométrie, quelle en eſt la maniere? C'eſt ce que ne dit pas le principe: & qu'on ne diſe pas que la proportion indiquée par le principe, a toujours lieu, quel que puiſſe être le mouvement, pourvu qu'il ſoit poſſible, c'eſt-à-dire compatible avec l'impénétrabilité des corps; car ce ſeroit une erreur; & nous ferons voir dans la ſuite que ces mouvements ſont aſſujettis à certaines conditions, en conſéquence deſquelles j'ai cru devoir leur donner le nom de *mouvements géométriques*.

On peut faire la même remarque ſur tous les principes où l'on propoſeroit de conſidérer la Machine dans deux états infiniment proches l'un de l'autre; car pour déterminer quels ſont ces deux états, c'eſt-à-dire quel mouvement il faudroit que la Machine prît pour paſſer de l'un à l'autre, il faut ou employer de nouveaux principes méchaniques conjointement avec celui qu'on propoſe; ce qui rendroit celui ci inſuffiſant; ou la géométrie ſuffit; & dans ce cas c'eſt un défaut dans le principe, de ne pas faire connoître les conditions géométriques auxquelles ce mouvement eſt aſſujetti.

VI. Les deux loix dont on vient de parler, ſont bornées l'une & l'autre au cas de l'équilibre; on paſſe aiſément de ce cas à celui du mou-

Finally, [there is] a criticism which it seems to me one may level against *Descartes*'s Principle, as well as against any other principle that considers systems wherein tiny motions are generated. [The defect is that] when the equilibrium has been disturbed [which in turn leads to the birth of tiny motions], [the Principle of *Descartes*] does not indicate [how] these tiny motions [their strengths and directions] shall be determined. If, however, [in order to determine these motions] one must have recourse to some new principle of mechanics, then this just goes to show that the starting principle (that of *Descartes*) was not sufficient; &, if one could determine [the tiny, nascent motions] by pure geometry, how is this to be done? This is [exactly] what *Descartes*'s Principle doesn't tell us: &, it also does not tell us that its predictions should always apply, whatever are the characteristics [strengths and directions] of these tiny motions - so long as they are possible motions, that is to say, ones that are compatible with the impenetrability of bodies. Otherwise, this [omission] would be an error; & we shall show in what follows that these [tiny, nascent] motions are subject to certain conditions, in consequence of which I believed it was necessary to give them [a special name,] the name, *geometric motions*.

One may make the same remark about all principles [where it has been proposed to] consider a system in two states, infinitesimally close together. In order to determine what are these two states, that is to say, what motion the Machine must undergo in order to pass from the one [state] to the other, it is necessary either to use new principles of mechanics in addition to those which have already been proposed (which then renders these [latter principles] insufficient), or the geometry [alone] suffices [to determine the states] (& this amounts to a defect in the principle - not to know the geometric conditions that the motion is subject to).

VI. The two laws we have just been talking about are both limited to the case of equilibrium. One may pass easily from this case to the mo-

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vement, par le principe de dynamique dû à *M. d'Alembert ;* mais on en a auſſi trouvé pluſieurs autres qui s'appliquent immédiatement au cas du mouvement ; tel eſt celui de la conſervation des forces vives dans le choc des corps parfaitement élaſtiques, lequel eſt d'autant plus général, qu'il s'étend au cas même où le mouvement paſſe bruſquement d'un état à l'autre ; mais il paroît qu'on n'a gueres ſongé à l'uſage qu'on en pouvoit faire dans la théorie des Machines proprement dites ; il eſt cependant évident que cette loi doit avoir ſon analogue dans le choc des corps durs ; & comme on prend ordinairement ceux-ci pour ſervir de terme de comparaiſon, ce principe transféré aux corps durs avec la modification qu'exige la différence de leur nature, ne peut manquer d'être plus utile que la conſervation même dont il s'agit : nous ferons voir en effet qu'on en déduit avec la plus grande facilité pluſieurs vérités capitales, & particuliérement la conſervation des forces vives dans un ſyſtême de corps durs dont le mouvement change par degrés inſenſibles ; principe dont l'utilité dans la théorie des Machines eſt ſi connue : on verra en même temps par-là une relation intime entre ces deux conſervations de forces vives ; on en tire également le principe de *Deſcartes*, & même, en le généraliſant, la loi d'équilibre dans les Machines à poids dont il a été queſtion ci-deſſus ; ce principe enfin, après lui avoir donné l'extenſion dont il eſt ſuſceptible, nous a paru renfermer toutes les loix de l'équilibre & du mouvement, & nous n'avons pas cru pouvoir en adopter un meilleur pour ſervir de baſe à notre théorie.

VII. Cet Eſſai ſera diviſé en deux parties ;

vement by means of the dynamic principle of *Mr. d'Alembert*. However we have also found several other principles which apply directly to the case of motion. One such is the principle of the conservation of live force [kinetic energy] in the collision of two perfectly elastic bodies. This principle is all the more general in that it extends even to the case where the motion passes suddenly from one state to the other. However it appears to me that the huge scope of it [this principle] in the theory of Machines proper has barely been imagined. It is nevertheless evident that this law must have an analogue in the collision of hard bodies & as one ordinarily takes these hard-body collisions as test cases, then the [conservation of live force] principle, applied to hard bodies with appropriate modifications [(taking into account the special characteristics of hard-body collisions)], cannot fail to be more enlightening than the mere fact of conservation. We will show, in effect, that one can deduce several capital truths with the greatest of ease, & in particular, the conservation of living force in a system of hard bodies having its motion change by insensible degrees[16] - a principle whose utility in the theory of Machines is well known. We will see at the same time an intimate relation between the two live force conservation principles. We will, likewise, draw out conclusions from the principle of *Descartes*, & even from the law of equilibrium science of weight Machines, discussed above, after it has been generalised. This principle [the law of equilibrium in weight Machines], after being given the extensions to which it is susceptible, is, finally, the one that seems to us to contain all the laws of equilibrium & motion, & we do not believe we could adopt a better one to serve as the basis of our theory.

VII. This Essay will be divided into two parts;

dans la premiere, on traitera des principes généraux de l'équilibre & du mouvement dans les Machines ; & dans la seconde, on recherchera les propriétés des Machines proprement dites, c'est-à-dire, de ce à quoi le nom de Machines a été plus spécialement affecté, sans cependant s'arrêter jamais à aucune Machine particuliere.

PREMIERE PARTIE.

Principes généraux.

VIII. Lorsqu'un corps agit sur un autre, c'est toujours immédiatement, ou par l'entremise de quelque corps intermédiaire ; ce corps intermédiaire est en général ce qu'on appelle une Machine ; le mouvement que perd à chaque instant chacun des corps appliqués à cette Machine, est en partie absorbé par la Machine même, & en partie reçu par les autres corps du systême ; mais comme il peut arriver que l'objet de la question soit uniquement de trouver l'action réciproque des corps appliqués aux corps intermédiaires, sans qu'on ait besoin d'en connoître l'effet sur le corps intermédiaire même, on a imaginé, pour simplifier la question, de faire abstraction de la masse même de ce corps, en lui conservant d'ailleurs toutes les autres propriétés de la matiere ; dès-lors la science des Machines est devenue en quelque sorte une branche isolée de méchanique, dans laquelle il s'agit de considérer l'action réciproque des différentes parties d'un systême de corps, parmi in the first, one will treat the general principles of equilibrium & motion in Machines; & in the second, one will research the properties of Machines proper - that is to say, those by which the name Machines has been especially assumed - but never pausing at one or another particular Machine.

FIRST PART.

General principles.

VIII. When one body acts on another body, it is always proximately, or by the interposition of some intermediate bodies. This intermediate body is, in the most general sense, what one denominates, Machine. The motion that is lost at each instant by each of the bodies applied to the Machine, is in part absorbed by the Machine itself, & in part received by other bodies in the system. But, as it may happen that the aim of the investigation is [solely] to find the reciprocal action between applied bodies via some intermediary body, but without needing to know the effect on the intermediate body itself, then, in order to simplify the investigation, we have abstracted [away] the mass of such intermediate bodies (while keeping all other properties of the system unaltered). Following from this [abstraction], the science of Machines[17] has become in some sense an isolated branch of mechanics, in which only the reciprocal action of different parts within a system of bodies is to be considered. Amongst

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lesquelles il s'en trouve qui, privés de l'inertie commune à toutes parties de la matiere telle qu'elle existe dans la nature, ont retenu le nom de Machines.

IX. Cette abstraction pouvoit simplifier dans certains cas particuliers, où les circonstances indiquoient ceux des corps dont il convenoit de négliger la masse, pour arriver plus facilement au but; mais on conçoit que la théorie des Machines en général est devenue réellement plus compliquée qu'auparavant; car alors cette théorie étoit renfermée dans celle du mouvement des corps tels que la nature nous les offre; mais à présent il faut considérer à la fois deux sortes de corps, les uns tels qu'ils existent réellement, les autres dépouillés en partie de leurs propriétés naturelles; or, il est clair que le premier de ces problêmes est un cas particulier de celui-ci; donc celui ci est plus compliqué que l'autre: aussi, quoiqu'on parvienne aisément par de pareilles hypotheses, à trouver les loix de l'équilibre & du mouvement dans chaque Machine particuliere, telle que le levier, le treuil, la vis, il en résulte un assemblage de connoissances dont la liaison s'apperçoit difficilement, & seulement par une espece d'analogie; ce qui doit nécessairement arriver tant qu'on aura recours à la figure particuliere de chaque Machine, pour démontrer une propriété qui lui est commune avec toutes les autres: ces propriétés communes étant celles que nous avons en vue dans cet Essai, il est clair que nous ne parviendrons à les trouver qu'en faisant abstraction des formes particulieres: commençons donc par simplifier l'état de la question, en cessant de considérer dans un même système des corps de these [system-of-bodies] one may find some which, when deprived of the inertia common to all material things such as exist in nature, have retained the name: Machines.

IX. This abstraction can simplify [the analysis] in certain particular cases - cases where the circumstances are such that it is proper to neglect the mass of these bodies in order to arrive more easily at the goal. However we realize that the theory of Machines in general has in actual practice become more complicated than in former times. Previously, this general theory was wrapped up with the consideration of moving bodies of whatever kind, but always such as are offered up in Nature. But now one must consider two kinds of body - those that actually exist in nature, and those in part stripped of their natural properties. However, it is clear that the first type of analysis [the one considering only actual bodies] is a particular case of this more general type of analysis. Thus this latter type of analysis is more complicated than the earlier one. In addition, although by similar hypotheses one can easily obtain laws of equilibrium & motion in any particular Machine - such as the lever, the winch, the screw - one ends up with an assembly of knowledge, in which the interconnections are not transparent, & [such links as are apparent are] only achievable by some kind of analogy. This [flaw] must necessarily ensue so long as we only consider configurations particular to this, that or the other specific Machine while [nevertheless] trying to demonstrate a property which is common to all. Since it is these common properties that we have in view in this Essay, it is clear that we will find them only by abstracting away the specificities of particular configurations. Let us start by simplifying the question, by ceasing to consider, in one system, bodies of différente nature ; rendons enfin aux Machines leur force d'inertie ; il nous ſera facile, après cela, d'en négliger la maſſe dans le réſultat ; nous ſerons maîtres d'y avoir égard ou non ; & partant, la ſolution du problême ſera auſſi générale, en même temps quelle ſera plus ſimple.

X. La ſcience des Machines en général ſe réduit donc à la queſtion ſuivante.

Connoiſſant le mouvement virtuel d'un ſyſtême quelconque de corps, (c'eſt à-dire celui que prendroit chacun de ces corps, s'il étoit libre) trouver le mouvement réel qui aura lieu l'inſtant ſuivant, à cauſe de l'action réciproque des corps, en les conſidérant tels qu'ils exiſtent dans la nature, c'eſt-à-dire comme doués de l'inertie commune à toutes les parties de la matiere.

XI. Or, cette queſtion renfermant évidemment toute la méchanique, il faut, pour procéder avec clarté, remonter juſqu'aux premieres loix que la nature obſerve dans la communication des mouvements : on peut les réduire en général à deux, que voici.

Loix fondamentales de l'équilibre & du mouvement.

Premiere loi. *La réaction eſt toujours égale & contraire à l'action.*

Cette loi conſiſte en ce que tout corps qui change ſon état de repos ou de mouvement uniforme & rectiligne, ne le fait jamais que par l'influence ou action de quelqu'autre corps auquel il imprime en même temps une quantité de mouvement égale & directement oppoſée à celle qu'il en reçoit ; c'eſt-à-dire que la vîteſſe different natures. Also, let us finally allow Machines their force of inertia - it will be [equivalent and just as] easy for us to [afterwards] disregard mass in the result[s]. We will [thereby] make ourselves the masters of whether we account for [inertia] or not; & in this way, the solution of the problem will at one and the same time gain in generality as it increases in simplicity.

X. The science of Machines in general therefore reduces to the following question[18]:
Knowing the virtual motion of an arbitrary system of bodies (that is to say, a system which takes on each [individual] *body as if that body was free), to determine the actual motion which will occur in the following instant, arising from the interactions between the bodies, and considering these bodies as they exist in nature, that is to say, as having the inertia common to all pieces of matter.*

XI. Now, this question evidently encompasses the whole of mechanics, and so it must, in order to proceed with clarity, reach back to the [elemental] first laws that nature observes in transfers of motion. One may reduce these [elemental laws], in general, to two, as below.

Fundamental laws of equilibrium & motion

First law. *The reaction is always equal & opposite to the action*[19].
This law asserts that any body which changes its state of rest or uniform motion in a straight line, never does this except by the influence or action of some other body to which it imparts at the same time a quantity of motion equal & directly opposite to that which it receives. That is to say, the speed

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qu'il prend réellement l'inſtant d'après, eſt la force réſultante de celle que lui imprime cet autre corps, & de celle qu'il auroit eue ſans cette derniere force Tout corps réſiſte donc à ſon changement d'état, & cette réſiſtance qu'on nomme force d'inertie, eſt toujours égale & directement oppoſée à la quantité de mouvement qu'il reçoit, c'eſt-à-dire à la quantité de mouvement qui, compoſée avec celle qu'il avoit immédiatement avant le changement, produit pour réſultante la quantité de mouvement qu'il doit réellement avoir immédiatement après; ce qui s'exprime encore en diſant que, dans l'action réciproque des corps, la quantité de mouvement perdue par les uns, eſt toujours gagnée par les autres, en même temps & dans le même ſens.

Seconde loi. *Lorſque deux corps durs agiſſent l'un ſur l'autre, par choc ou preſſion, c'eſt-à-dire en vertu de leur impénétrabilité, leur vîteſſe relative, immédiatement après l'action réciproque, eſt toujours nulle.*

En effet, on obſerve conſtamment que, ſi deux corps durs viennent à ſe choquer, leurs vîteſſes, immédiatement après le choc, eſtimées perpendiculairement à leur ſurface commune au point de contingence, ſont égales; de même que s'ils ſe tiroient par des fils inextenſibles, ou ſe pouſſoient par des verges incompreſſibles, leurs vîteſſes eſtimées dans le ſens de ce fil ou de cette verge, ſeroient néceſſairement égales: d'où il ſuit que leur vîteſſe relative, c'eſt-à-dire celle par laquelle ils s'approchent ou s'éloignent l'un de l'autre, eſt dans tous les cas nulle au premier inſtant.

De ces deux principes, il eſt aiſé de tirer les loix du choc des corps durs, & de conclure which it actually adopts in the following instant, is the resultant* of the speed imposed on it by the other body, & the speed it would have had without this imposition. All bodies, therefore, resist a change in their state, & this resistance, which we call the force of inertia[20], is always equal & directly opposite to the quantity of motion which the body receives - that is to say, the quantity of motion which, combined with that which it had immediately before the change, produces the resultant, actual quantity of motion which it must have immediately afterwards. This is expressed again in saying that, in the interaction between bodies, that quantity of motion lost by any one body, is always gained by the others, at the same time & in the same direction.

Second Law. *When two hard*[21] *bodies act on each other by collision or pressure - that is to say, they act on each other in virtue of their impenetrability - their relative speed, immediately after the interaction, is always null.*

In effect, one constantly observes that, if two hard [plastic] bodies collide, then their speeds - immediately after the collision, and estimated perpendicularly to their common surface and the point of contact - are equal. The same [is true] if they are pulled by inextensible threads, or pushed by incompressible rods - their speeds calculated along the line of the thread or rod will necessarily be equal. From this it follows that their relative speed - that is to say, that by which each approaches or moves away from the other - is in all examples null in the first instant.

From these two principles, it is easy to derive the laws of collision of hard [plastic] bodies, & to deduce

*See *Errata*.

par conſéquent les deux autres principes ſecondaires dont l'uſage eſt continuel en méchanique : ſavoir.

1°. *Que l'intenſité du choc ou de l'action qui s'exerce entre deux corps qui ſe rencontrent, ne dépend point de leurs mouvements abſolus, mais ſeulement de leur mouvement relatif.* 2°. *Que la force ou quantité de mouvement qu'ils exercent l'un ſur l'autre, par le choc, eſt toujours dirigée perpendiculairement à leur ſurface commune au point de contingence.*

XII. Des deux loix fondamentales, la *premiere* convient généralement à tous les corps de la nature, ainſi que les deux loix ſecondaires qu'on vient de voir, & la *ſeconde* eſt ſeulement pour les corps durs; mais comme ceux qui ne le ſont pas ont des degrés d'élaſticité différents, on ramene ordinairement les loix de leur mouvement à celles des corps durs qu'on prend pour terme de comparaiſon, c'eſt-à-dire qu'on regarde les corps élaſtiques, comme compoſés d'une infinité de corpuſcules durs ſéparés par de petites verges compreſſibles, auxquelles on attribue toute la vertu élaſtique de ces corps; de ſorte qu'on ne conſidere, à proprement parler, dans la nature, que des corps animés de différentes forces motrices : nous ſuivrons cette méthode, comme la plus ſimple; ainſi nous réduirons la queſtion à la recherche des loix qu'obſervent les corps durs, & nous en ferons enſuite quelques applications aux cas où les corps ſont doués de différents degrés d'élaſticité.

XIII. Cet Eſſai ſur les Machines n'étant point un Traité de méchanique, mon but n'eſt pas d'expliquer en détail ni de prouver les loix as a consequence the two other subordinate principles whose usage is continual in mechanics. To state them,

1st *That the intensity of the impact or action* [that is exerted] *between two bodies that meet, does not depend on their absolute motion but only on their relative motion.*

2nd *That the force, or quantity of motion which each exerts on the other, due to the impact, is always directed perpendicularly to their common surface at the point of contact.*

XII. Of these two fundamental laws, the *first* is generally applied to all bodies in nature, as well as to the two subordinate laws which we are going to see, while the *second* is only for hard [plastic] bodies. However, since bodies that are not hard [plastic] have various degrees of elasticity, one usually falls back again to the hard-[plastic]-body laws and uses these as a bodies of comparison. In other words, one regards elastic bodies as being composed of an infinity of little hard bodies separated by little compressible rods, and attributes to this all the elastic properties of these bodies. Thus one does not think that, properly speaking, in nature, bodies are [ever] animated by different motive forces. We will follow this method, as being the simpler; thus we will reduce the question to researching the laws observed by hard bodies, & we will then make some applications to the case where the bodies have different degrees of elasticity.

XIII. The Essay on Machines not being a Treatise on Mechanics, my goal is not to explain in detail nor to prove the fundamental laws

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fondamentales que je viens de rapporter; ce ſont des vérités que tout le monde ſent très-bien, dont on convient généralement, & qui ſe manifeſtent avec la plus grande évidence dans tous les phénomenes de la nature; cela me ſuffit pour remplir mon objet, qui eſt uniquement de tirer de ces loix, une méthode ſimple & exacte pour trouver l'état de repos ou de mouvement qui en réſulte dans un ſyſtême quelconque de corps, c'eſt-à-dire de préſenter ces mêmes loix ſous une forme qui puiſſe en faciliter l'application à chaque cas particulier.

XIV. Imaginons donc un ſyſtême quelconque de corps durs dont le mouvement virtuel donné ſoit changé par leur action réciproque en un autre qu'il s'agit de trouver; & pour embraſſer la queſtion dans toute ſa généralité, ſuppoſons que le mouvement puiſſe changer ſubitement, ou varier par degrés inſenſibles; enfin, comme il peut ſe rencontrer des point fixes, ou obſtacles quelconques, conſidérons-les tels qu'ils ſont en effet, c'eſt-à-dire comme des corps ordinaires faiſant eux-mêmes partie du ſyſtême propoſé, mais fixément arrêtés dans le lieu où ils ſont placés.

XV. Pour parvenir à la ſolution de ce problême, obſervons d'abord que toutes les parties du ſyſtême étant ſuppoſées parfaitement dures, c'eſt-à-dire incompreſſibles & inextenſibles, on peut viſiblement, quel qu'il ſoit, le regarder comme compoſé d'une infinité de corpuſcules durs, ſéparés les uns des autres, ou par de petites verges incompreſſibles, ou par de petits fils inextenſibles; car, lorſque deux corps ſe choquent, ſe pouſſent, ou tendent en général à ſe rapprocher l'un de l'autre ſans pouvoir le faire, of which I shall speak. These are truths which are well-known to all, are generally applicable, & that are manifest, with much evidence, in all natural phenomena. This is sufficient for my purpose [to be fulfilled], which is only to draw from these laws a simple & exact method of finding the state of rest or of motion that occurs in an arbitrary system of bodies, that is to say, to present these laws in a form that facilitates application to each [and any] particular case.

XIV. Let us imagine then an arbitrary system of hard [plastic] bodies whose given virtual motions will be changed by their interactions with another hard [plastic] body, this last body being the subject of our enquiry; & to grapple with the question in all its generality, let us suppose that the motion can change suddenly, or vary by insensible degrees. Finally, as we may encounter fixed points or some other constraints, let us treat these as they appear by their effects, that is to say, as if they were ordinary bodies, making up a part of the proposed system, but [constrained to remain] fixed where they are placed.

XV. In order to arrive at the solution of this problem, let us first observe that, in taking all parts of the system to be perfectly hard, that is to say, incompressible & inextensible, then we may evidently - whatever is the actual case - regard it [(the system)] as composed of an infinity of hard particles separated from each other either by little incompressible rods or by little inextensible threads.

This [assumption may be made] because, when two bodies collide, push against each other, or have a general tendency to approach each other - but without being able to do so,

(25)

à cauſe de leur impénétrabilité, on peut concevoir entre les deux une petite verge incompreſſible, & ſuppoſer que le mouvement ſe tranſmet de l'un à l'autre ſuivant cette verge; & de même ſi deux corps tendent à ſe ſéparer, on peut concevoir qu'ils ſont retenus l'un à l'autre par un petit fil inextenſible, ſuivant lequel ſe propage le mouvement : cela poſé, conſidérons ſucceſſivement l'action de chacun de ces petits corpuſcules ſur tous ceux qui lui ſont adjacents, c'eſt-à-dire examinons deux à deux tous ces petits corpuſcules ſéparés l'un de l'autre par une petite verge incompreſſible ou par un petit fil inextenſible, & voyons ce qui en doit réſulter dans le ſyſtême général de tous ces corpuſcules : pour cela nommons

m' & m'' Les maſſes des corpuſcules adjacents.

V' & V'' Les vîteſſes qu'ils doivent avoir l'inſtant ſuivant.

F' L'action de m'' ſur m', c'eſt-à-dire la force ou quantité de mouvement que le premier de ces corpuſcules imprime à l'autre.

F'' La réaction de m' ſur m''.

q' & q'' Les angles formés par les directions de V' & F', & par celles de V'' & F''.

Cela poſé, la vîteſſe réelle de m' étant V', cette vîteſſe eſtimée dans le ſens de F' ſera V' coſ q', de même la vîteſſe de m'' eſtimée dans le ſens de F'' ſera V'' coſ q''; de même la vîteſſe de m'' eſtimée dans le ſens de F'' ſera V'' coſ q''. Donc, puiſque par la ſeconde loi fondamentale, les corps doivent aller de compagnie, on aura V' coſ $q' + V''$ coſ $q'' = 0$ (A); donc par la premiere loi fondamentale on aura auſſi $F'\,V'$ coſ q' $+ F''\,V''$ coſ $q'' = 0$ (B); car ſi m' & m''

because of their impenetrability, then one may conceive that between the two bodies there is a small incompressible rod, & that the motion is transmitted along this rod. Similarly, if two bodies tend to separate, one may conceive that they are held to each other by a small inextensible thread along which the motion propagates. With this assumed, let us consider successively the action of each of these little particles on all those which are adjacent. That is to say, let us examine them, two by two, all these little particles separated from each other by a small incompressible rod or by a small inextensible string, & let us see what must happen in the general system, the totality of these particles. For this let us call

$m'\&m''$	The masses of the adjacent particles.
$V'\&V''$	The speeds which they must have the following instant.
F'	The action of m'' on m', that is to say, the force or quantity of motion that the first of these particles imparts to the other.
F''	The reaction of m' on m''
$q'\&q''$	The angles formed by the directions of V' & F', & by V'' & F''.

This posed, the real speed of m' being V', then in the direction of F' the speed will be $V'cosq'$. Similarly, the speed of m'' calculated in the direction of F'' will be $V''cosq''$. Then, since by the second fundamental law[22] the bodies must move together, one will have:

$$V'cosq' + V''cosq'' = 0 \qquad \text{(A)}$$

and then by the first fundamental law one will have also:

$$F'V'cosq' + F''V''cosq'' = 0 \qquad \text{(B)}$$

because if m' & m''

ſont mobiles tous les deux, il eſt clair, par cette loi, qu'on a $F' = F''$, donc à cauſe de l'équation (A) on aura auſſi l'équation (B); & ſi l'un des deux, m' par exemple, eſt fixe ou fait partie d'un obſtacle, on aura $V' \text{ coſ } q' = 0$; donc à cauſe de l'équation (A) on aura auſſi $V'' \text{ coſ } q'' = 0$; donc l'équation (B) aura encore lieu; donc cette équation (B) eſt vraie pour tous les corpuſcules du ſyſtême pris deux à deux : imaginant donc une pareille équation pour tous ces corps pris en effet deux à deux, & ajoutant enſemble toutes ces équations, ou ce qui revient au même, intégrant l'équation (B), on aura pour tout le ſyſtême;
$\int F' V' \text{ coſ } q' + \int F'' V'' \text{ coſ } q'' = 0$: c'eſt-à-dire que la ſomme des produits des quantités de mouvement que s'impriment réciproquement les corpuſcules ſéparés par chacun des petits fils inextenſibles, ou des petites verges incompreſſibles, de ces quantités, dis-je, multipliées chacune par la vîteſſe du corpuſcule auquel elle eſt imprimée, eſtimée dans le ſens de cette force, eſt égale à zero.

Cela poſé, abandonnant les dénominations précédentes, nommons

La maſſe de chacun des corpuſcules du ſyſtême. m

Sa vîteſſe virtuelle, c'eſt-à-dire celle qu'il prendroit s'il étoit libre. W

Sa vîteſſe réelle. V

La vîteſſe qu'il perd, de ſorte que W ſoit la réſultante de V & de cette vîteſſe. U

La force ou quantité de mouvement qu'imprime à m chacun des corpuſcules adjacents, & par l'entremiſe deſquels il reçoit évidemment tout le mouvement qui lui eſt tranſmis des différentes parties du are both mobile, it is clear, by this law, that one has $F' = F''$. Then due to equation (A) one will also have equation (B); & if one of the two (m' for example) is fixed, or made part of an obstacle, then one will have $V'cosq' = 0$. Following from which, and using equation (A), one will also have $V''cosq'' = 0$; so equation (B) still applies. Therefore this equation (B) is valid for all particles of the system taken two by two[23]. Imagining then a similar equation for all the bodies taken in effect two by two, & adding together all equations or, which comes to the same thing, integrating equation (B), one will have for the whole system[*]:

$$\sum F'V'cosq' + \sum F''V''cosq'' = 0.$$

That is to say, the sum of the products of the quantities of motion which are imparted reciprocally on the particles separated by each of the small inextensible threads, or the small incompressible rods , of these quantities, I say, multiplied each by the speed of the particle which is imparted, estimated [hereafter also calculated] in the direction of this force, is equal to zero.

That posed, abandoning the preceding nomenclature, let us call:

The mass of each of the particles of the system	m
Its virtual speed[24], that is to say, that which it would take if it were free	W
Its real speed	V
The speed which it loses, (so that W is the resultant of V & this speed)	U
The force or quantity of motion imparted to m by each adjacent particle, & by the intervention of which it evidently receives all the motion transmitted to it by different parts of the system	F

[*] The elongated letter $\int$ is used by Carnot both for sums and integrals. In cases where there is no differential and so a true summation occurs we use the $\sum$ symbol.

ſyſtême. F

L'angle compris entre les directions de W & V. X

L'angle compris entre les directions de W & U. Y

L'angle compris entre les directions de V & U. Z

L'angle compris entre les directions de V & F. q

On aura donc pour tout le ſyſtême $\int F V \text{coſ}\, q = 0$, ou $\int V F \text{coſ}\, q = 0$ (C); à préſent il faut obſerver que la vîteſſe de m avant l'action réciproque, étant W, cette vîteſſe eſtimée dans le ſens de V ſera $W \text{coſ}\, X$; donc $V - W \text{coſ}\, X$, eſt la vîteſſe gagnée par M dans le ſens de V; donc $m\ (V - W \text{coſ}\, X)$ eſt la ſomme des forces F qui agiſſent ſur m eſtimées chacune dans le ſens de V; donc $m V (V - W \text{coſ}\, X)$ eſt la même ſomme multipliée par V; or, à chaque molécule répond une pareille ſomme, & de plus la ſomme totale de toutes ces ſommes particulieres eſt viſiblement pour tout le ſyſtême $\int V F \text{coſ}\, q$; donc $\int m V (V - W \text{coſ}\, X) = \int V F \text{coſ}\, q$; ajoutant à cette équation l'équation (C), il vient $\int m V (V - W \text{coſ}\, X) = 0$ (D): mais W étant la réſultante de V & U, il eſt clair qu'on aura $W \text{coſ}\, X = V + U \text{coſ}\, Z$; ſubſtituant donc cette valeur de $W \text{coſ}\, X$ dans l'équation (D), elle ſe réduira à $\int m V U \text{coſ}\, Z = 0$ (E); *premiere équation fondamentale.*

XVI. Imaginons maintenant qu'au moment où le choc va ſe faire, le mouvement actuel du ſyſtême ſoit tout à coup détruit, & qu'on lui faſſe prendre à la place ſucceſſivement deux autres mouvements arbitraires, mais égaux &

The angle included between the directions of W & V	X
The angle included between the directions of W & U	Y
The angle included between the directions of V & U	Z
The angle included between the directions of V & F	q

One will then have for the whole system

$$\sum F V \cos q = 0 \quad \text{or} \quad \sum V F \cos q = 0 \tag{C}$$

At present it must be observed that the speed of m before the reciprocal action being W, this speed calculated in the direction of V will be $W \cos X$. Thus $V - W \cos X$ is the speed gained by M in the direction of V. Then $m(V - W \cos X)$ is the sum of the forces, F, which act on m each calculated in the same direction as V. Therefore, $mV(V - W cos X)$ is the same sum multiplied by V. However, a similar sum applies to each molecule, & moreover the total sum of all the individual sums is obviously for the whole system[25], $\sum V f \cos q$. Then $\sum mV(V - W \cos X) = \sum V F \cos q$. Adding to this equation the equation (C) , it becomes

$$\sum mV(V - W \cos X) = 0 \tag{D}$$

However, W being the resultant of V & U, it is clear that one will have $W \cos X = V + U \cos Z$. Substituting this value of $W \cos X$ in equation (D) it reduces to

$$\sum mVU \cos Z = 0 \tag{E}$$

This is the *first fundamental equation*[26].

XVI. Let us imagine now that the moment when the collision takes place, the actual motion of the system will all of a sudden be destroyed, & that we will instead make it take on, one after the other, two arbitrary motions – but nevertheless equal &

directement oppoſés l'un à l'autre, c'eſt-à-dire qu'on le faſſe partir ſucceſſivement de ſa poſition actuelle, avec deux mouvements tels qu'en vertu du ſecond, chaque point du ſyſtême ait au premier inſtant une vîteſſe égale & directement oppoſée à celle qu'il auroit eue en vertu du premier de ces mouvements : cela poſé, il eſt clair, 1°. que la figure du ſyſtême étant donnée, cela peut ſe faire d'une infinité de manieres différentes, & par des opérations purement geométriques ; c'eſt pourquoi j'appellerai ces mouvements *mouvements géométriques* : c'eſt-à-dire que *ſi un ſyſtême de corps part d'une poſition donnée, avec un mouvement arbitraire, mais tel qu'il eût été poſſible auſſi de lui en faire prendre un autre tout à fait égal & directement oppoſé ; chacun de ces mouvements ſera nommé mouvement* (1) *géométrique* ; 2°. je dis qu'en

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(1) Pour diſtinguer par un exemple très-ſimple les mouvements que j'appelle *géométriques*, de ceux qui ne le ſont pas, imaginons deux globes qui ſe pouſſent l'un l'autre, mais du reſte libres & dégagés de tout obſtacle ; imprimons à ces globes des vîteſſes égales & dirigées dans le même ſens ſuivant la ligne des centres ; ce mouvement eſt *géométrique*, parce que les corps pourroient de même être mus en ſens contraire avec la même vîteſſe, comme il eſt évident : mais ſuppoſons maintenant qu'on imprime à ces corps des mouvements égaux & dirigés dans la ligne des centres, mais qui au lieu d'être, comme précédemment, dirigés dans le même ſens, tendent au contraire à les éloigner l'un de l'autre ; ces mouvements, quoique poſſibles, ne ſont pas ce que j'entends par *mouvements géométriques* ; parce que ſi l'on vouloit faire prendre à chacun de ces mobiles une vîteſſe égale & contraire à celle qu'il reçoit dans ce premier mouvement, on en ſeroit empêché par l'impénétrabilité des corps.

directly opposite to each other–. That is to say, we make it [the system] depart from its actual position by [taking on] two successive motions such that each point in the system ends up with a velocity equal & opposite to that which it would have had from the first of these motions alone. That posed, it is clear

1st that the configuration of the system being given, [nevertheless] it can occur in an infinite variety of different ways, & by purely geometric operations. This is why I will call these motions *geometric motions*[27]. That is to say, *if a system of bodies moves from a given position, with an arbitrary motion, but such that it was possible also to make another absolutely equal & directly opposite motion, then each of these motions will be called a geometric motion* (I); 2nd I say that,

(1) To distinguish, by a very simple example, between the motions which I call *geometric* from those which are not, let us imagine two globes which push against each other, but in all other respects remain free & unconstrained. Let us impose on these globes equal speeds in the same direction, along the line joining the [globe-]centres. Such a motion is *geometric* because the bodies could evidently be moved in the opposite direction. Let us suppose that one again imposes on these bodies equal motions directed along the line of their centres but instead of, as before, approaching, the bodies, on the contrary, tend to move away from each other. These motions, although possible, are not what I intend by *geometric motions* because if one wants to give to each of these moveable bodies a speed equal & opposite to that which it receives in the first motion, one will be stopped by the impenetrability of the bodies.

vertu de ce mouvement géométrique, les corpuſcules voiſins qui peuvent être cenſés ſe

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De même ſi deux corps ſont attachés aux extrêmités d'un fil inextenſible, & qu'on faſſe prendre au ſyſtême un mouvement arbitraire, mais tel que la diſtance des deux corps ſoit conſtamment égale à la longueur du fil, ce mouvement ſera *géométrique*, parce que les corps peuvent prendre un pareil mouvement dans un ſens tout contraire; mais ſi ces mobiles ſe rapprochent l'un de l'autre, le mouvement n'eſt point *géométrique*, parce qu'ils ne pourront prendre un mouvement égal & contraire, ſans s'éloigner l'un de l'autre; ce qui eſt impoſſible, à cauſe de l'inextenſibilité du fil.

En général il eſt évident que, quelle que ſoit la figure du ſyſtême, & le nombre des corps, ſi on peut lui faire prendre un mouvement tel qu'il n'en réſulte aucun changement dans la poſition reſpective des corps, ce mouvement ſera *géométrique;* mais il ne s'enſuit pas de là qu'il n'y ait aucun autre moyen de ſatisfaire à cette condition, comme nous allons le montrer par quelques exemples.

Imaginons un treuil à la roue & au cylindre duquel ſoient attachés des poids ſuſpendus par des cordes; ſi l'on fait tourner la machine, de maniere que le poids attaché à la roue deſcende d'une hauteur égale à ſa circonférence, tandis que celui du cylindre montera d'une hauteur égale à la ſienne, ce mouvement ſera *géométrique*, parce qu'il eſt également poſſible de faire deſcendre le poids attaché au cylindre d'une hauteur égale à ſa circonférence, tandis que le poids attaché à la roue monteroit d'une hauteur égale à la ſienne; mais ſi tandis qu'on fera deſcendre le poids attaché à la roue d'une hauteur égale à ſa circonférence, on faiſoit monter le poids attaché au cylindre d'une hauteur plus grande que ſa circonférence, le mouvement ne ſeroit pas *géométrique*, parce que le mouvement égal & contraire ſeroit viſiblement impoſſible.

Si pluſieurs corps ſont attachés aux extrêmités de différents fils réunis par les autres extrêmités à un même nœud, & qu'on faſſe prendre au ſyſtême un mouvement tel que chacun des corps reſte conſtamment éloigné du nœud d'une même quantité égale à la lonin virtue of this geometric motion, then neighbouring particles that are imagined as being

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Similarly, if two bodies are attached to the ends of an inextensible thread, & we give to the system an arbitrary motion – but a motion such that the distance of the two bodies is constantly equal to the length of the thread – then this motion will be *geometric* because the bodies could have made an equal motion in the opposite direction. However if these moveable bodies approach each other, the motion is not *geometric*, because they could not take on equal & opposite motions without separating from each other - which is not possible because of the inextensibility of the thread.

In general it is evident that, regardless of the configuration of the system, & the number of bodies, if one can make a motion such that there results no change in the respective positions of the bodies, then this motion is *geometric*. But [note that] it does not follow that there is no other means of satisfying this condition, as we are going to show by some examples.

Let us imagine a winch, and to the wheel & cylinder of this winch are attached weights suspended by ropes. If one makes the Machine turn in such a way that the weight attached to the wheel descends by a height equal to the wheel-circumference, while the weight [or in this case, also mass or body] at the cylinder rises by a height equal to the cylinder-circumference, then this motion will be *geometric* [and this is] because it is equally possible to make the weight [mass] attached to the drum descend by a height equal to the drum-circumference, while the mass attached to the wheel would rise by a height equal to the wheel-circumference. But if, while one makes the weight attached to the wheel descend by a height equal to the wheel-circumference, one makes the weight attached to the drum rise to a height *greater* than the drum-circumference, then the motion will not be geometric, because the equal & opposite motion is obviously impossible.

If several bodies are attached to the ends of different threads, with the other thread-ends all united together in one knot, & then one gives to the system a motion such that each of these bodies stays at a constant distance from the knot (& whereby each such fixed distance is equal to the length pouſſer par une verge, ou ſe tirer par un fil, ne ſe rapprocheront ni ne s'éloigneront l'un de l'autre au premier inſtant, c'eſt-à-dire qu'au premier inſtant de ce mouvement géométrique, la vîteſſe relative de ces corpuſcules voiſins ſera nulle; en effet, il eſt clair, premiérement, que ſi *m* eſt ſéparé d'un corpuſcule voiſin par une verge incompreſſible, il ne pourra s'en rappro-

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gueur du fil auquel il eſt attaché, ce mouvement ſera *géométrique*, quand même les différents corps ſe rapprocheroient les uns des autres : mais ſi quelqu'uns d'eux ſe rapprochoient du nœud, le mouvement ne ſeroit plus *géométrique*, parce que les fils étant ſuppoſés inextenſibles, le mouvement égal & contraire ſeroit viſiblement impoſſible.

Si deux corps ſont attachés aux extrêmités d'un fil dans lequel ſoit enfilé un grain mobile, il ſuffira, pour que le mouvement ſoit *géométrique*, que la ſomme des diſtances du grain mobile à chacun des deux autres corps, ſoit conſtamment égale à la longueur du fil; de ſorte que ſi ces deux corps ſont fixes, le grain mobile ne ſortira pas d'une courbe elliptique.

Si un corps ſe meut ſur une ſurface courbe, par exemple dans la concavité d'une calotte ſphérique, le mouvement ſera *géométrique*, tant que le corps ſe mouvra tangentiellement à la ſurface; mais s'il s'en écarte, le mouvement ceſſera d'être *géométrique*, parce que le mouvement égal & contraire eſt viſiblement impoſſible.

D'après tout cela, il eſt évident, que quoiqu'en faiſant prendre à un ſyſtême un mouvement *géométrique*, les différents corps de ce ſyſtême puiſſent ſe rapprocher les uns des autres, cependant on peut dire que les corpuſcules voiſins, conſidérés deux à deux, ne tendent au premier inſtant ni à ſe rapprocher ni à s'éloigner, comme je le prouve au long dans le texte : les corps n'exercent donc aucune action les uns ſur les autres, en vertu d'un pareil mouvement; ces mouvements ſont donc abſolument indépendants des regles de la dynamique; & c'eſt pour cette raiſon que je les ai appellés *géométriques*.

pushed by a rod, or pulled by a string, will neither approach or recede from each other in the first instant. In other words, in the first instant of the geometric motion, the relative speed of these particles will be zero[28]. In effect it is clear, firstly, that if m is separated from a neighbouring particle by an incompressible rod, it cannot approach

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of the corresponding thread, then this motion will be *geometric* even though the different bodies approach each other. But if some of them approach the knot, the motion will not be *geometric* because the threads, being supposed inextensible, the equal & opposite motion would obviously be impossible.

If two bodies are attached to the ends of a string on which is threaded a movable granule, it will suffice, for the motion to be *geometric*, that the sum of the distances of the mobile granule from the two bodies is constantly equal to the length of the string. Thus if the two bodies are fixed, the mobile granule will be constrained to stay on an elliptic curve.

If a body moves on a curved surface, for example in the concavity of a spherical cap, the motion will be *geometric* for as long as the body moves tangentially to the surface. But if it moves away [from the surface], the motion will cease to be *geometric*, because the equal & opposite motion is obviously impossible.

From all this, it is evident that, what with giving a system a *geometric* motion, the different bodies of the system may [nevertheless] approach each other. However one notes that neighbouring particles, considered two by two, tend in the first instant neither to approach or recede, as I prove through the course of the text. The bodies, therefore, do not exert any action on each other, in virtue of such a motion. These motions are then absolutely independent of the rules of dynamics, & it is for this reason that I have called them *geometric*.

(31)

cher; & que s'il en eſt ſéparé par un fil inextenſible, il ne pourra s'en éloigner : ſecondement, je dis que s'il en eſt ſéparé par une verge incompreſſible, il ne pourra non-plus s'en éloigner; car s'il s'en éloignoit, il eſt clair qu'en vertu du mouvement égal & directement oppoſé, lequel eſt auſſi poſſible, par hypotheſe, il s'en rapprocheroit; ce qui ne ſe peut à cauſe de l'incompreſſibilité de la verge; par la même raiſon enfin, il eſt viſible que ſi c'eſt un fil qui ſépare *m* du corpuſcule voiſin, il ne pourra s'en rapprocher, puiſqu'alors il ſeroit poſſible qu'il s'en éloignât par un mouvement égal & directement oppoſé; or, cela ne ſe peut, à cauſe de l'inextenſibilité du fil; donc, quel que ſoit le mouvement géométrique imprimé au ſyſtême, la vîteſſe relative de tous ces corpuſcules voiſins qui agiſſent les uns ſur les autres, pris deux à deux, ſera nulle au premier inſtant : cela poſé, nommons *u* la vîteſſe abſolue qu'aura *m* dans le premier inſtant, en vertu de ce mouvement géométrique, & *z* l'angle compris entre les directions de *u* & *U*; il eſt clair que les corpuſcules *m* ne tendront point à ſe rapprocher ni à s'éloigner les uns des autres, en vertu des vîteſſes *u*, ſi on les ſuppoſe animés en même temps de ces vîteſſes *u* & des vîteſſes *U*; ils ne tendront pas à ſe rapprocher ou à s'éloigner davantage que s'ils étoient animés des ſeules vîteſſes *U*; donc l'action réciproque exercée entre les différentes parties du ſyſtême ſera la même, ſoit que chaque molécule ſoit animée de la ſeule vîteſſe *U*, ou des deux vîteſſes *u* & *U*; mais ſi chaque molécule étoit animée de la ſeule vîteſſe *U*, il y auroit viſiblement équilibre; donc ſi elle eſt animée à la fois des deux vîteſſes

& if it is separated by an inextensible thread, it cannot move away. Secondly, I say that if it is separated by an incompressible rod, it cannot move away either because, if it moved away, it is clear that in virtue of an equal & opposite motion, which by hypothesis is also possible, it could get closer - but this cannot happen due to the incompressibility of the rod. Finally, for the same reason, it is obvious that if it is a thread which separates *m* from its neighbouring particle, it cannot get closer, since then it would be possible to move away by an equal & opposite motion - but this cannot happen because of the inextensibility of the thread. [Thus,] regardless of the geometric motion imparted to the system, the relative speeds of all the neighbouring particles which act on each other, taken two-by-two, will be zero in the first instant. That posed[29], let us name u as the absolute speed which m will have in the first instant in virtue of the geometric motion & z the angle between the directions of u & U. It is clear that the particles m will not tend to move closer together or further apart in virtue of the speeds u, [then][*] if one supposes them animated at the same time with the speeds u & the speeds U. They will not tend to get closer or move away more than if they were animated with the same speeds U. Thus the reciprocal action exercised between the different parts of the system will be the same, either each molecule will be animated by the same speed U, or the two speeds u & U. However, if each molecule was animated only by the speed U, there is obviously equilibrium. Therefore if it is animated at one and the same time by two speeds,

[*] See *Errata*.

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U & *u*, ou d'une vîteſſe unique qui en ſoit la réſultante, *U* ſera encore la vîteſſe perdue par *m*; & partant, *u* ſera la vîteſſe réelle, après l'action réciproque : donc, par la même raiſon qu'on a eu la premiere équation fondamentale (E), on aura auſſi $\int m\,u\,U \cos z = 0$ (F); *ſeconde équation fondamentale.*

Il eſt bien facile à préſent de réſoudre le problême que nous nous ſommes propoſés, car l'équation précédente devant avoir lieu, quelle que ſoit la valeur de *u*, & ſa direction, pourvu que le mouvement auquel elle ſe rapporte ſoit géométrique; il eſt clair qu'en attribuant ſucceſſivement à cette indéterminée différentes valeurs & directions arbitraires, on obtiendra toutes les équations néceſſaires entre les quantités inconnues, d'où dépend la ſolution du problême, & des quantités ou données ou priſes à volonté.

XVII. Pour achever de mettre cette ſolution dans tout ſon jour, il ſuffira d'en donner un exemple :

Suppoſons donc que tout le ſyſtême ſe réduiſe à un aſſemblage de corps liés entre eux par des verges inflexibles, de ſorte que toutes les parties du ſyſtême ſoient forcées de conſerver toujours leurs mêmes poſitions reſpectives; mais qu'il n'y ait aucun point fixe ou obſtacle quelconque; l'équation (F) va nous donner la ſolution de ce problême, en attribuant ſucceſſivement à *u* différentes valeurs & différentes directions.

1°. Comme les vîteſſes *u* ne ſont aſſujetties à aucune condition, ſinon que le mouvement du ſyſtême, en vertu duquel les corpuſcules *m* ont ces vîteſſes, ſoit *géométrique*, il eſt évident

U & u, or by one single speed which would be the resultant, then U will again be the speed lost by m, &, from this, u will be the actual speed after reciprocal action. Therefore, for the same reason that one arrived at the first fundamental equation (E), one will also arrive at

$$\sum muU \cos z = 0 \qquad \text{(F);}$$

the *second fundamental equation*.

It is very easy now to solve the problem that we proposed because the preceding equation must hold whatever the magnitude & direction of u, provided that the motion to which it relates is geometric. It is clear that in successively attributing to this variable [u] different magnitudes & arbitrary directions, one will obtain all the necessary equations between the unknown quantities - on which the solution depends – & the quantities, whether given or chosen at will.

XVII. In order to put this solution in plain view, for all to see, it will suffice to give an example.

Let us suppose then that the whole system reduces to a group of bodies joined by inflexible rods, in such a way that all parts of the system are forced to retain their respective positions – but with no fixed point or constraint of any kind –. The equation (F) will give us the solution to this problem, in successively assigning to u different values & different directions.

1st. As the velocities, u, are not subject to any conditions – except that the motion of the system, in virtue of which the particles m have their speeds, should be *geometric* –, it is evident

dent que nous pouvons d'abord les ſuppoſer toutes égales & paralleles à une même ligne donnée; alors u étant conſtante, ou la même pour tous les points du ſyſtême, l'équation (F) ſe réduira à $\int m U \cos z = 0$; ce qui nous apprend, que la ſomme des forces perdues par l'action réciproque des corps, dans le ſens arbitraire de u, eſt nulle, & que par conſéquent celle qui reſte eſt la même que ſi chaque corps eût été libre : *principe très-connu.*

2°. Imaginons maintenant qu'on faſſe tourner tout le ſyſtême au tour d'un axe donné, de ſorte que chacun des points décrira une circonférence autour de cet axe, & dans un plan qui lui ſera perpendiculaire; ce mouvement eſt viſiblement géométrique; donc l'équation (F) a lieu; mais alors, en nommant R la diſtance de m à l'axe, il eſt clair qu'on a $u = AR$, A étant la même pour tous les points; donc l'équation (F) ſe réduit à $\int m R U \cos z = 0$; c'eſt-à-dire que la ſomme des moments des forces perdues par l'action réciproque, relativement à un axe quelconque, eſt nulle : *autre principe très-connu.*

3°. Nous pourrions encore attribuer à u d'autres valeurs; mais cela ſeroit inutile & meneroit à des équations déjà renfermées dans les précédentes; car on ſait que celles-ci ſuffiſent pour réſoudre la queſtion, ou du moins pour la réduire à une affaire de pure géométrie.

Remarque I.

XVIII. Le but qu'on ſe propoſe, en imprimant un mouvement géométrique, eſt de changer l'état du ſyſtême, ſans cependaut altérer that we can first suppose them [the velocities] all equal & parallel to the same given line. Then u being constant, or the same for all points of the system, the equation (F) reduces to $\sum mU\cos z = 0$. This teaches us that the sum of the forces lost because of the interaction of bodies, taken in the arbitrary direction of u, is zero, & that in consequence what remains is the same as if each body had been free – a *well-known principle*[30].

2nd. Let us imagine now that one makes the system turn around a given axis, so that each of the points describes a circumference around this axis, & in the plane perpendicular to it; this motion is obviously geometric; then the equation (F) holds. But then, in naming R the distance of m to the axis, it is clear that we have $u = AR$, A [angular speed] being the same for all points. Then the equation (F) reduces to $\sum mRU cos z = 0$; that is to say that the sum of the moments of forces lost by mutual interactions, relative to any axis, is null: *another well-known principle*[31].

3rd. We can again assign other values[32] to u, but that would serve no purpose & would lead to equations already contained within the preceding [equations]. [And anyhow] one knows that these [previous equations] [already] suffice to resolve the question, or at least to reduce it to a matter of pure geometry.

Remark I.

XVIII. The goal that we have proposed, in imparting a geometric motion, is to change the state of the system, without however altering

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l'action réciproque des corps qui le composent; afin de se procurer par-là des rapports entre ces forces exercées & inconnues, & les vitesses arbitraires que prennent les corps, en vertu de ces différents mouvements géométriques; mais il faut remarquer qu'il y a un cas où les mouvements géométriques ne sont pas les seuls qui puissent remplir le même objet, & où quelques autres mouvements peuvent s'employer de même, pour tirer de l'équation générale (F) des équations déterminées; ce cas arrive lorsque ces autres mouvements, sans être absolument géométriques, le deviennent cependant, en supprimant seulement quelques-uns des petits fils ou verges que nous avons imaginés interposés entre les particules adjacentes du systême, lors dis-je que ces fils ou verges qui étoient supposés transmettre le mouvement d'un corpuscule à l'autre n'en transmettent en effet aucun; c'est-à-dire lorsque la tension de quelques-uns de ces fils, ou la pression de quelques-unes de ces verges, est égale à zero : car alors, en supprimant ces fils & verges, dont les tensions ou pressions sont nulles, on ne change évidemment rien du tout à l'action réciproque des corps, & cependant il est possible qu'on rende par-là le systême susceptible de quelques mouvements géométriques, qui ne pourroient avoir lieu sans cela : rien n'empêche donc alors qu'on ne regarde comme anéantis ces fils & verges, puisqu'ils n'influent en rien sur l'état du systême, & qu'on n'emploie par conséquent comme géométriques, les mouvements qui, sans l'être effectivement, le deviennent cependant par cette suppression.

De plus, lorsque deux corps sont contigus l'un à l'autre, c'est la même chose évidemment de supprimer la petite verge que nous avons the mutual interaction of the bodies which compose it, and then to derive by this means the relations between the applied & unknown forces, & the arbitrary speeds that the bodies must adopt in virtue of the different geometric motions. However, it must be said that there is a case where the geometric motions are not the only ones which can fulfill this objective, & where some other motions can equally well be used in order to draw out from the general equation (F) the determined equations. This case arises when these other motions, without being absolutely geometric, nevertheless do become so, in removing just a few of the little threads or rods which we have imagined interposed between the adjacent particles of the system. From which I declare that the threads or rods which we assumed had the effect of transmitting the motion from one particle to another in fact have no such effect, [because, that is to say,] when the tension of some of these threads, or the pressure of some of these rods, is equal to zero, then, in removing these threads & rods – the ones having zero tension or pressure – one evidently changes nothing at all in the interaction between these bodies. However it is [nevertheless] possible that, by this means [the neglect of some threads & rods], one makes the system susceptible to certain geometric motions which could not have taken place otherwise. Nothing prevents us therefore from regarding these threads & rods as having been annihilated, since they have no influence on the state of the system. Consequently, we can treat as geometric those motions which, without actually being so, nevertheless become geometric as a result of this suppression [of the rods & threads].

Furthermore, when two bodies are contiguous to each other it is evidently the same thing as removing the little rod that we have imaginée interpoſée entre deux, pour les empêcher de ſe rapprocher, ou de ſuppoſer que ces corps ſoient perméables l'un à l'autre, c'eſt-à-dire qu'ils puiſſent ſe pénétrer auſſi facilement que l'eſpace vuide eſt pénétré par tous les corps ; d'où il ſuit évidemment qu'en général, dans un ſyſtême quelconque de corps agiſſant les uns ſur les autres, ſoit immédiatement, ſoit par des fils & verges, c'eſt-à-dire par l'entremiſe d'une Machine quelconque, s'il ſe trouve quelque fil, verge ou autre partie quelconque de la Machine qui n'exerce aucune action ſur les corps qui lui ſont appliqués, c'eſt-à-dire qui puiſſe être anéantie, ſans qu'il en réſulte aucun changement dans l'action réciproque de ces corps, on pourra traiter comme géométriques tous les mouvements qui, ſans l'être effectivement, le deviendroient par cette ſuppreſſion, de même que ceux qui le deviendroient auſſi, en regardant comme librement perméables l'un à l'autre ceux des corps entre leſquels il ne s'exerce aucune preſſion, quoiqu'ils ſoient adjacents. Voici maintenant quelle eſt l'utilité de cette obſervation.

Si lorſqu'on entreprend la ſolution de quelque problême, on ſait d'avance que telle partie de la Machine n'exerce aucune action ſur les autres parties du ſyſtême, on pourra ſuppoſer que cette partie de Machine eſt totalement anéantie, & chercher le mouvement du ſyſtême d'après cette hypotheſe, c'eſt-à-dire en traitant comme géométriques tous les mouvements qui le deviendroient réellement par cette ſuppoſition ; & de même, ſi l'une des conditions données du problême, eſt que tels corps adjacents n'exercent l'un ſur l'autre aucune preſſion, on exprimera cette condition, en regardant ces imagined interposed between the two – to prevent them getting closer – or to suppose that these bodies are permeable to each other, that is to say, they can penetrate each other as easily as empty space is penetrated by all bodies. From this it evidently follows that in general, in whatever system of bodies that act on each other, either immediately or by threads & rods – that is to say via some sort of Machine – then if it is found that some thread, rod, or any other part of the Machine exerts no action on the bodies which are applied to it – that is to say [these threads] can be disregarded – [and all this] without resulting in any change in the mutual interaction of the bodies, then one can treat as geometric all those motions which, without actually being geometric, would become so by this annihilation. Similarly, for those [motions] which would also become so, in taking as freely permeable those bodies exerting no pressure on each other despite the fact that they are adjacent. Here now, is the usefulness of this observation.

If, when one undertakes the solution [33] of some problem, one knows in advance that a certain part of the Machine exerts no action whatsoever on other parts of the system, then one can imagine this certain part of the Machine to be totally annihilated, & search for the motion of the system that follows on from this hypothesis, (that is to say, in treating as geometric all the motions which really become it by this supposition). Similarly, if one of the given conditions of the problem is that such adjacent bodies exert no pressure on each other, one will express this condition by regarding these

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deux corps comme perméables l'un à l'autre ; c'eſt-à-dire en traitant comme géométriques les mouvements qui le deviendroient en effet par cette ſuppoſition.

Mais s'il arrivoit qu'on ignorât ſi cette preſſion eſt réelle ou nulle, il faudroit chercher le mouvement du ſyſtême, en ſuppoſant d'abord à volonté l'un ou l'autre ; on ſuppoſera donc, par exemple, que cette preſſion eſt réelle ; alors ſi en cherchant, d'après cette hypotheſe, la valeur de cette preſſion, on la trouve réelle & poſitive, on conclura que l'hypotheſe eſt légitime, & le réſultat exact ; ſinon on ſeroit aſſuré que la preſſion en queſtion eſt nulle, & qu'on peut par conſéquent traiter comme géométriques les mouvements qui le deviendroient en effet, ſi les deux corps dont il s'agit étoient librement perméables l'un à l'autre.

De même, s'il y avoit dans le ſyſtême une Machine, un fil par exemple, & qu'on ignorât ſi la tenſion de ce fil eſt nulle ou réelle, on pourroit faire le calcul, en ſuppoſant d'abord qu'il y a réellement tenſion ; alors, ſi l'on trouve pour la valeur de cette tenſion une quantité réelle & poſitive, on conclura que la ſuppoſition étoit légitime, & que le réſultat eſt exact ; ſinon il faudra recommencer le calcul, en partant de la ſuppoſition contraire, c'eſt-à-dire en ſuppoſant que la tenſion du fil ſoit égale à zero ; ce qui ſe fera, en ſuppoſant le fil anéanti, c'eſt-à-dire en traitant comme géométriquesles mouvements qui le ſeroient effectivement, ſi le fil en queſtion n'exiſtoit pas.

Il ſuit delà que pour tirer dans chaque cas particulier de l'équation générale (F), toutes les équations déterminées qu'elle peut donner, il two bodies as permeable to each other; that is to say, in treating the motions as if they were geometric, they in effect become geometric by this very supposition[34].

But if it happens that we do not know whether this pressure is real or null, it is necessary to search for the [appropriate] motion of the system by freely imagining one or the other case. We will suppose then, for example, that this pressure is real; then if by assuming this hypothesis, and in searching the value of this pressure, one finds that it is real & positive, one will conclude that the hypothesis is correct & the result exact. If not, one will be assured that the pressure in question is null, & that one may in consequence treat as geometric the motions which would in effect become geometric, so long as the two bodies were freely permeable, each by the other.

Similarly, if there is in the system a Machine – a thread for example – & one does not know if the tension of this thread is null or real, then one can carry out the calculation by supposing, first, that there is actual tension. Then, if one finds from this that the value of the tension is a real & positive quantity, one will conclude that the supposition was correct, & the result exact. Otherwise, one must restart the calculation with the contrary supposition, that is to say, supposing that the tension of the thread is equal to zero. This would be the case if we supposed the thread to be annihilated. In other words, we treat the motions as geometric, which they would be if the thread in question did not exist.

It follows from this that to draw out in each particular case of [applying] the general equation (F), all the determined equations that it can give, one

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faut, 1°. faire prendre au ſyſtême tous les mouvements géométriques dont il eſt ſuſceptible; 2°. traiter encore comme tels tous ceux qui le deviendroient, en ſupprimant quelque Machine ou partie de Machine, dont l'action ſur le reſte du ſyſtême ſoit nulle, ou en regardant comme perméables l'un à l'autre les corps entre leſquels, quoiqu'adjacents, il ne s'exerce aucune preſſion; 3°. enfin, ſi l'on eſt en doute que tel fil, verge ou partie quelconque de Machine ait ou non une action réelle ſur les autres parties du ſyſtême, ou qu'il y ait preſſion réelle entre deux corps adjacents, il faut éclaircir d'abord ce doute, en ſuppoſant la choſe en queſtion, comme on l'a expliqué ci-deſſus, & traitant comme géométriques les mouvements que ces ſuppoſitions auront fait découvrir pouvoir être pris pour tels.

D'après cette remarque, il paroît donc à propos d'étendre le nom de *géométriques* à tous les mouvements, qui ſans l'être effectivement, le deviennent, en ſupprimant quelque Machine ou partie de Machine qui n'influe en rien ſur l'état du ſyſtême, & en regardant auſſi comme parfaitement perméables l'un à l'autre les corps qui ſe touchent, ſans qu'il s'exerce entre eux aucune preſſion, c'eſt-à-dire ſans qu'il y ait autre choſe qu'une ſimple juxtapoſition: ainſi nous comprendrons dorénavant tous ces mouvements, ſous le nom commun de *mouvements géométriques*, puiſqu'en effet ils ſe déterminent également par des opérations purement géométriques, & s'emploient de même pour tirer de l'équation générale (F), des équations déterminées, attendu que la propriété générale &

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must, 1st make the system undertake all the *geometric* motions it is susceptible to; 2nd treat as such all those motions which will become so [become *geometric*]) by removing whatever Machine or parts of a Machine that have no action on the rest of the system, or by regarding as inter-permeable all those bodies which, despite being adjacent, exert no pressure on each other; 3rd finally, if one is in doubt as to whether [such and such] a thread, rod, or any part of the Machine, has a real action on the other parts of the system, or whether a real pressure exists between two adjacent bodies, it is first necessary to resolve this doubt, by supposing the thing in question (as one has explained above), & then treating as *geometric* the motions that these suppositions will have made it possible to treat as such.

After this remark, it seems, then, appropriate to extend the name of *geometric* to all those motions that, without actually being *geometric*, become so in the removal of whatever Machine or part of a Machine that has no influence on the state of the system; & also, by regarding as perfectly inter-permeable all those bodies which are touching but without exerting any pressure, that is to say, without there being anything but simple juxtaposition. In this way, we will, from now on, understand all the motions [as coming] under the common name of *geometric motions*, as they are all, in effect, equally determined by purely geometric operations, & are all employed in the same fashion for drawing out of the general equation (F) the determined equations, appreciating the fact that the general &

excluſive (1) de ces mouvements, eſt de changer l'état du ſyſtême, ſans altérer l'action réciproque des corps qui le compoſent; cependant, pour laiſſer entre eux quelque diſtinction, on peut appeller les premiers, *mouvements géométriques abſolus*, & les autres, *mouvements géométriques par ſuppoſition*; mais lorſque je parlerai ſimplement de mouvements géométriques, ſans les déſigner autrement, on entendra indifféremment les uns & les autres.

Cela poſé, puiſque nous avons expliqué comment on peut déterminer, ſans le ſecours d'aucun principe méchanique, tous les mouvements géométriques dont un ſyſtême donné eſt ſuſceptible, il s'enſuit que le problême général que nous nous étions propoſé, ſe trouve entiérement réduit par l'équation générale (F), à des opérations purement géométriques & analytiques; il faut cependant obſerver qu'il ne ſuffit pas d'attribuer aux arbitraires u, différentes valeurs, mais qu'il faut auſſi leur attribuer différents rapports ou directions; car ſi l'on ſe contentoit de leur attribuer différentes valeurs,

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(1) Il eſt évident que cette propriété appartient ſucceſſivement aux mouvements que j'appelle ici géométriques, & que ce ſeroit par conſéquent en avoir une idée très-fauſſe, que de les regarder comme des mouvements ſimplement poſſibles, c'eſt-à-dire compatibles avec l'impénétrabilité de la matiere: car, ſuppoſons par exemple que tout le ſyſtême ſe réduiſe à deux globes adjacents, & ſe pouſſant l'un l'autre; il eſt clair que ſi l'on force ces corps à ſe ſéparer, ou à ſe mouvoir en ſens contraire l'un de l'autre, ce mouvement ne ſera pas impoſſible, mais qu'en même temps les corps ne peuvent le prendre ſans ceſſer d'agir l'un ſur l'autre: ce mouvement n'eſt donc pas propre à remplir le but qu'on ſe propoſe, qui eſt de ne rien changer à l'action réciproque des corps.

exclusive (I) property of these motions is to change the state of the system – but without altering the mutual interaction between the bodies that make up the system. However, in order to make some distinction between them, one may call the first – *absolute geometric motions*, & the others – *motions geometric by supposition*. But, when I shall speak simply of geometric motions, without any other designation, one will understand [that I refer to] either kind of motion – with no distinction made between one or the other.

That posed, since we have explained how one can determine, without the help of any mechanical principle, all the geometric motions to which a given system is susceptible, it follows that the general problem which we have proposed is found to be completely reduced – by use of the general equation (F) – to purely geometric & analytic operations. It must however be observed that it is not enough to assign different values to the variable u, it is [also] necessary to assign different relations or directions. This is because, if one was merely content to assign different values,

(I) It is evident that this property belongs exclusively [See the *Errata* for this correction] to motions which I call here geometric, & therefore it would be a very false idea if one regarded them simply as possible motions, that is to say, [motions] compatible with the impenetrability of matter. For, let us suppose for example, that the whole system reduces to two adjacent globes, each pressing on the other. It is clear that if one forces the globes to separate, or move way from one another, this motion will not be impossible but at the same time the bodies cannot adopt it without ceasing to act upon each other: Such a motion is therefore not qualified to fulfill the proposed goal, which is *not* to change the mutual interaction between the bodies.

ſans rien changer aux rapports ni aux directions, on obtiendroit différentes équations toutes juſtes à la vérité, mais qui ſe réduiroient évidemment à la même, en les multipliant par différentes conſtantes.

Remarque II.

XIX. Comme il n'eſt encore queſtion juſqu'ici que de corps durs, il eſt clair que parmi les différentes valeurs qu'on peut attribuer à u, la vîteſſe V eſt elle-même compriſe, c'eſt-à-dire que le mouvement réel du ſyſtême eſt lui-même un des mouvements géométriques dont il eſt ſuſceptible; la premiere équation (E) eſt donc contenue dans l'équation indéterminée (F), & par conſéquent on peut réduire à cette ſeule équation (F) toutes les loix de l'équilibre & du mouvement dans les corps durs.

Or, on vient de voir que cette équation n'eſt autre choſe que la premiere (E), à laquelle on eſt parvenu à donner plus d'extenſion, par le moyen des mouvements géométriques; mais, comme on le verra bientôt (XXIV), l'analogie de cette équation (E) avec le principe de la conſervation des forces vives dans le choc des corps parfaitement élaſtiques, devient frappante, par une légere transformation; & nous verrons (XXVI), qu'en effet ce n'eſt autre choſe que ce principe lui-même transféré aux corps durs, avec la modification qu'exige la différente nature de ces corps: c'eſt donc cette conſervation de forces vives, qui ſervira, comme nous en avions prévenu, de baſe à toute notre théorie des Machines, ſoit en repos, ſoit en mouvement.

D'après ces remarques, on va récapituler briéwithout changing either the relations or the directions, one would obtain different equations, all just as true, but which would evidently reduce to the [original equations] after multiplying by different constants.

Remark II.

XIX. As, so far, only hard [plastic] bodies have been considered, it is clear that among the different values that one can attribute to u, the speed V is itself included, that is to say, the real motion of the system is itself one of the geometric motions to which it is susceptible. The first equation (E) is then contained in the indeterminate equation (F) & by consequence one can reduce to this single equation (F) all the laws of equilibrium & of motion for hard [plastic] bodies.

Yet, one comes to see that this equation is none other than the first (E), to which we have given a greater range of applicability by means of geometric motions. However, as we will come to see later (XXIV), the analogy of this equation (E) with the principle of the conservation of live force in the collision of perfectly elastic bodies becomes striking, after a small transformation, & we shall see (XXVI), that in effect it is nothing other than the same principle transferred to hard bodies, with the modification that is required by the different nature of the bodies. It is then this principle of the conservation of live force which will serve us – as we forecast – as the base of all our theory of Machines, either at rest or in motion.

After these remarks, we are going to briefly recapitulate

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vement la ſolution du problême précédent, pour faire voir d'un coup d'œil la ſuite des opérations qu'on vient d'indiquer.

Problême.

XX. *Connoiſſant le mouvement virtuel d'un ſyſtême quelconque donné de corps durs (c'eſt-à-dire celui qu'il prendroit, ſi chacun des corps étoit libre) trouver le mouvement réel qu'il doit avoir l'inſtant ſuivant.*

Solution. Nommons

Chaque molécule du ſyſtême, m

Sa vîteſſe virtuelle donnée, W

Sa vîteſſe réelle cherchée, V

La vîteſſe qu'elle perd, de ſorte que W ſoit la réſultante de V & de cette vîteſſe, U

Imaginons maintenant qu'on faſſe prendre au ſyſtême un *mouvement géométrique* arbitraire, & ſoit la vîteſſe qu'aura alors m, u

L'angle formé par les directions de W & V, X

L'angle formé par les directions de W & U, Y

L'angle formé par les directions de V & U, Z

L'angle formé par les directions de W & u, x

L'angle formé par les directions de V & u, y

L'angle formé par les directions de U & u, z

Cela poſé, on aura l'équation $\int m\, u\, U \cos z = 0$ (F), par le moyen de laquelle on trouvera dans tous les cas l'état du ſyſtême, en attribuant ſucceſſivement aux indéterminées u, difthe solution of the preceding problem, to show at a glance the order of operations we have indicated.

Problem.

XX. *Knowing the virtual motion of any given system of hard bodies (that is to say, those which would be followed if each of the bodies were free) to find the real motion which it must have in the following instant.*

Solution. Let us name

Each molecule of the system,	m
Its given speed,	W
Its real speed to be found,	V
The speed which is lost, so that W will be the resultant of V & this speed,	U

Let us now imagine that one gives to the system an arbitrary *geometric motion*, & that the speed of m will then be,	u
The angle between the directions of W & V,	X
The angle between the directions of W & U,	Y
The angle between the directions of V & U,	Z
The angle between the directions of W & u,	x
The angle between the directions of V & u,	y
The angle between the directions of U & u,	z

This assumed, one will arrive at the equation

$$\sum muU\cos z = 0 \tag{F}$$

by means of which one will find [solutions for] all possible states of the system, after letting the variable u take on all the différents rapports & directions arbitraires.

Définitions.

XXI. Imaginons un ſyſtême de corps en mouvement d'une maniere quelconque : ſoient m la maſſe de chacun de ces corps, & V ſa vîteſſe ; ſuppoſons maintenant qu'on faſſe prendre au ſyſtême un mouvement quelconque géométrique, & ſoient u la vîteſſe qu'aura alors m (& que j'appellerai ſa *vîteſſe géométrique*) & y l'angle compris entre les directions de V & u ; cela poſé, la quantité $m\,u\,V$ coſ y ſera nommée moment de la quantité de mouvement $m\,V$, à l'égard de la vîteſſe géométrique u, & la ſomme de toutes ces quantités, c'eſt-à-dire $\int m\,u\,V$ coſ y, ſera nommée moment de la quantité de mouvement du ſyſtême à l'égard du mouvement géométrique, qu'on lui a fait prendre : ainſi *le moment de la quantité de mouvement d'un ſyſtême de corps, à l'égard d'un mouvement quelconque géométrique, eſt la ſomme des produits des quantités de mouvement des corps qui le compoſent, multipliées chacune par la vîteſſe géométrique de ce corps, eſtimée dans le ſens de cette quantité de mouvement.* De ſorte qu'en conſervant les dénominations du problême, $\int m\,u\,W$ coſ x eſt le moment de la quantité de mouvement du ſyſtême avant le choc ; $\int m\,u\,V$ coſ y eſt le moment de la quantité de mouvement du même ſyſtême après le choc ; & $\int m\,u\,U$ coſ z eſt le moment de la quantité de mouvement perdu dans le choc : (tous ces moments étant rapportés au même mouvement géométrique). Ainſi de l'équation fondamentale (F) on peut conclure que *dans le choc des corps durs, ſoit que ces corps ſoient tous* ferent relations & arbitrary directions.

Definitions.

XXI. Let us imagine an arbitrary system of bodies in motion, letting m be the mass[35] of each of these bodies, & V its speed. Let us now suppose that one gives the system any geometric motion whatever, & let u be the speed that m will have (& which I will call the *geometric speed*) & y the angle between the directions of V & u; this supposed, the quantity $muV\cos y$ will be named the moment of the quantity of motion mV, with respect to the geometric speed[36] u, & the sum of all these quantities, that is to say, $\sum muV\cos y$, will be named the 'moment of the quantity of motion of the system' with respect to the geometric motion that we gave to the system. In this way *the moment of the quantity of motion of a system of bodies, with respect to any geometric motion whatever, is the sum of the products of the quantities of motion of the bodies which compose it, each multiplied by the geometric speed of the bodies, estimated in the direction of this quantity of motion*. So, using the same denominations, $\sum muW cosx$ is the moment of the quantity of motion before the collision; $\sum muV cosy$ is the moment of the quantity of motion after the collision; & $\sum muU cosz$ is the moment of the quantity of motion lost in the collision (all the moments being related to the same geometric motion). Thus from the fundamental equation (F) one can conclude that *in the collision of hard* [plastic] *bodies, whether these bodies are all*

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mobiles, ou qu'il y en ait de fixes, ou ce qui revient au même, ſoit que ce choc ſoit immédiat, ou qu'il ſe faſſe par le moyen d'une Machine quelconque ſans reſſort, le moment de la quantité de mouvement perdue par le ſyſtême général eſt égal à zero.

W étant la réſultante de V & U, il eſt clair qu'on à W coſ $x =$ coſ $y + U$ coſ z, ou $m\,u\,W$ coſ $x = m\,u\,V$ coſ $y + m\,u\,U$ coſ z, ou enfin $\int m\,u\,W$ coſ $x = \int m\,u\,V$ coſ $y + \int m\,u\,U$ coſ z; or, nous avons trouvé $\int m\,u\,U$ coſ $z = 0$; donc $\int m\,u\,W$ coſ $x = \int m\,u\,V$ coſ y, c'eſt-à-dire qu'*à l'égard d'un mouvement quelconque géométrique, le moment de quantité de mouvement du ſyſtême, immédiatement après le choc, eſt égal au moment de quantité de mouvement immédiatement avant le choc.*

Lorſqu'on décompoſe la vîteſſe que prendroit un corps s'il étoit libre, en deux, dont l'une ſoit la vîteſſe qu'il prend réellement, l'autre eſt la vîteſſe qu'il perd; & réciproquement ſi l'on décompoſe la vîteſſe qu'il prend, en deux, dont l'une ſoit celle qu'il auroit priſe s'il eût été libre, l'autre ſera la vîteſſe qu'il gagne : d'où il ſuit viſiblement que ce qu'on entend par la vîteſſe gagnée par un corps, & ce qu'on entend par ſa vîteſſe perdue, ſont deux quantités égales & directement oppoſées : cela poſé, le moment de la quantité de mouvement perdue par m, à l'égard de la vîteſſe géométrique u, étant, ſuivant la définition précédente $m\,u\,U$ coſ z, le moment de la quantité de mouvement gagnée par le même corps ſera $- m\,u\,U$ coſ z; car il n'y a de différence entre ces deux quantités, qu'en ce que l'angle compris entre u & la vîteſſe gagnée, eſt le ſupplément de celui compris entre u & U; de ſorte que l'un de ces angles étant *mobile, or whether some are fixed, or - what comes to the same thing - whether the collision is immediate, or done by means of any Machine without springs, the moment of the quantity of motion lost by the general system is equal to zero.*

W being the resultant of V & U, it is clear that we have $Wcosx = Vcosy + Ucosz$,* or $muWcosx = muVcosy + muUcosz$, or finally $\sum muWcosx = \sum muVcosy + \sum muUcosz$. However, we have found $\sum muUcosz = 0$, and therefore $\sum muWcosz = \sum muVcosy$, that is to say, *with regards to any geometric motion, the moment of the quantity of motion of the system immediately after the collision is equal to the moment of the quantity of motion immediately before the collision.*

When one decomposes the speed that the body would take if it were free into two parts, one being the speed which it actually takes, the other the speed which is lost (& reciprocally, if one decomposes the speed which it takes into two parts, one of which being that which it would have taken if it were free, the other the speed which it gains) then it obviously follows that what is meant by the speed gained by a body & what is meant by the speed lost are two equal & directly opposite quantities. That posed, the moment of the quantity of motion lost by m with respect to the geometric speed u is (following the preceding definition) $muUcosz$. Then the moment of the quantity of motion gained by the same body will be $-muU\cos z$, as there is no difference between these two quantities except that the angle between u & the speed gained is the supplement of that between u & U. So that one of these angles being

*See *Errata*.

aigu, l'autre ſera obtus, & ſon coſinus égal au coſinus de l'autre, pris négativement.

Il ſuit delà que le moment de la quantité de mouvement perdue par le ſyſtême général, à l'égard d'un mouvement quelconque géométrique, (lequel eſt nul, comme on l'a vu ci-deſſus), eſt la même choſe que la différence entre le moment de quantité de mouvement perdue par une partie quelconque des corps qui le compoſent, & le moment de la quantité de mouvement gagnée par les autres corps du même ſyſtême; donc cette différence eſt égale à zero; donc l'une de ces deux quantités eſt égale à l'autre, c'eſt-à-dire que *le moment de quantité de mouvement perdue dans le choc par une partie quelconque des corps du ſyſtême, à l'égard d'un mouvement quelconque géométrique, eſt égal au moment de quantité de mouvement gagnée par les autres corps du même ſyſtême.*

On peut donc, de la définition précédente, recueillir les trois propoſitions contenues dans le théorême ſuivant.

Théorême.

XXII. *Dans le choc des corps durs, ſoit que ce choc ſoit immédiat, ou qu'il ſe faſſe par le moyen d'une Machine quelconque ſans reſſort, il eſt conſtant qu'à l'égard d'un mouvement quelconque géométrique :*

1°. *Le moment de la quantité de mouvement perdue par-tout le ſyſtême, eſt égal à zero.*

2°. *Le moment de la quantité de mouvement perdue par une partie quelconque des corps du ſyſtême, eſt égal au moment de la quantité de mouvement gagnée par l'autre partie.*

acute, the other will be obtuse, & its cosine will be equal to the cosine of the other, taken negatively.

It follows that the moment of the quantity of motion* lost by the general system, taken with respect to any geometric motion – which is null, as we have seen above – is the same thing as the difference between the moment of the quantity of motion lost by any part of the bodies which compose it, & the moment of the quantity of motion gained by the other parts of the same system. In other words, this difference is equal to zero; so one of these two quantities is equal to the other, that is to say, *the moment of the quantity of motion lost in the collision of one part of the bodies of the system – with respect to any geometric motion – is equal to the moment of the quantity of motion gained by the other bodies of the same system.*

We may then, from the preceding definition, collect the three propositions contained in the following theorem.

Theorem.

XXII. *In the collision of hard bodies, whether the collision is immediate, or whether it is done by means of any Machine without spring* [compression or stretching], *the following are constant with respect to any geometric motion:*

1st *The moment of the quantity of motion lost by the whole system is equal to zero.*

2nd *The moment of the quantity of motion lost by one part of the bodies of the system is equal to the moment of the quantity of motion gained by the other part.*

* 'Moment of the quantity of motion' is, in modern terms, the component of the momentum in a given direction - for example the direction of a geometric motion.

3°. *Le moment de la quantité de mouvement réelle du ſyſtême général, immédiatement après le choc, eſt égal au moment de la quantité de mouvement du même ſyſtême, immédiatement avant le choc.*

Il eſt clair, par la définition précédente, que ces trois propoſitions ſont identiques au fonds, & ne ſont autre choſe que l'équation même fondamentale (F) exprimée de diverſes manieres.

On peut remarquer auſſi que ces propoſitions ont beaucoup de rapport à celles que l'on tire de la conſidération des moments, relativement à différents axes ; mais celles-ci ſont moins générales, & ſe tirent aiſément de celles qu'on vient d'établir (XVII).

Il y a donc, comme on voit, par la troiſieme propoſition de ce théorême ; il y a, dis-je, dans toute percuſſion ou communication de mouvement, ſoit immédiate, ſoit faite par l'entremiſe d'une Machine, une quantité qui n'eſt point altérée par le choc : cette quantité n'eſt pas, comme l'avoit penſé *Deſcartes*, la ſomme des quantités de mouvement ; ce n'eſt pas non-plus la ſomme des forces vives, car celle-ci ne ſe conſerve que dans le cas où le mouvement change par degrés inſenſibles, comme on verra plus bas, & elle diminue toujours lorſqu'il y a percuſſion, comme on le prouvera dans le corollaire ſecond : lorſque le ſyſtême eſt libre, la quantité de mouvement eſtimée dans un ſens quelconque, eſt à la vérité la même avant & après la percuſſion ; mais cette conſervation n'a plus lieu, s'il y a des obſtacles, non-plus que celle des moments de quantité de mouvements rapportés à différents axes : toutes ces quantités

3rd. *The moment of the actual quantity of motion of a general system, immediately after the collision, is equal to the moment of the quantity of motion of the same system, immediately before the collision*.

It is clear, by the preceding definition, that these three propositions are basically identical, & are nothing other than the same fundamental equation (F) expressed in various ways.

One may also remark that these propositions bear much relation to those which we draw from consideration of the moments, relative to different axes; but those are less general & are easily deduced from those which we have established (XVII).

There is therefore, as we see (by the third proposition of this theorem), there is, I say, in all percussion or communication of motion, whether immediate or by the intervention of a Machine, a quantity which is not affected by the collision. This quantity is not, as was thought by *Descartes*, the sum of the quantities of motion. Neither is it the sum of the live force, because that is not conserved in the case where the motion changes by insensible degrees – as we will see below – & is always diminished when there is percussion, as we prove in the second corollary. When the system is free, the quantity of motion measured in any direction is truly the same before & after the percussion. But this conservation [37] [of the moment of the quantity of motion] is no longer valid when there are barriers, any more than when it is referred to different axes. All these quantities

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ſont donc altérées par le choc, ou du moins ne ſe conſervent que dans quelques cas particuliers; mais il y a une autre quantité que ni les divers obſtacles qui s'oppoſent au mouvement, ni les Machines qui le tranſmettent, ni l'intenſité des différentes percuſſions ne peuvent changer; c'eſt le moment de quantité de mouvement du ſyſtême général, à l'égard de chacun des mouvements géométriques dont il eſt ſuſceptible, & ce principe renferme en lui ſeul toutes les loix de l'équilibre & du mouvement dans les corps durs; nous verrons même dans le corollaire IV, que cette loi s'étend également aux autres eſpeces de corps, quelle qu'en ſoit la nature & le degré d'élaſticité.

Si le choc détruiſoit tous les mouvements, on auroit $V = 0$, ainſi l'équation ſe réduiroit à $\int m\, W\, u \cos x = 0$, qui nous apprend que ce cas arrive, c'eſt-à-dire que tous les mouvements ſe détruiſent réciproquement par le choc, dans le cas où immédiatement avant ce choc, le *moment de la quantité de mouvement* du ſyſtême général eſt nul relativement, à tous les mouvements géométriques dont il eſt ſuſceptible.

Corollaire I.

XXIII. *Parmi tous les mouvements dont eſt ſuſceptible un ſyſtême quelconque de corps durs agiſſants les uns ſur les autres, ſoit par un choc immédiat, ſoit par des Machines quelconques ſans reſſort, celui de ces mouvements qui aura lieu réellement, l'inſtant d'après, ſera le mouvement géométrique, qui eſt tel que la ſomme des produits de chacune des maſſes, par le carré de la vîteſſe qu'elle perdra, eſt un* minimum, *c'eſt-à-dire moindre que* are then altered by the collision, or at least only conserved in some specific cases. But there is one other quantity which neither the various obstacles which obstruct the motion, nor the Machines which transmit it, nor the intensity of the different percussions, can change. This is the moment of the quantity of motion of the general system, with regard to each of the geometric motions to which it is susceptible, & this principle includes on its own all the laws of equilibrium & motion for hard bodies. We will even show in Corollary IV that this law applies equally to other types of body, whatever their nature & degree of elasticity.

If the collision destroys all motion, we will have $V = 0$, and then the equation reduces to $\sum mWucosx = 0$, which tells us that this case can happen. That is to say, all the motions are destroyed reciprocally by the collision in the case where immediately before the collision the *moment of the quantity of motion* of the general system is null relative to all the geometric motions to which it is susceptible.

Corollary I.

XXIII. *Amongst all the motions to which an arbitrary system of hard bodies interacting with each other is susceptible, whether by an immediate collision, or by Machines without springs, those of these motions will actually take place that, the instant afterwards, are geometric motions such that the sum of the products of each of the masses, by the square of the speed which it will lose, is a* minimum, *that is to say, less than* *la ſomme des produits de chacun de ces corps, par la vîteſſe qu'il auroit perdue, ſi le ſyſtême eût pris un autre mouvement quelconque géométrique.*

Sur quoi il faut remarquer qu'en donnant pour *minimum* la ſomme des produits de chaque maſſe, par le carré de ſa viteſſe perdue, j'entends ſeulement que la différentielle de cette ſomme eſt nulle, c'eſt-à-dire que ſa différence avec ce qu'elle ſeroit ſi le ſyſtême avoit un mouvement géométrique infiniment peu différent du premier, eſt égal à zero : ainſi cette ſomme peut être quelquefois un *maximum*, ou même n'être ni un *maximum* ni un *minimum*, & j'ai ſeulement à établir que $d \int m\, U^2 = 0$.

Démonſtration. Il eſt d'abord évident que le vrai mouvement du ſyſtême après le choc doit être géométrique, car les mouvements géométriques étant ceux qui n'alterent point l'action qui s'exerce entre les corps, il eſt clair que le premier en ordre eſt le mouvement même que prend le ſyſtême : il s'agit donc de ſavoir quel eſt, parmi tous les mouvements géométriques poſſibles, celui qui doit avoir lieu : or, ſuppoſons que s'il en prenoit un autre infiniment peu différent de celui qu'on cherche, la vîteſſe de chaque molécule m fût alors V'; décompoſons V' en deux, dont l'une ſoit V; c'eſt-à-dire la vîteſſe réelle, & l'autre V'', cela poſé, il eſt évident que ſi les corps n'avoient pas d'autres vîteſſes que ces dernieres V'', le mouvement ſeroit encore géométrique, car V'' eſt viſiblement la réſultante de V' & d'une vîteſſe égale & directement oppoſée à V; or, par hypotheſe, les molécules priſes deux à deux ne tendent ni en vertu de V', ni en vertu de $-V$, à ſe rapprocher ou à s'éloigner, puiſque dans ces deux *the sum of the products of each of the bodies, by the square of speed*[*] *which it would have lost, if the system had taken another geometric motion*.

It must be remarked that in giving for the *minimum* the sum of the products of each mass by the square of the speed lost, I mean only that the differential of this sum is null, that is to say, it is the same as if the system had a geometric motion infinitesimally different to the first. Therefore this sum may be sometimes a *maximum*, or even neither a *maximum* or a *minimum*, & I only have to establish that $d\sum mU^2 = 0$.

Demonstration. It is evident from the outset that the true motion of the system after the collision must be geometric, as geometric motions are those [sorts of motions] that can in no way alter the action [internal forces] that exist between the bodies of the system, [and thus] it is clear that of primary importance is the very motion that the system [as a whole] undertakes. It is a question of knowing which geometric motions, amongst all possible ones, are the ones which do take place.

Now, let us suppose that it [the system] takes another motion only infinitesimally different from the one we seek, such that the speed of each molecule m is V'. Let us decompose V' into two parts: V (that is to say, the actual speed), & V''. That supposed, it is evident that if the bodies have no other speeds apart from these last speeds (V'') then the motion would again be geometric because V'' is obviously the resultant of V' & a speed equal & directly opposite to V. However, by hypothesis, the molecules taken two by two tend neither by virtue of V', nor by virtue of $-V$, to approach or depart from each other, since in these two

[*] See *Errata*.

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cas le mouvement eſt géométrique ; donc, en ſuppoſant que les molécules m aient à la fois les vîteſſes V' & $-V$, ou leur réſultante V'', ils ne tendront non-plus ni à ſe rapprocher ni à s'éloigner ; & partant, le mouvement ſera alors géométrique : donc, ſi l'on appelle z'' l'angle compris entre les directions de V'' & U, on aura par l'équation fondamentale (F) $\int m U V''$ coſ $z = 0$; d'un autre côté, nommons U' la vîteſſe que perdroit m ſi ſa vîteſſe effective étoit V', de ſorte que W ſoit la réſultante de V' & de U', il faudra néceſſairement que U' ſoit compoſée de U & d'une vîteſſe égale & directement oppoſée à V'' ; d'où il ſuit évidemment que $U' - U$ ou $dU = - V''$ coſ z'' ; donc l'équàtion $\int m U V''$ coſ $z'' = 0$, trouvée ci-deſſus, devient $\int m U dU = 0$ ou $d \int m U^2 = 0$.

Je ſuppoſe, par exemple, que deux globes A & B, venant à ſe choquer obliquement, on demande leurs mouvements après le choc.

Suppoſons que la vîteſſe de A, eſtimée ſuivant la ligne des centres, ſoit avant le choc a, & après le choc V ; que celle de B, auſſi eſtimée ſuivant la ligne des centres, ſoit avant le choc b, & après le choc u ; que celle de A, eſtimée perpendiculairement à la même ligne, ſoit avant le choc a', & après le choc V' ; qu'enfin celle de B, auſſi eſtimée perpendiculairement à cette ligne des centres, ſoit avant le choc b', & après le choc u' ; cela poſé, par notre propoſition, le mouvement devant être géométrique, il faut d'abord qu'on ait $V = u$, ainſi la vîteſſe perdue par A, ſuivant la ligne des centres, ſera $a - u$, & celle perdue par B, dans le même ſens, ſera $b - u$; de plus, dans le ſens perpendiculaire à la ligne des centres, la vîteſſe perdue par A

cases the motion is geometric. Therefore, in supposing that the molecules, m, have at the same time the speeds V' & $-V$, or their resultant, V'', they tend neither to approach or move apart & thus the motion will be geometric. Therefore, if we call z'' the angle between the directions V'' & U, we will have, by the fundamental equation (F), the result $\sum mUV^{''}cosz = 0$. On the other hand, let us name U' the speed that would be lost if the effective speed were V', so that W is the resultant of V' & of U'. It is essential that U' is composed of U & a speed equal & directly opposite to V'', from which it follows evidently that $U' - U$ or $dU = -V''cosz''$. Then the equation $\sum mUV^{''}cosz^{''} = 0$, found above, becomes $\sum mUdU = 0$ or $d\sum mU^2 = 0$.

I suppose, for example, that two globes A & B, come to collide obliquely. We ask for their motions after the collision.

Let us suppose that the speed of A, calculated along the line of the centres before the collision is a, & after the collision, V; that of B, also calculated along the line of centres, is b before the collision, & after the collision, u; that of A, estimated perpendicularly to the same line, before the collision is a', & after the collision, V'; and finally, that of B, also estimated perpendicularly to the line joining the centres, is b' before the collision, & after the collision, u'. That supposed, by our proposition, before being geometric, the motion must initially satisfy $V = u$. Thus the speed lost by A, along the line of the centres, will be $a - u$, & that lost by B, in the same direction, will be $b - u$. Furthermore, in the direction perpendicular to the line joining the centres, the speed lost by A

ſera $a' - V'$, & celle perdue par B, ſera $b' - u'$; donc $\sqrt{(a-u)^2 + (a' - V')^2}$ ſera la vîteſſe abſolue perdue par A, & celle perdue par B ſera $\sqrt{(b-u)^2 + (b' - u')^2}$; donc, ſuivant la propoſition, on doit avoir $d(A(a - u)^2 + A(a' - V')^2 + B(b-u)^2 + B(b' - u')^2) = 0$, ou $A(a-u)du + A(a' - V')dV' + B(b-u)du + B(b' - u')du' = 0$, équation qui doit avoir lieu généralement, c'eſt-à-dire, quelles que ſoient les valeurs de du, dV', & du'; il faut donc que le coefficient de chacune de ces différentielles ſoit égal à zero; ce qui donne $V' = a'$, $u' = b'$, & $u = \frac{Aa + Bb}{A + B}$; *ce qu'il falloit trouver.*

Il eſt clair que cette propoſition renferme toutes les loix du choc des corps durs, ſoit que ce choc ſoit immédiat, ou qu'il ſe faſſe par le moyen d'une Machine quelconque, puiſqu'il aſſigne le caractere auquel on reconnoîtra parmi tous les mouvements qui ſont poſſibles, celui qui doit avoir lieu réellement à chaque inſtant : ce principe a beaucoup d'analogie avec celui que *M. de Maupertuis* a trouvé & nommé *principe de la moindre action.* (*Eſſai de coſmologie*).

Corollaire II.

XXIV. *Dans le choc des corps durs, ſoit qu'il y en ait de fixes, ou qu'ils ſoient tous mobiles (ou ce qui revient au même), ſoit que ce choc ſoit immédiat, ou qu'il ſe faſſe par le moyen d'une Machine quelconque ſans reſſort ; la ſomme des forces vives avant le choc, eſt toujours égale à la ſomme des forces vives après le choc, plus*

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will be $a' - V'$, & that lost by B, will be $b' - u'$. Then $\sqrt{(a-u)^2 + (a' - V')^2}$ will be the absolute speed lost by A , & that lost by B will be $\sqrt{(b-u)^2 + (b' - u')^2}$. Then, following the proposition, one must have $d(A(a-u)^2 + A(a' - V')^2 + B(b-u)^2 + B(b' - u')^2) = 0$, or $A(a-u)du + A(a' - V')dV' + B(b-u)du + B(b' - u')du' = 0$, an equation which must hold generally, that is to say, whatever may be the values of du, dV', & du'. Therefore the coefficients of each of these differentials must be equal to zero, which gives $V' = a'$, $u' = b'$, & $u = \frac{Aa+Bb}{A+B}$, *which it was necessary to find.*

It is clear that this proposition includes all the laws of collision of hard [plastic] bodies, whether the collision is immediate, or whether it is done by means of whatever kind of Machine, since it determines the telltale characteristics by which we recognize - amongst all the possible motions - the actual one that takes place at each instant. This principle is very analogous to the one discovered by *M. de Maupertuis*[38] & named the *principle of least action*[39]. *(Essay on Cosmology).*

Corollary II.

XXIV. *In the collision of hard* [plastic] *bodies, whether some are fixed, or all are mobile (or, which comes to the same), whether the collision is immediate, or whether it happens by whatever Machine* [so long as it is] *without compression or extension, the sum of live forces before the impact is always equal to the sum of live forces after the impact; moreover* *la somme des forces vives qui auroit lieu si la vîtesse qui reste à chaque mobile, étoit égale à celle qu'il a perdue dans le choc.*

C'est-à-dire qu'il faut prouver l'équation suivante $\int m \mathrm{W}^2 = \int m V^2 + \int m U^2$; or, elle se déduit facilement de l'équation fondamentale (E), car W étant résultante de V & U, il est clair que W V & U sont proportionnelles aux trois côtés d'un certain triangle : donc, par la trigonométrie, on a $\mathrm{W}^2 = V^2 + U^2 + {}_2 VU$ cos$Z$: donc, $\int m \mathrm{W}^2 = \int m V^2 + \int m U^2 + {}_2 \int m VU$ cos Z : or, par l'équation (E) on a $\int m VU$ cos $Z = 0$; donc l'équation précédente se réduit à $\int m \mathrm{W}^2 = \int m V^2 + \int m U^2$; *ce qu'il falloit prouver.*

On voit donc, comme nous l'avons dit (XXI), que par cette transformation l'analogie de l'équation (E) avec la conservation des forces vives, devient frappante ; aussi peut-on aisément démontrer l'une par l'autre, comme on verra (XXVI).

L'analogie de cette même équation avec la conservation des forces vives dans un systême de corps durs dont le mouvement change par degrés insensibles, est encore plus évidente, puisqu'il s'agit alors d'un cas particulier de celui que nous venons d'examiner ; c'est en effet visiblement le cas particulier ou U est infiniment petite, & partant U^2 infiniment petite du second ordre ; ce qui réduit l'équation à $\int m \mathrm{W}^2 = \int m V^2$; mais cette conservation sera expliquée plus au long dans le corollaire suivant.

Corollaire III.

XXV. *Lorsqu'un systême quelconque de corps durs change de mouvement par degrés insensibles ;*

the [principle of] *the sum of live forces would apply if, for each mobile body, its remaining speed was equal to that which it lost in the collision.*

That is to say, it is required to prove the following equation $\sum m W^2 = \sum m V^2 + \sum m U^2$. However, this can easily be deduced from the fundamental equation (E), because W being the resultant of V & U, it is clear that W, V & U are proportional to three sides of a certain triangle. Then, by trigonometry, we have $W^2 = V^2 + U^2 + 2VU \cos Z$. Therefore $\sum m W^2 = \sum m V^2 + \sum m U^2 + 2 \sum m V U \cos Z$. However, by equation (E) we have $\sum m V U \cos Z = 0$, so the preceding equation reduces to $\sum m W^2 = \sum m V^2 + \sum m U^2$, *which it was necessary to prove.*

We see then, as we have said (XIX)*, that by this transformation the analogy between equation (E) and the conservation of live forces becomes striking. Also we can easily derive one from the other, as we will see (XXVI).

The analogy of this same equation with the conservation of live forces in a system of hard [plastic] bodies whose motion changes by insensible degrees is even more evident since it is then a specific case of that which we have just examined. It is in effect obviously the specific case where U is infinitely small, & thus U^2 is infinitely small to second order. This reduces the equation to $\sum m W^2 = \sum m V^2$; but this conservation will be explained further and at length in the following corollary.

Corollary III.

XXV. *When any system of hard* [plastic] *bodies changes its motion by insensible degrees*,

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*See *Errata*.

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ſi pour un inſtant quelconque on appelle m *la maſſe de chacun des corps*, V *ſa vîteſſe*, p *ſa force motrice*, R *l'angle compris entre les directions de* V *&* p, u *la viteſſe qu'auroit* m, *ſi on faiſoit prendre au ſyſtêmeun mouvement quelconque géométrique*, r *l'angle formé par* u *&* p, y *l'angle formé par* V *&* u, d t *l'élément du temps, on aura ces deux équations*

$\int$ m V p d t *coſ* R — $\int$ m V d V = o.

$\int$ m u p d t *coſ* r — $\int$ m u d (V *coſ* y) = o.

Démonſtration. Premiérement, *p d t* coſ *R* eſt viſiblement la vîteſſe que la force motrice *p* auroit imprimée à *m* dans le ſens de *V*, ſi ce corps eût été libre ; de plus, *d V* eſt la vîteſſe qu'il reçoit réellement dans le même ſens ; donc *p d t* coſ *R* — *d V* eſt la vîteſſe perdue par *m* dans le ſens de *V*, en vertu de l'action réciproque des corps : c'eſt donc cette quantité qu'il faut mettre pour *U* coſ *Z* dans l'équation fondamentale (E), laquelle devient par cette ſubſtitution $\int$ *m V p d t* coſ *R* — $\int$ *m V d V* = o, qui eſt la premiere des deux équations que nous avions à démontrer.

Secondement, *p d t* coſ *r* eſt la vîteſſe que la force motrice *p* auroit imprimée à *m* dans le ſens de *u*, ſi ce corps eût été libre ; de plus, *V* coſ *y* étant la vîteſſe de *m* dans le ſens de *u*, *d*(*V* coſ *y*) eſt la quantité dont cette vîteſſe eſtimée dans le même ſens augmente ; donc *p d t* coſ *r* — *d*(*V* coſ *y*) eſt la vîteſſe perdue par *m* dans le ſens de *u*, en vertu de l'action réciproque des corps : c'eſt donc cette quantité qu'il faut mettre pour *U* coſ *z* dans la ſeconde équation (F), laquelle devient par cette ſubſtitution $\int$ *m u p d t* coſ *r* — $\int$ *m u d*(*V* coſ *y*) = o, qui eſt la ſeconde des deux équations que nous avions à démontrer.

Ces équations ne ſont donc autre choſe que les équations fondamentales (E) & (F) appli-

then if for some instant we call m the mass of each of the bodies, V *its speed*, p *its motive force*, R *the angle between the directions of* V & p, *u the speed which m would have, and if we imparted whatsoever geometric motion to the system*, [and called] r, *the angle formed by u* & p, y, *the angle formed by* V & u, dt *the element of time, [then] we will have these two equations*:

$$\begin{aligned}\sum mVpdtcosR - \sum mVdV &= 0,\\ \sum mupdtcosr - \sum mud(Vcosy) &= 0.\end{aligned}$$

Demonstration. Firstly, $pdtcosR$ is obviously the speed which the motive force p would have imparted to m in the direction of V if the body would have been free; moreover, dV is the speed which it actually receives in the same direction. Then $pdtcosR - dV$ is the speed lost by m in the direction of V, by virtue of mutual interactions between the bodies. It is then this quantity which must be substituted for $UcosZ$ in the fundamental equation (E), which becomes by this substitution $\int mVpdtcosR - \int mVdV = 0$,[40] and this is the first of the two equations we have to prove.

Secondly, $pdtcosr$ is the speed that the motive force p would have imparted to m in the direction of u if this body had been free. Moreover, $Vcosy$ being the speed of m in the direction of u, then $d(Vcosy)$ is the quantity by which this speed increases when calculated in the same direction. Therefore $pdtcosr - d(Vcosy)$ is the speed lost by m in the same direction as u, by virtue of mutual interactions between the bodies. It is then this quantity which must be substituted for $Ucosr$ in the second equation (F), which becomes by this substitution $\int mupdtcosr - \int mud(Vcosy) = 0$,[41] which is the second of the two equations we have to show.

These equations are then nothing other than the fundamental equations (E) & (F) applied quées au cas où le mouvement change par degrés inſenſibles ; & partant, elles renferment toutes les loix de ce mouvement : on peut remarquer de plus, que la premiere de ces deux équations n'eſt qu'un cas particulier de la ſeconde, par la même raiſon que l'équation (E) d'où elle eſt tirée, eſt contenue dans celle (F) d'où eſt tirée la ſeconde ; mais cette premiere équation $\int m\, V p\, d t \operatorname{coſ} R - \int m\, V\, d V = 0$ mérite une attention particuliere ; parce qu'elle renferme le fameux principe de la conſervation des forces vives dans un ſyſtême de corps durs dont le mouvement change par degrés inſenſibles, comme on va l'expliquer.

Nommons d'abord $d s$ l'élément de la courbe décrite par le corpuſcule m pendant $d t$; cela poſé, nous aurons $V d t = d s$; & partant, l'équation précédente prend cette forme $\int m\, p\, d s \operatorname{coſ} R - \int m\, V d V = 0$: maintenant ſuppoſons pour un inſtant que la courbe décrite par m ſoit une ligne inflexible, que m ſoit un grain mobile enfilé dans cette courbe, qu'il la parcourt librement, c'eſt-à-dire ſans être gêné par les réactions des autres parties du ſyſtême, qu'il éprouve à chaque point de cette courbe la même force motrice que celle dont il étoit animé dans le premier cas, & qu'enfin dans ce premier cas la vîteſſe initiale de m ſoit κ, tandis que dans le ſecond elle ſera nulle au premier inſtant, & V′ après un temps indéterminé t ; cela poſé, en intégrant l'équation précédente pour avoir l'état du ſyſtême au bout du temps t ; nous aurons pour le premier cas $\int' \int m\, p\, d s \operatorname{coſ} R - \int' \int m\, V d V = 0$, $\int'$ déſignant le ſigne d'intégration relatif à la durée du mouvement, tandis que $\int$ eſt le ſigne d'intégration relatif à la figure to the case where the motion changes by insensible degrees[42], & thus it includes all the laws of this motion. We can moreover remark that the first of these two equations is only a specific case of the second, by the same reasoning that equation (E) (from which it is derived) is contained in (F) from which the second is derived. However this first equation, $\sum mVpdtcosR - \sum mVdV = 0$, merits specific attention because it includes the famous principle of the conservation of live forces in a system of hard [plastic] bodies whose motion changes by insensible degrees, as we are going to explain.

Let us name by ds the element of the curve described by particle m during dt. That supposed, we will have $Vdt = ds$, & thus the preceding equation takes the form $\sum mpdscosR - \sum mVdV = 0$. Now let us suppose for the moment that the curve followed by m is an inflexible line, that m is a movable granule threaded on this curve, that it runs freely, that is to say, without being hindered by the reactions of other parts of the system, that it experiences at each point of the curve the same motive force as that which drove it in the first place, & finally that in the first case the initial speed of m being K, while in the second it will be null in the first instant & V' after an indeterminate time t. [All] this [being] supposed, on integrating the preceding equation in order to obtain the state of the system at the end of time t, we will have for the first case, $\int \sum mpdscosR - \int \sum mVdV = 0$, where $\int$ designates the integration sign relative to the duration of the motion, while $\sum$ is the integration sign relative to the configuration

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du ſyſtême ; or, $\int'\int m V d V = \frac{\int m V^2}{2}$: donc l'équation peut ſe mettre ſous cette forme $\int'\int m p\, d s$ coſ $R - \int m V^2 + C = o$; C étant une conſtante ajoutée pour compléter l'intégrale, pour la déterminer, on obſervera qu'au premier inſtant on a $V = K$ & $\int'\int m p\, d s$ coſ $R = o$; donc $C = \frac{\int m K^2}{2}$; donc $2\int'\int m p\, d s$ coſ $R - \int m V^2 + \int m K^2 = o$; par les mêmes raiſons on a pour le ſecond cas $2\int'\int m p\, d s$ coſ $R - \int m V'^2 = o$, ſans conſtante, parce qu'on ſuppoſe V' nulle au premier inſtant ; ôtant donc cette équation de la précédente, réduiſant, & tranſpoſant, on a $\int m V^2 = \int m K^2 + \int m V'^2$; c'eſt-à-dire que *dans un ſyſtême quelconque de corps durs, dont le mouvement change par degrés inſenſibles, la ſomme des forces vives au bout d'un temps quelconque, eſt égale à la ſomme des forces vives initiales, plus la ſomme des forces vives qui auroit lieu, ſi chaque mobile avoit pour vîteſſe celle qu'il auroit acquiſe en parcourant librement la courbe qu'il a décrite, en ſuppoſant d'ailleurs qu'il eût été animé à chaque point de cette courbe, de la même force motrice qu'il y éprouve réellement, & que ſa vîteſſe au premier inſtant eût été nulle.*

C'eſt cette propoſition qu'on appelle principe de la conſervation des forces vives, & d'où l'on peut conclure que,

Dans un ſyſtême de corps durs dont le mouvement change par degrés inſenſibles, & qui ne ſont animés d'aucune force motrice, la ſomme des forces vives eſt une quantité conſtante, c'eſt-à-dire la même pour tous les inſtants.

Car dans ce cas on a par hypotheſe $p = o$, ce qui donne $V' = o$, & partant $\int m V^2 =$

of the system. However, $\int \sum mVdV = \sum \frac{mV^2}{2}$, and so the equation can be put in the form $\int \sum mpdscos R - \sum mV^2 + C = 0$, C being an additive constant to complete the integral. For determining it, we will observe that at the first instant we have $V = K$ & then $C = \sum \frac{mK^2}{2}$; then $2 \int \sum mpdscos R - \sum mV^2 + \sum mK^2 = 0$. By the same reasoning we have for the second case $2 \int \sum mpdscos R - \sum mV'^2 = 0$, without a constant, because we have supposed that V' is null at the first instant. Then, subtracting this equation from the preceding one, collecting & transposing, we have $\sum mV^2 = \sum mK^2 + \sum mV'^2$. That is to say, *in any system of hard* [plastic] *bodies, when the motion changes by insensible degrees, the sum of the live forces at the end of any time, is equal to the sum of the initial live forces plus the sum of the live forces which would exist if each mobile body would have for its speed that which it would acquire in running freely along the curve that it describes (assuming amongst other things that it is driven at each point of this curve by the same motive force which it actually experiences, & that its speed in the first instant is null).*

This [is the] proposition which we call the principle of conservation of live force, & from which we may conclude that,

In a system of hard [plastic] *bodies whose motion changes by insensible degrees, & which is not driven by any motive force, the sum of live forces is a constant quantity, that is, the same for all instants.*

Because in this case we have by hypothesis $p = 0$, which gives $V' = 0$ & thus $\sum mV^2 =$

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$\int m K^2$; équation qui ſe tire d'ailleurs immédiatement de celle $\int m p V d t \operatorname{coſ} R - \int m V d V =$ o trouvée (XXIV), laquelle à cauſe de $p = o$, ſe réduit à $\int m V d V = o$, dont l'intégrale complétée eſt $\frac{1}{2} \int m V^2 - \frac{1}{2} \int m K^2 = o$; d'où ſuit l'équation $\int m V^2 = \int m K^2$: qu'il falloit prouver.

Corollaire IV.

XXVI. J'ai prouvé (XIX), que l'équation indéterminée (F) renferme toutes les loix de l'équilibre & du mouvement dans les corps durs; je vais maintenant plus loin, & je dis que cette équation convient également aux corps qui ne le ſont pas, & que par conſéquent cette loi générale s'étend indiſtinctement à tous les corps de la nature : en effet, lorſque pluſieurs corps qui ne ſont pas durs agiſſent les uns ſur les autres d'une maniere quelconque, ſi l'on conçoit le mouvement qu'auroit pris chaque mobile s'il eût été libre, décompoſé en deux, dont l'un ſoit celui qu'il prendra réellement, l'autre ſera détruit; d'où il ſuit viſiblement que ſi les corps euſſent été durs & n'euſſent eu d'autres mouvements que ce dernier, il y auroit eu équilibre : ces mouvements détruits ſont donc aſſujettis aux mêmes loix, ont entre eux les mêmes rapports, & peuvent enfin ſe déterminer de la même maniere que ſi les corps étoient durs, c'eſt-à-dire par l'équation générale (F); cette équation (F) n'eſt donc point bornée aux corps durs, elle appartient également à tous les corps de la nature, & contient par conſéquent toutes les loix de l'équilibre & du mouvement, non-ſeulement pour les pre-

$\sum mK^2$. [This] equation, drawn directly from that found in (XXIV), $\sum mVpdtcosR - \sum mVdV = 0$, which, due to $p = 0$, reduces to $\sum mVdV = 0$, whose complete integral* is $\frac{1}{2}\sum mV^2 - \frac{1}{2}\sum m$ $K^2 = 0$. From this we obtain $\sum mV^2 = \sum mK^2$ – which is what had to be proved.

Corollary IV.

XXVI. I have proved, (XIX), that the indeterminate equation (F) includes all the laws of equilibrium & motion for hard [plastic] bodies. I now go further & declare this equation to be equally appropriate for bodies which are not hard, & that in consequence this law extends to all naturally–occurring bodies without distinction. In effect, when several bodies which are not hard [plastic] interact with each other in any manner whatever, one imagines the motion of each freely–moving body as being decomposed into two parts – one of which the body will actually take, the other will be destroyed. From this it obviously follows that if the bodies had been hard [plastic] & had only these last motions, then there would be equilibrium. These destroyed motions are then subject to the same laws, have the same relations between them, & finally, they can be determined in the same way as if the bodies were hard [plastic], that is to say, by equation (F). This equation (F) is therefore not restricted to hard bodies, it applies equally to all bodies in nature, & contains in consequence all the laws of equilibrium & of motion, not only for the first case [hard plastic bodies], D iij

*Although Carnot uses the word 'integral' as well as the symbol $\int$, in his calculations he will have carried out summations rather than integrations. The evidence is that there are no differential quantities, 'dt' or 'dV'.

miers, mais même pour tous les autres, quel que puiſſe être leur degré de compreſſibilité; mais la différence conſiſte en ce que l'on peut, dans le cas où il s'agit de corps durs, ſuppoſer $u = V$; de ſorte qu'alors $\int m\, V\, U \cos Z = 0$, devient une des équations déterminées du problême, au lieu que cela n'eſt pas lorſque les corps ſont d'une nature différente : c'eſt donc cette équation déterminée, laquelle eſt la même que la premiere équation fondamentale (E), c'eſt dis-je cette équation déterminée qui caractériſe les corps durs, & par conſéquent il eſt abſolument néceſſaire de l'employer au moins implicitement dans toutes les queſtions qui concernent ces corps; & lorſqu'il s'agit de corps d'une autre eſpece, il faut, outre les équations déterminées, qu'on peut obtenir en attribuant à u dans l'équation indéterminée, (F) différentes valeurs connues, il faut, dis je en tirer encore une qui ſoit analogue à l'équation (E), & qui exprime en quelque ſorte la nature de ces corps, de même que celle-ci (E) exprime celle des corps durs; mais comme cette recherche n'a qu'un rapport fort indirect aux Machines proprement dites, nous nous bornerons ici à examiner le cas où le degré d'élaſticité eſt le même pour tous les corps, c'eſt-à-dire que nous ſuppoſerons qu'en vertu de l'élaſticité, les corps exercent les uns ſur les autres des preſſions n fois auſſi grandes que ſi les corps étoient durs, n étant la même pour tous les corps du ſyſtême; nous ſuppoſerons de plus que la preſſion & la reſtitution ſe faſſent dans un inſtant indiviſible, quoiqu'en rigueur cela ſoit impoſſible. Cela poſé :

Les preſſions réciproques F devenant $n F$, auront entre elles les mêmes rapports que ſi les but even for all the other [cases], whatever the degree of compressibility [of the bodies]. But the difference consists in that one may, in the case of hard bodies, suppose $u = V$, which means that $\sum mVU cos Z = 0$ becomes one of the determined equations of the problem, instead of those [other equations] which are not [determined] when the bodies are of different natures. It is then this determinate equation, which is the same as the first fundamental equation (E), it is, I say, this determinate equation which characterizes hard [plastic] bodies, & in consequence it is absolutely necessary [for it] to be used, at least implicitly, in all questions that are concerned with these bodies. When it comes to [the consideration of] other types of bodies, it must (outside of the determinate equations, which one can obtain by attributing different known values to u in the indeterminate equation), it must [I repeat], [be possible to] draw out as before [a solution] which is the analogue of equation (E), & which expresses in some way the nature of the bodies, in the same way that (E) expresses that of hard [plastic] bodies.

However, as this research applies only very indirectly to Machines proper, we will limit ourselves to examining the case where the degree of elasticity is the same for all the bodies, that is to say, we will suppose that by virtue of their elasticity, the bodies exert pressures on each other n times as large as if the bodies were hard, n being the same for all the bodies of the system. We will suppose furthermore that the pressure & the restitution happen instantaneously, even though rigorously this is impossible. That posed:

The reciprocal pressures F becoming nF, will have between them the same relations as if the corps étoient durs ; donc leurs résultantes mU n'auront point changé de directions, mais seront seulement devenues n fois aussi grandes qu'elles auroient été si les corps avoient été durs ; cela posé, puisque W est la résultante de V & U, on a $V \cos Z = W \cos Y - U$; ainsi l'équation (E) à laquelle nous cherchons une analogue, peut se mettre sous cette forme $\int m W U \cos Y - \int m U^2 = 0$; or, suivant ce qu'on vient de dire, il faut, pour appliquer cette équation au cas dont il s'agit ici, mettre $\frac{U}{n}$ au lieu de U, sans rien changer à Y ; donc pour le cas que nous examinons, l'équation sera $\int m W \frac{U}{n} \cos Y - \int \frac{m U^2}{n^2} = 0$: ou en multipliant par n^2, $n \int m W U \cos Y - \int m U^2 = 0$, ou à cause de $W \cos Y = V \cos Z + U$ on aura $\frac{n}{1-n} \int m V U \cos Z = \int m U^2$; ainsi cette équation sera pour les corps dont il s'agit ce qu'est l'équation (E) pour les corps durs, & celle-ci même en est le cas particulier où l'on a $n = 1$, comme il est évident.

Lorsque $n = 2$ c'est le cas des corps parfaitement élastiques, & l'équation devient $2 \int m V U \cos Z + \int m U^2 = 0$; mais cette équation relative aux corps parfaitement élastiques, peut s'exprimer d'une maniere connue & plus simple, comme il suit : puisque W est la résultante de V & U, on a par la trigonométrie $W^2 = V^2 + U^2 + 2 V U \cos Z$; & partant $\int m W^2 = \int m V^2 + \int m U^2 + 2 \int m V U \cos Z$; ajoutant à cette équation celle trouvée ci-dessus, & réduisant, on a $\int m W^2 = \int m V^2$, qui est prébodies were hard [plastic]. Therefore their resultants, mU, will not have changed directions but will only have become n times as great as they would have been if the bodies had been hard [plastic]. That posed, since W is the resultant of V & U, we have $V cos Z = W cos Y - U$[cosX]. Thus the equation (E) for which we are searching for an analogue may be put in the form $\sum mWUcosY - \sum mU^2 = 0$. However, following on from what we have said, it is necessary, to apply this equation to the case here, to put $\frac{U}{n}$ in place of U without any change to Y. Then in the case we are examining, the equation will be $\sum mW(U/n)cosY - \sum \frac{mU^2}{n^2} = 0$, or on multiplying by n^2, it will be $n \sum mWUcosY - \sum mU^2 = 0$, where, because of $WcosY = VcosZ + U$, one will have $\frac{n}{1-n} \sum mVUcosZ = \sum mU^2$. Therefore this equation will be for [such] bodies what equation (E) is for hard [plastic] bodies, & this same equation applies in the specific case where one has $n = 1$; as is evident.

When $n = 2$, this is the case of perfectly elastic bodies, & the equation becomes $2 \sum mVUcosZ + \sum mU^2 = 0$. However this equation, as applied to perfectly elastic bodies, may be expressed in a well-known & simpler manner, as follows: since W is the resultant of V & U, one has by trigonometry $W^2 = V^2 + U^2 + 2VUcosZ$, & so $\sum mW^2 = \sum mV^2 + \sum mU^2 + 2 \sum mVUcosZ$. Adding to this equation that found above, & simplifying, one has $\sum mW^2 = \sum mV^2$, which is precisely

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cisément le principe de la conservation des forces vives, c'est-à-dire que cette conservation est pour les corps parfaitement élastiques, ce qu'est l'équation (E) pour les corps durs, comme nous avions promis de le prouver.

Remarque I.

XXVII. Je ne m'arrêterai point aux conséquences particulieres que je pourrois tirer de la solution du problême précédent; je remarquerai seulement que les vîtesses W, V, U, étant toujours proportionnelles aux trois côtés d'un triangle, la trigonométrie peut fournir les moyens de donner un grand nombre de formes différentes aux équations fondamentales (E) & (F), & je me contenterai d'en indiquer une qui est remarquable, à cause de la méthode imaginée par les Géometres, de rapporter les mouvements à trois plans perpendiculaires entre eux; ce qui donne aux solutions beaucoup d'élégance & de simplicité.

Imaginons donc à volonté trois axes perpendiculaires entre eux, & concevons que les vîtesses W, V, U & u, soient décomposées chacune en trois autres paralleles à ces axes. Cela posé. Nommons

Celles qui répondent à W, W', W'', W'''.
Celles qui répondent à V, V', V'', V'''.
Celles qui répondent à U, U', U'', U'''.
Celles qui répondent à u, u', u'', u'''.

Maintenant, pour peu qu'on y fasse attention, on verra aisément que la premiere équation fondamentale (E) peut se mettre sous cette forme $\int mV'U' + \int mV''V'' + \int mV'''U''' = 0$, & la seconde (F) sous celle-ci $\int mu'U'$

the principle of the conservation of live force. In other words, this conservation [principle] is for perfectly elastic bodies what equation (E) is for hard bodies, as we had promised to prove.

Remark I.

XXVII. I will not linger on the specific consequences I could draw from the solution of the preceding problem; I will only remark that the speeds W, V, U being always proportional to the three sides of a triangle, trigonometry can furnish the means of giving a great number of different forms to the fundamental equations (E) & (F), & I will be content to indicate one which is remarkable, due to the method imagined by Geometers, of considering the motions in three mutually perpendicular planes - which gives much elegance & simplicity to the solutions.

Let us imagine then three mutually perpendicular axes, & let us suppose that the speeds W, V, U & u, are decomposed into three components parallel to these axes. With that assumed, Let us name

Those corresponding to W: W', W'', W'''
Those corresponding to V: V', V'', V'''
Those corresponding to U: U', U'', U'''
Those corresponding to u: u', u'', u'''

Now, for those few who pay attention [to such details], one will easily see that the first fundamental equation (E) may be put in the form $\sum mV^{'}U^{'} + \sum mV^{''}V^{''} + \sum mV^{'''}U^{'''} = 0$, & the second, (F), in the form $\sum mu^{'}U^{'}+$

$+\int m\,u''\,U''+\int m\,u'''\,U'''=0$, parce qu'en général toute quantité qui eſt le produit de deux vîteſſes A & B, par le coſinus de l'angle compris entre elles, eſt égale à la ſomme de trois autres produits $A'\,B'+A''\,B''+A'''\,B'''$; A', A'', A''', étant la vîteſſe A eſtimée de ces trois axes, & B' B'' B''' étant la vîteſſe B eſtimée dans le ſens de ces mêmes axes; c'eſt-à-dire A$'$ étant la vîteſſe A, & B' la vîteſſe B, eſtimées parallelement au premier de ces axes; A'' & B'' les mêmes vîteſſes A & B eſtimées parallelement au ſecond axe; A''' & B''' les mêmes vîteſſes eſtimées parallelement au troiſieme axe: ce qui ſe prouve aiſément par les éléments de géométrie.

Dans le cas d'équilibre, la premiere de ces équations transformées ſe réduit à $0=0$, & la ſeconde, à cauſe que dans ce cas W $=U$ devient $\int m\,u'\,W'+\int m\,u''\,W''+\int m\,u'''\,W'''=0$, laquelle exprime toutes les conditions de l'équilibre.

Lorſque le mouvement change par degrés inſenſibles, nous avons trouvé (XXV) que les équations fondamentales deviennent $\int m\,V\,p\,d\,t$ coſ $R-\int m\,V\,d\,V=0$, & $\int m\,u\,p\,d\,t$ coſ $r-\int m\,u\,d\,(V$ coſ $y)=0$; donc en décompoſant p en trois autres forces paralleles aux trois axes, ſi ces forces compoſantes ſont déſignées par p', p'', p''', les équations précédentes deviendront, la premiere, $\int m\,V'\,p'\,d\,t+\int m\,V''\,p''\,d\,t+\int m\,V'''\,p'''\,d\,t=\int m\,V'\,d\,V'+\int m\,V''\,d\,V''+\int m\,V'''\,d\,V'''$, & la ſeconde, $\int m\,u'\,p'\,d\,t+\int m\,u''\,p''\,d\,t+\int m\,u'''\,p'''\,d\,t=\int m\,u'\,d\,V'+\int m\,u''\,d\,V''+\int m\,u''\,d\,V'''$; enfin, dans le cas d'équilibre, la premiere s'évanouira, & la ſeconde ſe réduira à $\int m\ u'\,p'+\int m\ u''\ p''+\int\ m\ u'''\ p'''=0$.

$+\sum mu^{''}U^{''}+\sum mu^{'''}U^{'''}=0$, because in general any quantity which is the product of two speeds, A & B, with the cosine of the angle between them, is equal to the sum of the three products $A'B'+A''B''+A'''B'''$ (A', A'', A''' being the speed [components of] A referred to the three axes, & B', B'', B''', being the speed [components of] B referred to the same [three] axes. That is, A' being the speed [component of] A, & B' the speed [component of] B, estimated parallel to the first of these axes; A'' & B'' the same speed [components of] A & B estimated parallel to the second axis; A''' & B''' the same speed [components] estimated parallel to the third axis - all of which is easily proved from the elements of geometry–.

In the case of equilibrium, the first of these transformed equations reduces to $0=0$, & the second (because in this case $W=U$) becomes $\sum mu^{'}W^{'}+\sum mu^{''}W^{''}+\sum mu^{'''}W^{'''}=0$ - which expresses all the conditions of equilibrium.

When the motion changes by insensible degrees, we have found (XXV) that the fundamental equations become $\sum mVpdtcosR-\sum mVdV=0$, & $\sum mupdtcosr-\sum mud(Vcosy)=0$. Therefore, in decomposing p into three other forces parallel to three axes, if these component-forces are designated by p', p'', p''', the preceding equations become: the first, $\sum mV^{'}p^{'}dt+\sum mV^{''}p^{''}dt+\sum mV^{'''}p^{'''}dt=\sum mV^{'}dV^{'}+mV^{''}dV^{''}mV^{'''}dV^{'''}$, & the second $\sum mu^{'}p^{'}dt+\sum mu^{''}p^{''}dt+\sum mu^{'''}p^{'''}dt=\sum mu^{'}dV^{'}+\sum mu^{''}dV^{''}+\sum mu^{'''}dV^{'''}$.* Finally, in the case of equilibrium, the first [equation] vanishes, & the second reduces to $\sum mu^{'}p^{'}+\sum mu^{''}p^{''}+\sum mu^{'''}p^{'''}=0$.

*Typographic error corrected from $u^{''}$ to $u^{'''}$ in the preceding expression.

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Remarque II.

XXVIII. Juſqu'ici j'ai regardé les fils, verges, leviers, *&c.* comme des corps faiſant eux-mêmes partie du ſyſtême. Et cette hypotheſe eſt entiérement conforme à la nature ; mais une choſe qu'il eſt indiſpenſablement néceſſaire d'obſerver, c'eſt qu'à parler ſtrictement, il n'y a probablement dans l'univers aucun point abſolument fixe, aucun obſtacle abſolument immobile ; l'hypomochlion d'un levier ne paroît tel, que parçe qu'il eſt appuyé ſur la terre qui n'eſt point fixe elle-même, mais dont la maſſe eſt preſque infiniment grande en comparaiſon de celles dont on conſidere ordinairement dans les Machines l'action & la réaction les unes ſur les autres : pour déplacer l'hypomochlion d'un levier, il faut donc auſſi mettre en mouvement le globe de la terre ; & il y eſt en effet, quelque foibles que ſoient les puiſſances qui agiſſent ſur la Machine ; la quantité de mouvement qu'elles lui procurent, eſt égale à la réſiſtence de l'hypomochlion ; mais cette quantité finie de mouvement, ſe diſtribuant dans une maſſe preſque infiniment grande, il en réſulte à cette maſſe une vîteſſe preſque infiniment petite, & voilà pourquoi ce mouvement n'eſt pas ſenſible, & peut ſe négliger dans la pratique.

Il ſuit de-là que ce qu'on appelle obſtacles immobiles en méchanique, ne ſont autre choſe que des corps dont la maſſe eſt ſi conſidérable, & par conſéquent la vîteſſe ſi petite, que leur mouvement ne peut être obſervé : ce ſera donc ſe rapprocher de la nature, que de conſidérer les obſtacles ou points fixes, comme des corps mobiles auſſi bien que tous les autres, mais d'une

Remark II.

XXVIII. Until now I have regarded the threads, rods, levers, &c. as bodies being themselves part of the system. And this hypothesis conforms completely to nature. However one thing which it is indispensably necessary to take note of is the fact that, strictly speaki-ng, there is probably no absolutely fixed point in the universe, and no absolutely immovable obstacle. The fulcrum of a lever does not appear [as an absolutely fixed point] because it pushes against the Earth which is itself not fixed, but whose mass is almost infinitely great in comparison with those one ordinarily considers in Machines acting on each other. In order to displace the fulcrum of a lever one requires, therefore, to put in motion as well the [whole] globe [(spherical mass)] of the Earth. There are, in effect, certain weak forces that act on this Machine [the lever]; the quantity of motion which they provide is equal to the [reaction at] the fulcrum, and this [quantity of motion] being distributed in a mass almost infinitely great results in a speed almost infinitely small in that mass - & that is why this motion is not detectable, & may be neglected in practice.

It follows from this that what one calls immovable obstacles in mechanics are nothing else than bodies whose mass is so great, & consequentially whose speed is so small, that their motion cannot be observed: it will be closer to nature to consider these obstacles or fixed points as bodies, mobile like all the others, but of

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maſſe infiniment grande, ou ce qui revient au même, comme des corps d'une denſité infinie, & qui ne différent qu'en ce point de tous les autres corps du ſyſtême. Il réſultera de-là un avantage conſidérable, c'eſt qu'on pourra faire prendre au ſyſtême où entreront ces corps, des mouvements quelconques géométriques; car dès qu'on ſuppoſera ces obſtacles mobiles comme tous les autres corps, ils deviendront ſuſceptibles de prendre des mouvements quelconques; & le ſyſtême général devra être regardé comme un aſſemblage de corps parfaitement mobiles: en conſéquence, les quantités de mouvement, abſorbées par les obſtacles, pourront s'évaluer comme pour toutes les autres parties du ſyſtême; de ſorte que ſi l'on appelle R la réſiſtance d'un point fixe donné, cette quantité R ſera dans l'équation (F) pour le point en queſtion, ce qu'eſt $m\,U$ pour le corps m: on trouvera donc par cette équation cette même quantité R comme toutes les autres forces $m\,U$, ce qui n'auroit pu ſe faire en conſidérant les obſtacles comme abſolument immobiles, ſans avoir recours à quelque nouveau principe méchanique, qu'il auroit fallu faire concourir avec l'équation générale (F) pour parvenir à la ſolution complete de chaque problême particulier: ainſi cette maniere de conſidérer les points fixes, eſt non-ſeulement la plus conforme à la nature, comme nous l'avons dit ci-deſſus, mais encore la plus ſimple & la plus facile.

Quant aux fils, verges ou autres portions quelconques du ſyſtême dont les maſſes pourront être ſuppoſées infiniment petites, on pourra les négliger, c'eſt-à-dire, ſuppoſer chacune de leurs molécules m égale à zéro, ou ce qui revient au infinitely great mass or, what comes to the same thing, as bodies of infinite density & which differ only in this regard from all the other bodies in the system. This yields a considerable advantage, which is that one can give a system which includes such bodies any geometric motions [one likes]; because as soon as one supposes these mobile obstacles to have the same properties as all other bodies, they will be susceptible to take up whatever motions, & [then] the general system may be considered as an assemblage of perfectly mobile bodies. In consequence, the quantities of motion absorbed by the obstacles can be evaluated in the same way as for other parts of the system.

Thus, calling the resistance of a given fixed point R, then this quantity R will be, for the point in question, what mU is for the body m. One will therefore find R by this equation in the same way as we find all the other forces, mU, which we would not have been able to do in considering the obstacles as absolutely immobile, without having recourse to some new principle of mechanics, which must then be used concurrently with the general equation (F) to arrive at the complete solution of each specific problem. Therefore this method of considering fixed points is not only the most congruent with nature, as we have said above, but is as well the simplest & the easiest.

As for the threads, rods or whatever other kind of system-parts whose masses are supposed to be infinitely small, one will be able to neglect them [the masses], that is to say, to suppose that each of their molecules's m equals zero, or what comes to même, regarder leur densité comme infiniment petite ou nulle; notre équation (F) deviendra donc ainsi indépendante de ces quantités, c'est-à-dire la même que si l'on eût fait abstraction de la masse de ces corps; & c'est ainsi qu'on trouvera aisément la théorie mathématique de chaque Machine, c'est-à-dire en faisant les abstractions dont on a parlé (VIII).

XXIX. De cette remarque, il résulte que quoiqu'il n'y ait qu'une seule espece de corps dans la nature, on les distingue cependant, pour la facilité des calculs, en trois classes différentes, qui sont, 1°. ceux qu'on considere tels qu'ils sont en effet & que la nature nous les offre, c'est-à-dire qui sont d'une densité finie; 2°. ceux auxquels on attribue une densité infiniment grande, & qui par cette raison, doivent être regardés comme sensiblement fixes & immobiles; 3°. ceux auxquels on attribue une densité infiniment petite ou nulle, & qui par conséquent n'opposent par leur inertie aucune résistance à leur changement d'état: on regarde ordinairement comme tels dans la pratique, les fils, verges, leviers & généralement tous les corps qui n'influent pas sensiblement par leur propre masse, aux changements qui arrivent dans le systême, mais qui sont seulement regardés comme des moyens de communication entre les différents agents qui le composent.

Remarque III.

XXX. Après avoir traité de l'équilibre & du mouvement en général, autant que mon objet principal pouvoit le permettre, je vais passer à ce qui regarde plus particuliérement ce qu'on the same thing, to regard their density as infinitely small or null. Our equation (F) will then become independent of these quantities, that is to say, it is the same as if we had ignored the mass of these bodies. It is in this way that we will easily find the mathematical theory of each Machine, that is [by making] the abstractions of which we spoke [earlier] (VIII).

XXIX. From this remark, there follows the consequence that, although there is only one type of body in nature, one can [nevertheless] divide them up, for ease of calculation, into three different classes, which are: 1st, those that one considers as they in fact are & as nature offers them up to us (that is to say, [those] which have finite density); 2nd, those to which one attributes an infinitely great density & which, for this reason, have to be regarded, for all practical purposes, as fixed & immobile; 3rd, those to which one attributes an infinitely small or null density & in consequence of which have no [inertial] resistance to changing their state. In practice, one ordinarily regards as such: threads, rods, levers, & generally all bodies which (by virtue of their [relatively miniscule] mass) offer no detectable influence to changes which occur in the system but which are regarded as the means of communication between the different agents which compose it.

Remark III.

XXX. After having treated equilibrium & motion in general, as much as my main aim allows me, I am going to pass on to the particular consideration of what [exactly] is entend communément par Machines ; car quoique la théorie de toute espece d'équilibre & de mouvement rentre toujours dans les principes précédents, puisqu'il n'y a, suivant la premiere loi, que des corps qui puissent détruire ou modifier le mouvement des autres corps ; cependant il y a des cas où l'on fait abstraction de la masse de ces corps, pour ne considérer que l'effort qu'ils font : par exemple, lorsqu'un homme tire un corps par un fil, ou le pousse par une verge, on n'introduit point dans le calcul la masse de cet homme, ni même l'effort dont il est capable, mais seulement celui qu'il exerce en effet sur le point auquel il est appliqué ; c'est-à-dire la tension du fil, si c'est en tirant qu'il agit, ou la pression, si c'est en poussant ; & sans considérer si c'est un homme ou un animal, un poids, un ressort, une résistance occasionnée par un obstacle ou par la force d'inertie d'un mobile (1), un frottement, une impulsion causée par le vent ou par un courant, *&c.* On donne en général le nom de puissance à l'effort exercé par l'agent, c'est-à-dire à cette pression ou tension par la-

(1) Un corps qu'on force à changer son état de repos ou de mouvement, résiste (XI) à l'agent qui produit le changement ; & c'est cette résistance qu'on appelle force d'inertie : pour évaluer cette force, il faut donc décomposer le mouvement actuel du corps en deux, dont l'uu soit celui qu'il aura l'instant d'après ; car l'autre sera évidemment celui qu'il faudra détruire, pour forcer le cops à son changement d'état ; c'est-à-dire la résistance qu'il oppose à ce changement ou sa force d'inertie, d'où il est aisé de conclure, que *la force d'inertie d'un corps, est la résultante de son mouvement actuel, & d'un mouvement égal & directement opposé à celui qu'il doit avoir l'instant suivant.*

commonly understood by [the term] Machine because, although the theory of any type of equilibrium & of motion always comes back to the preceding principles, by the first law) only bodies can destroy or modify the motion of other bodies. However, there are cases where one neglects the mass of these bodies, and considers only the effort that they make. For example, when a man pulls a body by a thread, or pushes it with a rod, one does not include the mass of this man in the calculation, nor the effort [force] of which he is capable, but only that which he exerts in fact at the point of application. That is to say, the tension of the thread (if pulling is taking place), or pressure (if pushing), & without considering whether [the force arises from] a man or an animal, a weight, a spring, a resistance occasioned by an obstacle or by the force of inertia of a movable [body] (I), a rubbing, an impulse caused by the wind or by a current, &c. One gives in general, the name puissance to the effort exerted by the agent, that is to say, to this pressure or tension by which

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(I) A body that one forces to change its state of rest or of motion resists (XI) the agent which produces the change, & it is this resistance which one calls the force of inertia. To evaluate this force it is necessary then to decompose the actual motion of the body into two parts, one of which is [the motion] that it will have the instant afterwards. [because] The other [part of the motion] will evidently be that which must be destroyed in order to force the body's change of state, that is to say, the 'resistance with which it opposes this change' or its 'force of inertia', from which it is easy to conclude, that *The force of inertia of a body is the resultant of its actual motion* & *the motion equal* & *opposite to that which it must have the following instant*.

quelle il agit ſur le corps auquel il eſt appliqué ; & l'on compare ces différents efforts ſans égard aux agents qui les produiſent, parce que la nature des agents ne peut rien changer aux forces qu'ils ſont obligés d'exercer pour remplir les différents objets auxquels ſont deſtinées les Machines : la Machine elle-même, c'eſt-à-dire le ſyſtême des points fixes, obſtacles, verges, leviers & autres corps intermédiaires qui ſervent à tranſmettre ces différents efforts d'un agent à l'autre ; la Machine, dis-je, elle-même eſt conſidérée comme un corps dépouillé d'inertie ; ſa propre maſſe, lorſqu'il eſt néceſſaire d'y avoir égard, ſoit à cauſe du mouvement qu'elle abſorbe, ſoit à cauſe de ſa peſanteur ou des autres forces motrices dont elle peut être animée, eſt regardée comme une puiſſance étrangere appliquée au ſyſtême ; en un mot, une Machine proprement dite, eſt un aſſemblage d'obſtacles immatériels, & de mobiles incapables de réaction, ou privés d'inertie, c'eſt-à-dire (XXIX) un ſyſtême de corps dont les denſités ſont infinies ou nulles : à ce ſyſtême, on imagine que différents agents extérieurs, au nombre deſquels on comprend la maſſe même de la Machine, ſont appliqués, & ſe tranſmettent leur action réciproque par l'entremiſe de cette Machine : c'eſt la preſſion ou autre effort exercé par chaque agent ſur ce corps intermédiaire, qu'on appelle force ou puiſſance, & c'eſt la relation qui exiſte entre ces différentes forces, dont la recherche eſt l'objet de la théorie des Machines proprement dites. Or, c'eſt ſous ce point de vue, que nous allons maintenant traiter de l'équilibre & du mouvement ; mais une force priſe dans ce ſens, n'en eſt pas moins une quantité de mouvement perdue par l'agent it acts on the body to which it is applied; & one compares these different efforts without regard to the agents that produce them as the nature of these agents can change nothing about the forces which they are obliged to exert in order to fulfill the various purposes that the Machines are [designed] for. The Machine itself, that is to say, the system of fixed points, barriers, rods, levers & other intermediary bodies which serve to transmit these different efforts from one agent to another - the Machine, I say, is of itself considered as a body stripped of inertia. Its real mass, when it is necessary to consider it, either because of the motion which it absorbs, or because of the weightiness or other motive forces which can be active, is regarded as a foreign puissance applied to the system. In a word, a Machine proper is an assemblage of immaterial barriers & movable bodies incapable of reaction or deprived of inertia, that is to say (XXIX), a system of bodies having infinite or zero density. To this system one imagines that different external agents, among which one considers the mass of the Machine, are applied & transmit their reciprocal action by the intervention of this Machine. It is the pressure or other effort exerted by each agent on this intermediate body that one calls force or puissance, & it is the relationship between these different forces the study of which is the object of the theory of Machines proper. Well, it is from this point of view, that we are now going to treat equilibrium & motion; but a force taken in this sense is nonetheless a quantity of motion lost by the agent

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qui l'exerce, quel que puiſſe être d'ailleurs cet agent; qu'il agiſſe ſur la Machine en la tirant par un cordon, ou en la pouſſant par une verge, la tenſion de ce cordon, ou la preſſion de cette verge, exprime également & l'effort qu'il exerce ſur la Machine, & la quantité de mouvement qu'il perd lui-même par la réaction qu'il éprouve: ſi donc on appelle F cette force, cette quantité F ſera la même choſe que celle qui eſt exprimée par $m\,U$ dans nos équations (1); donc ſi l'on appelle auſſi Z, l'angle compris entre cette force F & la vîteſſe u, qu'auroit le point où on la ſuppoſe appliquée, ſi l'on faiſoit prendre au ſyſtême un mouvement quelconque géométrique, l'équation générale (F) deviendra $\int F u \cos Z =$ o (AA). C'eſt donc ſous cette forme que nous emploierons déſormais cette équation, au moyen de quoi on pourra appliquer ce que nous dirons, à quelle eſpece de force on voudra imaginer; & les principes expoſés dans cette premiere partie, nous ſerviront à développer les pro-

(1) Il eſt évident que la quantité de mouvement perdue $m\,U$, eſt la réſultante du mouvement qu'auroit eu l'inſtant d'après le corps m, s'il eût été libre, & du mouvement égal & directement oppoſé à celui qu'il prendra réellement; or, le premier de ces deux mouvements, eſt lui-même la réſultante du mouvement actuel de m, & de ſa force motrice abſolue; donc $m\,U$ eſt la réſultante de trois forces qui ſont: ſa force motrice abſolue, ſa quantité actuelle de mouvement, & la quantité de mouvement égale & directement oppoſée à celle qu'il doit avoir l'inſtant d'après; mais ſuivant la note précédente, ces deux dernieres quantités de mouvement ont pour réſultante la force d'inertie; donc $m\,U$ ou F eſt la réſultante de la force motrice de m & de ſa force d'inertie; c'eſt-à-dire que *la force exercée par un corps quelconque, à chaque inſtant eſt la réſultante de ſa force motrice abſolue, & de ſa force d'inertie.*

which exerts it, no matter whatever this agent might be in other respects. Whether it acts on the Machine in pulling it by a cord or in pushing it by a rod, the tension of this cord or the pressure of this rod expresses equally the effort which it exerts on the Machine, & the quantity of motion that it itself loses by the reaction which it experiences. If therefore one calls this force F then this quantity F will be the same thing as that which is expressed by mU in our equations (I). Therefore if one calls Z the angle between this force F & the speed u, at what would be the point where it is supposed to be applied, [then] if one were to give the system any geometric motion whatever, the general equation (F) becomes $\sum FucosZ = 0$ (AA). It is therefore in this form that we will employ this equation from now on, by means of which we can apply what we say to any type of force we can imagine; & the principles displayed in this first part, will serve us to develop the

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(I) It is evident that the quantity of motion lost, mU, is the resultant of the motion which the body m would have had the instant after if it had been free, & motion equal & directly opposite to that which it really takes. Now the first of these two motions is itself the resultant of the actual motion of m & of the absolute motive force. Thus mU is the resultant of three forces which are its absolute motive force, its actual quantity of motion, & the quantity of motion equal & directly opposite to that which it must have the instant after. However, following the preceding note, the resultant of these last two quantities of motion is the force of inertia; thus mU or F is the resultant of the motive force of m & of its force of inertia. In other words, *the force exerted by any body, at each instant, is the resultant of its absolute motive force, & of its force of inertia*.

priétés générales des Machines proprement dites, qui font l'objet de la ſeconde.

SECONDE PARTIE.

Des Machines proprement dites.

DÉFINITIONS.

XXXI. PARMI les forces appliquées à une Machine en mouvement, les unes ſont telles, que chacune d'entr'elles fait un angle aigu avec la vîteſſe du point où elle eſt appliquée; tandis que les autres forment des angles obtus avec les leurs: cela poſé, j'appellerai les premieres *forces mouvantes* ou *ſollicitantes*; & les autres, *forces réſiſtantes*: par exemple, ſi un homme fait monter un poids par le moyen d'un levier, d'une poulie, d'une vis, *&c.* il eſt clair que la peſanteur & la vîteſſe du poids forment néceſſairement, par leur concours, un angle obtus; autrement il eſt viſible que le poids deſcendroit au lieu de monter; mais la puiſſance motrice & ſa vîteſſe forment un angle aigu; ainſi, ſuivant notre définition, le poids ſera la *force réſiſtante*, & la *force* de l'homme ſera *ſollicitante*: il eſt viſible en effet, que celle-ci tend à favoriſer le mouvement actuel de la Machine, tandis que l'autre s'y oppoſe.

On obſervera que les forces ſollicitantes peuvent être dirigées dans le ſens même de leurs vîteſſes, puiſqu'alors l'angle formé par leurs concours eſt nul, & par conſéquent aigu; & que les forces réſiſtantes peuvent agir dans le ſens general properties of Machines proper, which is the object of the second part.

SECOND PART.

Machines Proper.

DEFINITIONS.

XXXI. Among the forces applied to a Machine in motion, some are such that they make an acute angle with the speed (at the point where they are applied), while the others form obtuse angles with theirs. This posed, I will call the first – *moving forces* or *soliciting forces*, & the others – *resisting forces*. For example, if a man makes a weight rise by means of a lever, a pulley, a screw &c., it is clear that the force-weight & speed of the weights necessarily form an obtuse angle by the fact that they oppose each other (they are in competition) – were this not the case, it is obvious that the weight would descend instead of rising – whereas [in the case of motive force] the motive force & its speed form an acute angle. Therefore, by our definition, the weight will be the *resisting force* & the man will be the *soliciting force*. It is indeed obvious that this [(*soliciting force*)] tends to further the actual motion of the Machine, while the other [(*resisting force*)] opposes it.

We will observe that the soliciting forces can be directed in the same directions as their speeds, (as then the angle formed by their paths is null & in consequence acute), & that the resisting forces can act in the

sense

ſens directement oppoſé à celui de leurs vîteſſes ; puiſqu'alors l'angle formé par leurs concours, eſt de 180°, & par conſéquent obtus.

Il eſt à remarquer encore, que telle force qui eſt ſollicitante, pourroit devenir réſiſtante, ſi le mouvement venoit à changer ; que telle force qui eſt réſiſtante à un certain inſtant, peut devenir ſollicitante à un autre inſtant, & qu'enfin pour en juger à chaque inſtant, il faut conſidérer l'angle qu'elle fait avec la vîteſſe du point où on la ſuppoſe appliquée ; ſi cet angle eſt aigu, la force ſera ſollicitante ; & s'il eſt obtus, elle ſera réſiſtante, juſqu'à ce que l'angle en queſtion vienne à changer. On voit par-là, que ſi on fait prendre un mouvement géométrique à un ſyſtême quelconque de puiſſances, chacune d'elles ſera *ſollicitante* ou *réſiſtante* à l'égard de ce mouvement géométrique, ſuivant que l'angle formé par cette force, & ſa vîteſſe géométrique ſera aigu ou obtus.

XXXII. Si une force P ſe meut avec la vîteſſe u, & que l'angle formé par le concours de u & P ſoit z, la quantité $P \cos z\, u\, dt$ dans laquelle dt exprime l'élément du temps, ſera nommée *moment d'activité*, conſommé par la force P pendant dt ; c'eſt-à-dire que le *moment d'activité*, conſommé par une force P, dans un temps infiniment court, eſt le produit de cette force eſtimée dans le ſens de ſa vîteſſe, par le chemin que décrit dans ce temps infiniment court, le point où elle eſt appliquée.

J'appellerai *moment d'activité*, conſommé par cette force, dans un temps donné, la ſomme des *moments d'activité*, conſommés par elle à chaque inſtant, de ſorte que $\int P \cos z\, u\, dt$ eſt le *moment d'activité*, conſommé dans un temps directly opposite to their speeds, – as then the angle formed by their paths is 180°& consequently obtuse.

Again, it is to be noticed that a force that is soliciting could become resisting if the motion changed, and that a force that is resisting at a certain instant may become soliciting in another instant; & finally, in order to determine which it is at any instant, it is necessary to consider the angle which it [the force] makes with the speed (at the point where it is supposed applied); if this angle is acute, the force will be soliciting; & if it is obtuse, it will be resisting, until the angle in question changes. One sees from this that if one gives a geometric motion to any system of forces, each of these will be *soliciting* or *resisting* in regard to the geometric motion, depending on whether the angle formed by this force, & its geometric speed will be acute or obtuse.

XXXII. If a force P moves with speed u, & the angle formed by the directions of u & P is z, the quantity $P cos z u dt$ in which dt expresses the element of time, will be named *moment of activity* [43] consumed by the force P during dt. That is to say, the moment of activity consumed by a force, P, in an infinitely short time, is the product of this force (in the direction of the speed) and the distance undertaken (from the point of application) in this infinitely short time.

I will denominate by *moment of activity*, consumed by this force, in a given time, the sum of moments of activity, consumed by it at each instant, such that $\sum P cos z u dt$ is the *moment of activity*, consumed in some indeterminate time.[44]

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indéterminé par elle; par exemple, si P est un poids, le *moment d'activité*, consommé dans un temps indéterminé t, sera $P \int u\,dt \cos z$; supposons donc qu'après le temps t, le poids P soit descendu de la quantité H, on aura évidemment $dH = u\,dt \cos z$; donc le *moment d'activité*, consommé pendant dt sera $P \int dH = PH$.

XXXIII. Lorsqu'il s'agira d'un systême de forces appliquées à une Machine en mouvement, j'appellerai *moment d'activité*, consommé par toutes les forces du systême, la somme des *moments d'activité*, consommés en même temps par chacune des forces qui le composent; ainsi le *moment d'activité*, consommé par les forces sollicitantes, sera la somme des *moments d'activité*, consommés en même temps par chacune d'elles, & le moment d'activité, consommé par les forces résistantes, sera la somme des *moments d'activité*, consommés par chacune de ces forces: & comme chaque force résistante fait un angle obtus avec la direction de sa vîtesse; le cosinus de cet angle est négatif; le *moment d'activité*, consommé par les forces résistantes, est donc aussi une quantité négative; & partant, *le moment d'activité*, consommé par toutes les forces du systême, est la même chose que la différence entre le *moment d'activité*, consommé par les forces sollicitantes, & le *moment d'activité*, consommé en même temps par les forces résistantes, considéré comme une quantité positive.

Une force estimée dans un sens directement opposé à celui de sa vîtesse, & multipliée par le chemin que décrit dans un temps infiniment court, le point où elle est appliquée, s'appellera *moment d'activité produit* par cette force dans

For example, if P is a weight, the *moment of activity*, consumed in some indeterminate time t, will be $P \sum udtcosz$. Let us therefore suppose that after a time t the weight P descends by a quantity H. Then one evidently has $dH = udtcosz$, and therefore the *moment of activity* consumed during dt will be $P \sum dH = PH$.

XXXIII. When a system of forces is applied to a Machine in motion, I will call *moment of activity* consumed by all forces of the system, the sum of *moments of activity* consumed at the same time by each of these system-forces. Thus the *moment of activity* consumed by the soliciting forces will be the sum of the *moments of activity* consumed at the same time by each of them, & the *moment of activity* consumed by the resisting forces will be the sum of the *moments of activity* consumed by each of these forces: & as each resisting force makes an obtuse angle with the direction of its speed, the cosine of this angle is negative, and the *moment of activity* consumed by these resisting forces is therefore a negative quantity. Thus the *moment of activity* consumed by *all* the forces of the system is the same thing as the difference between the *moments of activity* consumed by the soliciting forces & the *moment of activity* consumed at the same time by the resisting forces, considered as a positive quantity.

A force calculated in the direction [directly] opposed to that of its speed, & multiplied by the path (at the point of application) described in an infinitely small time, will be called the *moment of activity* produced by this force in ce temps infiniment court : de ſorte que le *moment d'activité, conſommé*, & le *moment d'activité, produit*, ſont deux quantités égales, mais de ſignes contraires ; & qu'il y a entr'elles une différence analogue à celle qu'on trouve (XXI) entre les *moments de quantité de mouvement*, *gagnées & perdues*, par un corps, à l'égard d'un mouvement géométrique.

Je donnerai auſſi le nom de *moment d'activité, exercé* par une force, à ce que j'ai appellé ſon *moment d'activité, conſommé*, ſi elle eſt ſollicitante, & à ce que j'ai appellé ſon *moment d'activité, produit*, ſi elle eſt réſiſtante ; ainſi le *moment d'activité, exercé* par une force quelconque, dans un temps infiniment court, eſt en général le produit de cette force, par le chemin qu'elle décrit dans ce temps infiniment court, & par le coſinus du plus petit des deux angles formés par les directions de cette force & de ſa vîteſſe ; d'où il ſuit évidemment que ce *moment d'activité, exercé*, eſt toujours une quantité poſitive.

On fera, à l'égard des quantités que nous venons d'appeller *moments d'activité, produits*, & *moments d'activité, exercés*, les mêmes remarques ſemblables à celles que nous avons faites ci-deſſus, au ſujet du *moment d'activité, conſommé* par une puiſſance ou un ſyſtême de puiſſances, dans un temps donné.

Ces définitions admiſes, je paſſe au principe général de l'équilibre & du mouvement dans les Machines proprement dites, & dont la recherche a été le principal objet de cet Eſſai.

this infinitely short time. So the *moments of activity, consumed* & the *moment of activity, produced* are two equal quantities but with contrary signs. There is between them a difference analogous to that which one finds (XXI) between *moments of the quantity of motion, gained & lost* by a body - with respect to its geometric motion.

I will also give the name *moment of activity, exerted* by a force, to that which I have called *moment of activity, consumed* if it is soliciting, & to that which I have called its *moment of activity, produced* if it is resisting. Thus the *moment of activity, exerted* by any force, in an infinitely short time, is in general the product of this force and the distance which it describes in this infinitely short time, & by the cosine of the smaller of the two angles formed by the directions of this force & its speed - from which it evidently follows that this moment of activity, exerted, is always a positive quantity.

We will make, in respect of the quantities which we have just denominated *moments of activity, produced*, & *moments of activity, exerted*, similar remarks to those which we have made above, on the subject of the *moment of activity, consumed* by a force or a system of forces in a given time.

Admitting these definitions, I pass on to the general principle of equilibrium & motion in Machines proper, & whose study has been the principal object of this Essay.

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THÉORÊME FONDAMENTAL.

Principe général de l'équilibre & du mouvement dans les Machines.

XXXIV. *Quel que ſoit l'état de repos ou de mouvement où ſe trouve un ſyſtême quelconque de forces appliquées à une Machine, ſi on lui fait prendre tout-à-coup un mouvement quelconque géométrique, ſans rien changer à ces forces, la ſomme des produits de chacune d'elles, par la vîteſſe qu'aura dans le premier inſtant le point où elle eſt appliquée, eſtimée dans le ſens de cette force, ſera égale à zéro.*

C'eſt-à-dire donc qu'en nommant F chacune de ces forces (1), u la vîteſſe qu'aura au pre-

(1) Il ne ſera peut-être pas inutile de prévenir une objection qui pourroit ſe préſenter à l'eſprit de ceux qui n'auroient pas fait attention à ce qui a été dit (XXX) ſur le vrai ſens qu'on doit attacher au mot *force* : imaginons, par exemple, dira-t-on, un treuil à la roue & au cylindre duquel ſoient ſuſpendus des poids par des cordes; s'il y a équilibre, ou que le mouvement ſoit uniforme, le poids attaché à la roue, ſera à celui du cylindre, comme le rayon du cylindre eſt au rayon de la roue; ce qui eſt conforme à la propoſition. Mais il n'en eſt pas de même lorſque la Machine prend un mouvement accéléré ou retardé ; il paroît donc qu'alors les forces ne ſont pas en raiſon réciproque de leurs vîteſſes eſtimées dans le ſens de ces forces, comme il ſuivroit de la propoſition. La réponſe à cela, eſt que dans le cas où ce mouvement n'eſt pas uniforme, les poids en queſtion ne ſont pas les ſeules forces exercées dans le ſyſtême, car le mouvement de chaque corps, changeant continuellement, il oppoſe auſſi à chaque inſtant, par ſon inertie, une réſiſtance à ce changement d'état ; il faut

FUNDAMENTAL THEOREM.

General Principle of equilibrium & of motion in Machines.

XXXIV. *Whatever the state of rest or of motion found in any system of forces applied to a Machine, if we suddenly give it* [the system] *an arbitrary geometric motion, without changing these forces*, [then] *the sum of the products of each force and each* [initial] *speed* [when taken from the point of application, and in the direction of the force,] *will be equal to zero.*[45]

That is to say, therefore, that in naming by, F, each of these forces (1), u the speed

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(1) It will perhaps not be useless to forestall an objection that may present itself to the mind of those who have not paid attention to what has been said (XXX) on the true sense that must be attached to the word force. Let us imagine, for example, say, a winch to the wheel & drum of which weights are suspended by cords. If there is equilibrium, or the motion is uniform, then the ratio of weights attached to the wheel & to the cylinder, will be the same as the ratio of cylinder-radius to wheel-radius - and this is consistent with the proposition. But it is not the same when the Machine has an accelerated or a retarded motion. It appears then that these forces are [no longer] in the reciprocal ratio of their speeds (speeds as estimated [calculated] in the direction of these forces), as would follow from the proposition. The reply to this is that, in the case where the motion is not uniform, the weights in question are not the only forces exerted in the system, because the motion of each body, changing continually, opposes at each instant, by its inertia, a resistance to this change of state. We must

mier inſtant le point où elle eſt appliquée, ſi l'on fait prendre à la Machine un mouvement géométrique, & z l'angle compris entre les directions de F & de u, il faut prouver qu'on aura pour tout le ſyſtême $\int F u \cos z = 0$. Or, cette équation eſt préciſément l'équation (AA) trouvée (XXX) laquelle n'eſt autre choſe au fond que l'équation même fondamentale (F), préſentée ſous une autre forme.

Il eſt aiſé d'appercevoir que ce principe général n'eſt, à proprement parler, que celui de *Deſcartes*, auquel on donne une extenſion ſuffiſante, pour qu'il renferme non-ſeulement toutes les conditions de l'équilibre entre deux forces, mais encore toutes celles de l'équilibre & du mouvement, dans un ſyſtême compoſé d'un nombre

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donc auſſi tenir compte de cette réſiſtance. Nous avons déjà dit (XXX. *V.* la note), comment cette force doit s'évaluer, & nous verrons plus bas (XLI), comment on doit la faire entrer dans le calcul. En attendant, il ſuffit de remarquer que les forces appliquées à la Machine dont il eſt ici queſtion, ne ſont pas les poids même, mais les quantités de mouvement perdues par ces poids (XXX), leſquelles doivent s'eſtimer par les tenſions des cordons auxquels ils ſont ſuſpendus : or, que la Machine ſoit en repos ou en mouvement, que ce mouvement ſoit uniforme ou non, la tenſion du cordon attaché à la roue, eſt à celle du cordon attaché au cylindre, comme le rayon du cylindre eſt au rayon de la roue, c'eſt-à-dire que ces tenſions ſont toujours en raiſon réciproque des viteſſes des poids qu'ils ſoutiennent; ce qui eſt d'accord avec la propoſition. Mais ces tenſions ne ſont pas égales aux poids; elles ſont (XXX. *V.* la note) les réſultantes de ces poids & de leurs forces d'inertie, leſquelles ſont elles-même (XXX. *V.* la note) les réſultantes des mouvements actuels de ces corps, & des mouvements égaux & directement oppoſés à ceux qu'ils prendront réellement l'inſtant d'après.

at the first point of application, & z the angle between the directions of F & u, and if one imparts a geometric motion to the Machine, then it must be proved that for the whole system $\sum Fucosz = 0$. But, this equation is precisely equation (AA) found in (XXX), which is nothing else at root but the same fundamental equation (F), presented in a different form.

It is easy to perceive that this general principle is only, properly speaking, that of Descartes to which one gives a sufficient generalisation, for it to include not only all the conditions between two forces, but as well all those of equilibrium & motion in a system composed of any number

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therefore also take account of this resistance. We have already said (XXX - see the note), how this force must be evaluated , & we will see later (XLI), how one must make it enter into the calculation. Pending this, it suffices to remark that the forces applied to the Machine in question here are not given by the weights but by the quantities of motion lost by the weights (XXX) - to be estimated by the tensions of the cords by which they are suspended. However, whether the Machine is at rest or in motion, and whether the motion is uniform or not, the tension of the cord attached to the wheel is to that of the cord attached to the cylinder, as the radius of the cylinder is to the radius of the wheel - that is to say, the tensions are always in reciprocal ratio to the speeds of the weights they support. This is in accord with the proposition. But these tensions are not equal to the weights; they are (XXX - see the note) the resultants of these weights and their forces of inertia, which are themselves the resultants of the actual motions of the bodies & the equal & directly opposed motions to those they actually have the instant afterwards.

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quelconque de puiſſances : auſſi la premiere conſéquence de ce théorême, ſera ce principe de *Deſcartes*, rendu complet par les conditions que nous avons vu lui manquer (V).

Corollaire I.

Principe général de l'équilibre entre deux puiſſances.

XXXV. *Lorſque deux agents quelconques, appliqués à une Machine, ſe font mutuellement équilibre; ſi on fait prendre à cette Machine un mouvement géométrique, arbitraire; 1°. les forces exercées par les agents, ſeront en raiſon réciproque de leurs vîteſſes eſtimées dans le ſens de ces forces; 2°. l'une de ces puiſſances fera un angle aigu avec la direction de ſa vîteſſe, & l'autre, un angle obtus avec la ſienne.*

Car ſi les forces exercées par les agents, ſont nommées F & F', leurs vîteſſes u & u', les angles formés par ces puiſſances & leurs vîteſſes z & z', on aura par le théorême précédent, $F u \cos z + F' u' \cos z' = 0$; donc $F : F' :: - u' \cos z' : u \cos z$, qui eſt la proportion énoncée par la premiere partie de ce corollaire, & par laquelle on voit en même temps que le rapport de $\cos z$ à $\cos z'$, eſt négatif; d'où il ſuit que l'un de ces angles eſt néceſſairement aigu, & l'autre obtus.

of forces. Also, the first consequence of this theorem will be the principle of *Descartes*, rendered complete by the conditions that we have seen were omitted by him (V).

Corollary I.

General principle of equilibrium between two forces.

XXXV. *When any two agents*[46] *are applied to a Machine and are mutually in equilibrium, then, if one imparts to this Machine an arbitrary geometric motion: 1st, the forces exercised by these agents are in the reciprocal ratio of their speeds as estimated in the direction of these forces; 2nd, one of these forces will be at an acute angle with the direction of its speed, & the other, an obtuse angle with its speed.*

Because if these forces exerted by the agents are named F & F', their speeds u & u', the angles formed by these forces & their speeds z & z', one will have by the preceding theorem, $Fucosz + F'u'cosz' = 0$. Therefore $F : F' :: -u'cosz' : ucosz$, which is the proportion stated in the first part of this corollary, & by which one sees at the same time that the relationship of $cosz$ to $cosz'$ is negative; from which it follows that one of the angles is necessarily acute, & the other obtuse.

Corollaire II.

Principe général d'équilibre dans les Machines à poids.

XXXVI. *Lorſque pluſieurs poids appliqués à une Machine quelconque, ſe font mutuellement équilibre, ſi l'on fait prendre à cette Machine un mouvement quelconque géométrique, la vîteſſe du centre de gravité du ſyſtême, eſtimée dans le ſens vertical, ſera nulle au premier inſtant.*

Car ſi l'on appelle M la maſſe totale du ſyſtême, m celle de chacun des corps qui le compoſent, u la vîteſſe abſolue de m, V la vîteſſe du centre de gravité eſtimée dans le ſens vertical, g la gravité, z l'angle formé par u & par la direction de la peſanteur, on aura, ſuivant le théorême, $\int m\, g\, u \cos z = 0$, mais par les propriétés géométriques du centre de gravité, on a $\int m\, u\, d\, t \cos z = M\, V\, d\, t$, ou $\int m\, g\, u \cos z = M\, V\, g$; donc, puiſque le premier membre de cette équation eſt égal à zéro, le ſecond l'eſt auſſi; donc $V = 0$ *ce qu'il falloit prouver.*

Pour avoir toutes les conditions de l'équilibre dans une Machine à poids, il n'y a donc qu'à faire prendre ſucceſſivement à la Machine différents mouvements géométriques, & égaler dans chacun de ces cas, la vîteſſe verticale du centre de gravité à zéro.

Corollary II.

General principle of equilibrium in weight Machines.

XXXVI. *When several weights applied to any Machine are mutually in equilibrium, and one imparts any geometric motion to this Machine, then the speed of the centre of gravity of the system, estimated in a vertical sense, will be zero in the first instant.*

Because, if one calls M the total mass of the system, m that of each of the bodies that make it up, u the absolute speed of m, V the speed of the centre of gravity estimated in a vertical sense, g gravity, z the angle formed by u & by the direction of gravitational attraction, one will have (following the theorem), $\sum mgucosz = 0$, but by the geometric properties of the centre of gravity, one has $\sum mudtcosz = MVdt$, or $\sum mgucosz = MVg$. Therefore, since the first member of this equation is equal to zero, the second is also; therefore $V = 0$ *which is what had to be proved*.

To satisfy all the conditions for equilibrium in a weight Machine, it is therefore only necessary to consider successively different geometric motions of the Machine, & set the vertical speed of the centre of gravity to zero in each case.

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Corollaire III.

Principe général de l'équilibre entre deux poids.

XXXVII. *Lorſque deux poids ſe font mutuellement équilibre, ſi l'on fait prendre à la Machine un mouvement quelconque géométrique.*

1°. *Les vîteſſes de ces corps, eſtimées dans le ſens vertical, ſeront en raiſon réciproque de leurs poids.*

2°. *L'un de ces corps montera néceſſairement, tandis que l'autre deſcendra.*

Cette propoſition eſt une ſuite manifeſte du corollaire précédent, & ſe déduit plus évidemment encore du corollaire premier.

On peut remarquer en paſſant, combien il eſt eſſentiel pour l'exactitude de toutes ces propoſitions, que les mouvements imprimés à la Machine ſoient géométriques, & non pas ſimplement poſſibles; car la plus légere attention fera voir par quelque exemple particulier, que ſans cette condition, toutes ces propoſitions ſeroient abſurdes.

Remarque.

XXXVIII. On prend ordinairement pour principe de l'équilibre dans les Machines à poids, qu'alors le centre de gravité du ſyſtême eſt au point le plus bas poſſible; mais on ſait que ce principe n'eſt pas généralement vrai; car outre que ce point pourroit dans certains cas, être au point le plus haut, il y en a une infinité d'autres où il n'eſt ni au point le plus haut, ni au point

Corollary III.

General principle of equilibrium between two weights.

XXXVII. *When two weights are in mutual equilibrium, if one imparts any geometric motion to the Machine.*

1st *The speeds of these bodies, taken in the vertical sense, will be in the reciprocal ratio of their weights.*

2nd *One of the weights will necessarily rise, while the other descends.*

This proposition is one manifestly following from the preceding corollary, & is deduced even more evidently than the first corollary.

One may remark in passing, how very essential it is for the accuracy of all these propositions, that the motions imparted to the Machine are geometric & not simply possible; because the merest glance will show (looking at certain specific examples) that without this condition all the propositions will be absurd.

Remark.

XXXVIII. One ordinarily understands that for the principle of equilibrium in weight Machines, the centre of gravity of the system is at the lowest point possible; but one knows that principle is not generally true because, besides certain cases where it can be the highest point, there are an infinity of other [cases] where it is neither the highest point nor the lowest point.

le plus bas : par exemple, ſi tout le ſyſtême ſe réduit à un corps peſant, & que ce mobile ſoit placé ſur une courbe qui ait un point d'inflexion, dont la tangente ſoit horiſontale ; il reſtera viſiblement en équilibre, ſi on le met ſur ce point d'inflexion, qui n'eſt cependant ni le poids le plus bas, ni le point le plus haut poſſible.

On peut encore prendre pour principe de l'équilibre dans une Machine à poids, la propoſition que nous avons déjà donnée (II), & que nous allons rapporter encore, pour en donner la démonſtration rigoureuſe.

Pour s'aſſurer que pluſieurs poids appliqués à une Machine quelconque, doivent ſe faire mutuellement équilibre, il ſuffit de prouver que ſi l'on abandonne cette Machine à elle-même, le centre de gravité du ſyſtême ne deſcendra pas.

Pour le prouver, nommons M la maſſe totale du ſyſtême, m celle de chacun des poids qui le compoſent, g la gravité ; & ſuppoſons que ſi la Machine ne demeuroit pas en équilibre, comme je prétends qu'elle doit le faire, la vîteſſe de m après le temps t, fût V, la hauteur dont ſeroit deſcendu le centre de gravité au bout du même temps H, & celle dont ſeroit deſcendu le corps m h ; on aura donc, (XXIV) $\int m\,g\,d\,h - \int m\,V\,d\,V = 0$; donc en intégrant $M\,g\,H = \frac{1}{2}\int m\,V^2$; or par hypotheſe $H = 0$, donc $\int m\,V^2 = 0$; de plus V^2 eſt néceſſairement poſitive comme il eſt évident ; donc l'équation $\int m\,V^2 = 0$ ne peut avoir lieu ſans qu'on n'ait $V = 0$, c'eſt-à-dire ſans qu'il y ait équilibre ; *ce qu'il falloit prouver.*

Il ſuit de-là, comme nous l'avons dit (III), qu'il y a néceſſairement équilibre dans un ſyſtême de poids dont le centre de gravité eſt au point

For example, if the whole system reduces to a heavy[47] body, & this movable body is placed on a curve which has a point of inflection whose tangent is horizontal, then it will manifestly stay in equilibrium if it is placed at the point of inflection, which is not, however, either the lowest or the highest possible point [for this weight].

We may again take as the principle of equilibrium in a weight-Machine the proposition which we have already given (II), & that we are going to state again, in order to prove it rigorously[48].

To be sure that several weights applied to any Machine are in mutual equilibrium, it is sufficient to prove that if the Machine is left to itself, the centre of gravity of the system will not descend[49].

To prove it, let us name M the total mass of the system, m that of each of the weights that compose it, g the acceleration due to gravity; & let us suppose that if the Machine does not stay in equilibrium, as I claim, then the speed of m after time t, would be V, the height that the centre of gravity would have descended in the same time H, & the height through which the bodies descended mh; we will therefore have, (XXIV) $\sum mgdh - \sum mVdV = 0$, and therefore after integrating $MgH = \frac{1}{2}\sum mV^2$. Yet by hypothesis $H = 0$, thus $\sum mV^2 = 0$ which cannot hold unless $V = 0$, that is to say, without there being equilibrium - *which was what needed to be proved*.

It follows from this, as we have said (III), that there is necessarily equilibrium in a system of weights whose centre of gravity is at the le plus bas poſſible ; mais nous venons de voir (XXXVIII) que l'inverſe n'eſt pas toujours vraie, c'eſt-à-dire que toutes les fois qu'il y a équilibre dans un ſyſtême de poids, il ne s'enſuit pas toujours que le centre de gravité ſoit au point le plus bas poſſible.

Corollaire IV.

Loix particulieres d'équilibre dans les Machines.

XXXIX. *S'il y a équilibre entre pluſieurs puiſſances appliquées à une Machine, & qu'ayant décompoſé toutes les forces du ſyſtême, tant celles qui ſont appliquées à la Machine, que celles qui ſont exercées par les obſtacles mêmes ou points fixes qui en font partie ; ſi on les décompoſe, dis-je, chacune en trois autres paralleles à trois axes quelconques perpendiculaires entre eux ;*

1°. *La ſomme des forces compoſantes, qui ſont paralleles à un même axe, & conſpirantes vers un même côté, eſt égale à la ſomme de celles qui, étant paralleles à ce même axe, conſpirent vers le côté oppoſé :*

2°. *La ſomme des moments des forces compoſantes, qui tendent à faire tourner autour d'un même axe, & qui conſpirent dans un même ſens, eſt égale à la ſomme des moments de celles qui tendent à faire tourner autour du même axe, mais en ſens contraire.*

Pour démontrer cette propoſition, commençons par imaginer qu'à la place de chacune des forces exercées par la réſiſtance des obſtacles, on ſubſtitue une force active, égale à cette rélowest point possible; but we have seen (XXXVIII) that the inverse is not always true, that is to say, that every time there is equilibrium in a system of weights, it does not always follow that the centre of gravity is at the lowest point possible.

Corollary IV.

Specific laws of equilibrium in Machines.

XXXIX. *If there is equilibrium between several forces applied to a Machine &, having decomposed all the forces of the system (those applied to the Machine, & those exerted by the barriers or fixed points in the Machine), if we decompose them, I say, each into three other forces parallel to any three mutually perpendicular axes then*;

1st. *The sum of the component forces, which are parallel to the same axis, & directed to the same side, is equal to the sum of those which, being parallel to the same axis, are directed towards the opposing side.*

2nd. *The sum of the moments of the component forces, which tend to turn around the same axis & which act in the same direction, is equal to the sum of the moments of those which turn about the same axis, but in the opposite direction.*

To prove this proposition, let us start by imagining that in place of the forces exerted by the resistance of barriers, we substitute an active force, equal to this ſiſtance, & dirigée dans le même ſens ; ce changement n'altere point l'état d'équilibre, & fait de la Machine un ſyſtême de puiſſances parfaitement libre, c'eſt-à-dire dégagé de tout obſtacle : cela poſé, ſi l'on fait prendre à ce ſyſtême un mouvement quelconque géométrique, on aura par le théorême fondamental $\int F\, u \cos z = 0$, en nommant F chacune des forces, u ſa vîteſſe, & z l'angle compris entre F & u ; donc,

1°. Si l'on ſuppoſe que u ſoit la même pour tous les points du ſyſtême & parallele à l'un des axes quelconque, le mouvement ſera géométrique, & l'équation à cauſe de u conſtante, ſe réduira à $\int F \cos z = 0$: c'eſt-à-dire que la ſomme des forces du ſyſtême, eſtimées dans le ſens de la *vîteſſe u*, imprimée parallelement à cet axe, ſera nulle ; ce qui revient évidemment à la premiere partie de la propoſition.

2°. Si l'on fait tourner tout le ſyſtême autour de l'un, quelconque, des axes, ſans rien changer à la poſition reſpective des parties qui le compoſent, ce mouvement ſera encore géométrique ; u ſera proportionnelle à la diſtance de chaque puiſſance à l'axe ; & partant, pourra s'exprimer par $A\,R$, R exprimant cette diſtance, & A une conſtante ; donc, l'équation ſe réduira à $\int F\,R \cos z = 0$; ce qui, comme il eſt aiſé de le voir, revient à la ſeconde partie de la propoſition.

Corollaire V.

Loi particuliere concernant les Machines dont le mouvement change par degrés inſenſibles.

XLI. *Dans une Machine dont le mouvement* resistance, & directed in the same direction. This change does not alter the state of equilibrium, & makes the Machine into a system of perfectly free forces, that is to say, with no constraints operating. This assumed, if we impart to the system any geometric motion, we will have, by the fundamental theorem, $\sum Fucosz = 0$, (denominating the forces F, u the speed, & z the angle beween F & u; therefore

1st. If we suppose that u is the same for all points of the system & it is parallel to one or another axis, then the motion will be geometric, & the equation will reduce to $\sum Fcosz = 0$ due to u being constant - that is to say, the sum of the forces in the system, estimated in the direction of the *speed u* and acting parallel to this axis, will be null. This evidently amounts to the first part of the proposition.

2nd. If we turn the system around any of these [Cartesian] axes, without any change to the respective positions of the parts which compose it, then this motion will be geometric; u will be proportional to the distance of each force to the axis & so can be expressed by AR, where R expresses this distance & A is a constant. Thus, the equation reduces to $\sum FRcosz = 0$ which, as is easy to see, amounts to the second part of the proposition.

Corollary V.

Specific law concerning Machines whose motion changes by insensible degrees[50].

XLI. *In a Machine whose motion*

change par degrés insensibles, le moment d'activité, consommé *dans un temps donné par les forces sollicitantes*, *est égal* au moment d'activité, exercé *en même temps par les forces résistantes.*

C'est-à-dire (XXXIII), que le *moment d'activité, consommé* par toutes les forces du systême, pendant le temps donné, est égal à zéro; ce qui sera clair (XXXII), si l'on prouve que *le moment d'activité*, *consommé* à chaque instant par ces forces, est nul: or, F exprimant chacune de ces forces, V sa vîtesse, Z l'angle compris entre F & V, & $d\,t$ l'élément du temps, *le moment d'activité*, *consommé* par toutes les forces du systême pendant $d\,t$, est (XXXIII), $\int F\,V \cos Z\,d\,t$; il faut donc prouver qu'on a $\int F\,V \cos Z\,d\,t = 0$, ou $\int F\,V \cos Z = 0$; or, cela est clair par le théorême fondamental: donc, *&c.*

La loi particuliere dont il s'agit ici, est certainement la plus importante de toute la théorie du mouvement des Machines proprement dites: en voici quelques applications particulieres, en attendant le détail où nous entrerons à son sujet, dans le scholie qui succédera au corollaire suivant, & qui terminera cet Essai.

XLII. Supposons donc, par exemple, que les puissances appliquées à la Machine, soient des poids; nommons m la masse de chacun de ces corps, M la masse totale du systême, g la gravité, V la vîtesse actuelle du corps m, K sa vitesse initiale, t le temps écoulé depuis le commencement du mouvement, H la hauteur dont est descendu le centre de gravité du systême pendant le temps t, & enfin, W la vîtesse due à la hauteur H.

Cela posé, il faut considérer qu'il y a deux sortes de forces appliquées à la Machine; savoir:

changes by insensible degrees, the moment of activity, consumed in a given time by the soliciting forces, is equal to the moment of activity exerted in the same time by the resisting forces.

That is to say, (XXXIII), that the *moment of activity consumed* by all the forces of the system during the given time is equal to zero, which will be clear, (XXXII), if one proves that the *moment of activity consumed* at each instant by these forces is null. However, taking F as expressing each of these forces, V its speed, Z the angle between F & V, & dt the element of time, then the *moment of activity consumed* by all the forces of the system during dt is, (XXXIII), $\sum \int FVcosZdt$. It is necessary then to prove $\sum \int FVcosZdt = 0$ or $\sum FVcosZ = 0$ – but this is [already] clear from the fundamental theorem (and therefore, &c.).

The specific law stated here is without doubt the most important law in the whole of the theory of motion of Machines proper. Here are some particuar applications, still awaiting detail, which we will introduce in the scholium that will follow the succeeding corollary, & which will end this Essay.

XLII. Let us suppose, for example, that the forces applied to the Machine are weights. Let us name m the mass of each of the bodies, M the total mass of the system, g the acceleration due to gravity, V the actual speed of the bodies, K their initial speed, t the time elapsed since the start of the motion, H the height that the centre of gravity of the system descends during time t, & finally, W the speed due to the height H.

This assumed, it is necessary to consider that there are two sorts of forces applied to the Machine. These are

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celles qui viennent de la peſanteur des corps, & celles qui viennent de leur inertie ou réſiſtance qu'ils oppoſent à leur changement d'état, (note c (XXX)): or, (XXXII) le moment d'activité, conſommé pendant le temps t, par la premiere de ces forces, eſt pour tout le ſyſtême $M g H$, ou $\frac{1}{2} M W^2$; voyons maintenant quel eſt le moment d'activité, conſommé par la force d'inertie: la vîteſſe de m étant V, & devenant l'inſtant d'après $V + dV$, il eſt clair (note b (XXX)), que ſa force d'inertie eſtimée dans le ſens de V, eſt $m\, dV$, ou plutôt $m \frac{dV}{dt}$; donc, (XXX), le moment d'activité, exercé par cette force pendant dt, eſt $m \frac{dV}{dt} V dt$, ou $mVdV$; donc, le moment d'activité, conſommé par cette force d'inertie, pendant le temps t, eſt $\int m V dV$, ou en intégrant & complétant l'intégrale $\frac{1}{2} m V^2 - \frac{1}{2} m K^2$; donc le moment d'activité, conſommé en même temps par la force d'inertie, de tous les corps du ſyſtême, ſera $\frac{1}{2} \int m V^2 - \frac{1}{2} \int m K^2$; or, cette inertie eſt une force réſiſtante, puiſque c'eſt par elle que les corps réſiſtent à leur changement d'état: & la peſanteur eſt ici une force ſollicitante, puiſque le centre de gravité eſt ſuppoſé deſcendre; donc, par la propoſition de ce corollaire, on doit avoir $M W^2 = \int m V^2 - \int m K^2$, ou $\int m V^2 = \int m K^2 + M W^2$: c'eſt-à-dire que

Dans une Machine à poids, dont le mouvement change par degrés inſenſibles, la ſomme des forces vives du ſyſtême, eſt après un temps quelconque donné, égale à la ſomme des forces vives initiales; plus, la ſomme de force vive qui auroit lieu, ſi tous those that come from the weight of the bodies & those which come from their inertial resistance (which opposes any change of state) (note 3 (XXX))[*]. Now, (XXXII) the moment of activity consumed during time t by the first of these forces is, for the whole system, MgH, or $\frac{1}{2}MW^2$. Let us now see what is the moment of activity, consumed by the force of inertia. The speed of m being V, & becoming the instant after $V + dV$, it is clear (note b (XXX))that the force of inertia calculated in the direction of V is mdV or, rather, $m\frac{dV}{dt}$. Therefore, (XXX), the moment of activity exerted by this force during dt is $m\frac{dV}{dt}Vdt$ or $mVdV$. Therefore, the moment of activity consumed by the force of inertia during time t is $\sum mVdV$ or, on integrating & completing the integral, $\frac{1}{2}mV^2 - \frac{1}{2}mK^2$. Therefore, the moment of activity consumed at the same time by the force of inertia of all the bodies of the system will be $\frac{1}{2}\sum mV^2 - \frac{1}{2}\sum mK^2$. However, this inertia is a resisting force, as it is in this way that bodies resist their change of state: & the [heaviness] gravity is here a soliciting force, since the centre of gravity is assumed to descend; therefore, by the proposition of this corollary, we must have $MW^2 = \sum mV^2 - \sum mK^2$; *or* $\sum mV^2 = \sum mK^2 + MW^2$, that is to say

In a weight Machine whose motion changes by insensible degrees the sum of the live forces is, after a given time, equal to the sum of the initial live forces plus the sum of the live forces that would result if all

[*] See *Errata*.

les corps du ſyſtême étoient animés d'une vîteſſe commune, égale à celle qui eſt due à la hauteur dont eſt deſcendu le centre de gravité du ſyſtême.

XLIII. Si le mouvement de la Machine eſt uniforme, on aura continuellement $V = K$, & partant $W^2 = 0$, ou $H = 0$; ce qui nous apprend que

Dans une Machine à poids, dont le mouvement eſt uniforme, le centre de gravité du ſyſtême reſte conſtamment à la même hauteur.

XLIV. Puiſque $\frac{1}{2} M W^2$ ou $M g H$ eſt (XXXII) le moment d'activité, produit par un poids $M g$, qu'on fait monter à la hauteur H, il s'enſuit évidemment que

De quelque maniere qu'on s'y prenne pour élever un certain poids à une hauteur donnée, il faut que les forces qui ſont employées à produire cet effet, conſomment un moment d'activité, égal au produit de ce poids, par la hauteur à laquelle on doit l'élever.

XLV. De même, puiſque (XLII) le moment d'activité, produit dans un temps donné par la force d'inertie d'un corps, eſt égal à la moitié de la quantité dont ſa force vive augmente pendant ce temps; on peut conclure auſſi que

Pour faire naître un certain mouvement quelconque par degrés inſenſibles dans un ſyſtême de corps, ou changer celui qu'il a, il faut que les puiſſances deſtinées à cet effet, conſomment un moment d'activité, égal à la moitié de la quantité dont aura augmenté par ce changement la ſomme des forces vives du ſyſtême.

XLVI. Il ſuit évidemment de ces deux dernieres propoſitions, que pour élever un poids $M g$ à une hauteur H, & lui faire prendre en même temps une vîteſſe V, il faut, en ſuppoſant *the bodies of the system had a common speed, equal to that which is due to the fall in height of the centre of gravity of the system.*

XLIII. If the motion of the Machine is uniform we will have $V = K$ continually, & thus $W^2 = 0$ or $H = 0$, from which we deduce that:

In a weight Machine, where the motion is uniform, the centre of gravity of the system remains constantly at the same height.

XLIV. Because (by XXXII) the moment of activity produced by a weight Mg rising to a height H is given by $\frac{1}{2}MW^2$ or MgH, it evidently follows that:

No matter in what way one raises a certain weight to a given height, it is necessary that the forces which are used to produce this effect consume a moment of activity equal to the product of this weight and the height to which it must be raised.

XLV. Similarly, since ([by] XLII) the moment of activity produced in a given time by the force of inertia of a body is equal to half the quantity by which its live force will be increased during this time, we can also conclude that:

To generate, by insensible degrees, any motion in a system of bodies, or to change that which it has, it is necessary that the forces that produce this effect consume a moment of activity equal to half the quantity by which the live force of the system will be increased by this change.

XLVI. It evidently follows from the last two propositions that in order to raise a weight Mg by a height H, & at the same time give it a speed V, it is necessary, supposing ce corps en repos au premier inſtant, que les forces employées à produire cet effet, conſomment elles-mêmes un moment d'activité égal à $M g H + \frac{1}{2} M V^2$.

LXVII. On ſuppoſe dans tout ce qui vient d'être dit, comme l'annonce le titre de ce corollaire, que le mouvement change par degrés inſenſibles; mais, ſi chemin faiſant, il arrivoit un choc ou changement ſubit dans le ſyſtême, ce que nous venons de dire n'auroit plus lieu. Suppoſons, par exemple, qu'au moment où arrive le choc, le centre de gravité du ſyſtême ſoit deſcendu de la hauteur h; qu'à ce même inſtant, la ſomme des forces vives ſoit X immédiatement avant le choc, & Y immédiatement après; nommons Q le moment d'activité qu'auront à conſommer les forces mouvantes pendant tout le temps du mouvement, & q celui qu'elles auront à conſommer depuis le commencement juſqu'à l'époque de la percuſſion; ſuppoſons enfin, pour plus de ſimplicité, que le ſyſtême ſoit en repos au premier inſtant & au dernier, il eſt clair (XLVI) qu'on aura $q = M g h + \frac{1}{2} X$, & que par la même raiſon, le moment d'activité à conſommer par les forces mouvantes après le choc, c'eſt-à-dire $Q - q$ ſera $M g (H - h) - \frac{1}{2} Y$, donc $Q = M g H + \frac{1}{2} X - \frac{1}{2} Y$; or, (XXIII) il eſt clair que $X > Y$, donc, le moment d'activité à conſommer pour élever dans ce cas M à la hauteur H, eſt néceſſairement plus grand que s'il n'y avoit point de choc, puiſque dans ce cas, on auroit ſimplement $Q = M g H$ (XLIV).

Il ſuit de là, que ſans conſommer un plus grand moment d'activité, les forces mouvantes peuvent, en évitant qu'il y ait choc, élever le même poids à une hauteur plus grande H, car this body is at rest in the first instant, that the forces employed to produce this effect, themselves consume a moment of activity equal to $MgH + \frac{1}{2}MV^2$.

LXVII. We suppose that in everything that has just been said, as stated in the title of this corollary, the motion changes by insensible degrees. However, if in moving on its path, a collision or sudden change in the system happens, what we have just said will not apply. Let us suppose, for example, that at the moment the collision happens, the centre of gravity of the system has descended by a height h and that, at the same moment, the sum of live forces is X immediately before the collision, & Y immediately afterwards. Let us name as Q the moment of activity which will be consumed by the moving forces during the period of motion, & as q that which has been consumed since the beginning up to the moment of collision. Let us finally suppose, for greater simplicity, that the system is at rest in the first instant & at the last. Then it is clear (by XLVI) that we will have $q = Mgh + \frac{1}{2}X$ & that, for the same reason, the moment of activity consumed by the moving forces after the collision, that is to say, $Q - q$ will be $Mg(H - h) - \frac{1}{2}Y$, and so $Q = MgH + \frac{1}{2}X - \frac{1}{2}Y$. However, (by XXIII) it is clear that $X > Y$, and thus the moment of activity consumed in raising M to the height H, in this case, is necessarily greater than if there had been no collision, since in this case we would simply have $Q = MgH$ (XLIV).

It follows from this that without consuming a greater moment of activity the forces can, in avoiding a collision, raise the same weight to a greater height H because alors on aura (XLVI) $Q = M g H'$, ou $H' = \frac{Q}{Mg}$, tandis que dans le cas préſent, on a $H = \frac{Q - \frac{1}{2}(X - Y) :}{M g}$ d'où l'on voit que X étant plus grande que Y, il faut néceſſairement qu'on ait auſſi $H' > H$.

Corollaire V I.

Des Machines hydrauliques.

XLXVIII. On peut regarder un fluide comme l'aſſemblage d'une infinité de corpuſcules ſolides, détachés les uns des autres; on peut donc appliquer aux Machines hydrauliques tout ce que nous avons dit des autres Machines : ainſi, par exemple, du corollaire premier (XXXV), on peut conclure que ſi une maſſe fluide, ſans peſanteur, étant enfermée de tout côté dans un vaſe, & qu'ayant fait à ce vaſe deux petites ouvertures égales, on y applique des piſtons; les forces qui agiront ſur la maſſe fluide, en pouſſant ces piſtons, ne peuvent qu'être égales, ſi elles ſe font mutuellement équilibre; c'eſt-à-dire donc que dans une maſſe fluide, la preſſion ſe répand également en tout ſens; c'eſt le principe fondamental de l'équilibre des fluides, qu'on regarde ordinairement comme une vérité purement expérimentale : on prouvera de même (XXV) que la conſervation des forces vives a lieu dans les fluides incompreſſibles, dont le mouvement change par degrés inſenſibles; & généralement enfin tout ce que nous avons prouvé d'un ſyſtême de corps durs, eſt également vrai pour une maſſe de fluide incompreſſible.

Scholie.

we will then have (by XLVI) $Q = MgH'$, where $H' = Q/Mg$, whereas in the present case we have $H = \frac{Q - \frac{1}{2}(X-Y)}{Mg}$. From all this we see that X being greater than Y, it is necessary that we also have $H' > H$.

Corollary VI.

Hydraulic Machines

XLXVIII. We can regard a fluid as an assemblage of an infinite number of solid particles, disconnected from each other. We can then apply to hydraulic Machines all that we have said of other Machines. Thus, for example, in the first corollary (XXXV), we can conclude that if a fluid mass, without weight, were enclosed on all sides in a vessel, & that, having given this vessel two equal little openings, we apply pistons, then the forces which act on the fluid mass in pushing these pistons must be equal if they are in mutual equilibrium. That is to say, therefore, that in a fluid mass the pressure acts equally in all directions: this is the fundamental principle of the equilibrium of fluids, which we ordinarily regard as a purely experimental fact. We will prove also ([by] XXV) that the conservation of live force holds in incompressible fluids whose motion changes by insensible degrees. Finally, in general, everything that we have proved for a system of hard bodies is equally true for a mass of incompressible fluid. *Scholium.*

Scholie.

XLIX. Ce ſcholie eſt deſtiné au développement du principe énoncé dans le corollaire V ; cette propoſition renferme en effet la principale partie de la théorie des Machines en mouvement, parce que la plupart d'entr'elles ſont mues par des agents qui ne peuvent exercer que des forces mortes ou de preſſion ; tels ſont tous les animaux, les reſſorts, les poids, *&c.* ce qui fait que la Machine change ordinairement d'état par degrés inſenſibles. Il arrive même le plus ſouvent que cette Machine paſſe bien vîte à l'uniformité de mouvement ; en voici la raiſon :

Les agents qui font mouvoir cette Machine, ſe trouvant d'abord un peu au deſſus des forces réſiſtantes, font naître un petit mouvement qui s'accélere enſuite peu-à-peu ; mais ſoit que par une ſuite néceſſaire de cette accélération, la force ſollicitante diminue, ſoit que la réſiſtance augmente, ſoit enfin qu'il ſurvienne quelque variation dans les directions, il arrive preſque toujours que le rapport des deux forces s'approche de plus en plus de celui en vertu duquel elles pourroient ſe faire mutuellement équilibre : alors ces deux forces ſe détruiſent, & la Machine ne ſe meut plus qu'en vertu du mouvement acquis, lequel, à cauſe de l'inertie de la matiere, reſte ordinairement uniforme.

L. Pour comprendre encore mieux comment cela doit arriver, il n'y a qu'à faire attention au mouvement que prend un navire qui a le vent en poupe ; c'eſt une eſpece de Machine animée par deux forces contraires qui ſont l'impulſion du vent & la réſiſtance du fluide ſur lequel il

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Scholium.

XLIX. This scholium is intended for the development of the principle enunciated in corollary V. The proposition encompasses, in effect, the main part of the theory of Machines in motion, because most of them [the other propositions] are mute when it comes to agents that can only exert dead forces or pressure – such as [is the case for] all animals, springs, weights, &c – that ordinarily make the Machine change its state by insensible degrees. [However] It happens even more often that this Machine passes very quickly to [the state of] uniform motion; here is the reason

The agents [working substances] which move this Machine, finding themselves at the beginning a little greater than the resisting forces, give rise to a little motion which subsequently accelerates little by little. But, whether by a necessary consequence of this acceleration – the soliciting force diminishes, whether the resistance increases or whether, finally, some variation in the direction occurs), it almost always happens that the relation of the two forces [(the resisting force and the soliciting force)] approaches more and more to that needed to make them mutually in equilibrium. Therefore these two forces destroy each other & the Machine does not move except by virtue of the acquired motion – which, because of the inertia of matter, usually remains uniform.

L. In order to better understand how this must happen, one need only consider the motion taken up by a ship that has wind at the stern it is a type of Machine subject to two contrary forces – the propulsion of the wind, & the resistance of the fluid on which

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vogue : ſi la premiere de ces deux forces qu'on peut regarder comme ſollicitante, eſt la plus grande, le mouvement du navire s'accélérera ; mais cette accélération a néceſſairement des bornes, par deux raiſons ; car, plus le mouvement du navire s'accélere, 1°. plus il eſt ſouſtrait à l'impulſion du vent ; 2°. plus au contraire la réſiſtance de l'eau augmente : par conſéquent, ces deux forces tendent à l'égalité : lorſqu'elles y ſeront parvenues, elles ſe détruiront mutuellement ; & partant, le navire ſera mu comme un corps libre, c'eſt-à-dire que ſa vîteſſe ſera conſtante. Si le vent venoit à baiſſer, la réſiſtance de l'eau ſurpaſſeroit la force ſollicitante ; le mouvement du navire ſe ralentiroit ; mais par une ſuite néceſſaire de ce ralentiſſement, le vent agiroit plus efficacement ſur les voiles ; & la réſiſtance de l'eau diminueroit en même temps : ces deux forces tendroient donc encore à l'égalité, & la Machine arriveroit de même à l'uniformité de mouvement.

LI. La même choſe arrive lorſque les forces mouvantes ſont des hommes, des animaux ou autres agents de cette nature : dans les premiers inſtants, le moteur eſt un peu au deſſus de la réſiſtance ; de là naît un petit mouvement qui s'accélere peu-à-peu, par les coups répétés de la force mouvante ; mais l'agent lui-même eſt obligé de prendre un mouvement accéléré, afin de reſter attaché au corps auquel il imprime le mouvement. cette accélération qu'il ſe procure à lui-même, conſomme une partie de ſon effort ; de ſorte qu'il agit moins efficacement ſur la Machine, & que le mouvement de celle-ci s'accélérant de moins en moins, finit par devenir bientôt uniforme. Par exemple, un homme qui pourroit faire un it sails. If the first of these forces (the soliciting force), is the greater, then the motion of the ship will accelerate. But this acceleration necessarily has limits, for two reasons: the more the motion of the ship accelerates then; 1st, the more it reduces the impulse of the wind, & 2nd, the more the (oppositional) resistance of the water increases. In consequence, these two forces tend to equality. When they arrive at this equality, they will mutually cancel out & so the ship will be moving like a free body, that is to say, [the ship's] speed will be constant. If the wind dies, the resistance of the water exceeds the soliciting force and the motion of the ship slows. But, by a necessary consequence of this slowing, the wind will act more effectively on the sails, & the resistance of the water diminishes at the same time. These two forces will therefore again tend to equality, & the Machine will likewise come to uniformity of motion.

LI. The same thing happens when the moving forces are men, animals or other agents of this nature. In the first instants, the driving force is a little greater than the resistance; from this, a small motion is born, and this increases little by little, by repeated thrusts of the moving force. But the agent itself is obliged to speed up its motion in order to stay attached to the body to which it imparts the motion. This acceleration (which it procures from itself) consumes a part of its effort. As a result of this, the agent acts less effectively on the Machine – the motion increases less and less, soon ending up by becoming uniform. For example, a man who exerts a

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certain effort dans le cas d'équilibre, en feroit un beaucoup moindre, si le corps auquel il est appliqué lui céde, & qu'il soit obligé de le suivre pour agir sur lui : ce n'est pas que le travail absolu de cet homme soit moindre, mais c'est que son effort est partagé en deux, dont l'un est employé à mettre la masse même de l'homme en mouvement, & l'autre, transmis à la Machine. Or, c'est de ce dernier seul que l'effet se manifeste dans l'objet qu'on s'est proposé.

Je continuerai cependant de considérer les Machines sous un point de vue plus général : ainsi je placerai dans ce scholie plusieurs réflexions applicables au mouvement varié ; je supposerai seulement que cette variation se fait par degrés insensibles, & je prouverai que cela doit être en effet, lorsqu'on veut les employer de la maniere la plus avantageuse possible.

LII. Désignons donc par Q le moment d'activité, consommé par les forces sollicitantes dans un temps donné t, & par q le moment d'activité exercé en même temps par les forces résistantes : cela posé, quel que soit le mouvement de la Machine, nous aurons toujours, par le corollaire V, $Q = q$; de sorte, par exemple, que si chacune F des forces sollicitantes, est constante, sa vîtesse V uniforme, & l'angle Z formé par les directions de F & V, toujours nul, on aura au bout du temps t $\int F V t = q$; & si toutes les forces sollicitantes se réduisent à une seule, on aura par conséquent $F V t = q$ (XXXII & XXXIII).

LIII. On peut en général regarder le moment q d'activité, exercé par les forces résistantes, comme l'effet produit par les forces sollicitantes ; par exemple, lorsqu'il s'agit d'élever un poids P à une hauteur donnée H, il est tout simple de

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certain effort under equilibrium conditions would exert a lot less if the body to which he applies his effort yields to him, & if he must follow that body in order to act upon it. It is not that the man's absolute capacity to do work is less, but rather that his effort is shared into two parts - one part employed to put his own mass in motion, & the other part transmitted to the Machine. But, it is only this last part that is manifest for our proposed purposes.

I will continue however to consider Machines from the most general point of view. Thus I will place in this scholium several reflections applicable to varied motions. I will suppose only that this variation is made by insensible degrees, & will prove that this must be so if one wants to employ them in the most advantageous manner possible.

LII. Let us designate by Q the moment of activity consumed by the soliciting forces in a given time t, & by q the moment of activity exerted in the same time by the resisting forces. This assumed, then whatever the motion of the Machine we will always have, by corollary V, $Q = q$. Thus, for example, if each F of the soliciting forces is constant, the speed V uniform, & the angle Z formed by the directions of F & V always null, we will have at the end of time t, $\sum FVt = q$, & if all the soliciting forces reduce to a single one, one will have in consequence[51] $FVt = q$ ([by] XXXII & XXXIII).

LIII. We can in general regard the moment of activity q exerted by the resisting forces as the effect produced by the soliciting forces. For example, when the question concerns the raising of a weight P to a given height H, it is very simple

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regarder l'effet produit par la force mouvante, comme étant en raiſon composée du poids & de la hauteur à laquelle il a fallu l'élever ; de ſorte que $P\,H$ eſt ce qu'on entend alors naturellement par l'effet produit. Or, d'un autre côté, cette quantité $P\,H$ eſt préciſément ce que nous avons appellé moment d'activité, exercé par la force réſiſtante P ; donc ce moment d'activité, ou q, eſt ce qu'on entend naturellement, dans ce cas, par l'effet produit.

Or, dans les autres cas, il eſt évident que q eſt toujours une quantité analogue à celle dont il vient d'être queſtion ; c'eſt pourquoi j'appellerai ſouvent dans la ſuite cette quantité, q, *effet produit :* ainſi, par *effet produit*, j'entendrai le moment d'activité, exercé par les forces réſiſtantes ; de ſorte qu'en vertu de l'équation $Q = q$, on peut établir pour regle générale, que *l'effet produit dans un temps donné par un ſyſtême quelconque de forces mouvantes, eſt égal au moment d'activité conſommé en même temps par toutes ces forces.*

LIV. On voit par l'équation $F\,V\,t = q$, trouvée dans l'article précédent, qu'il eſt inutile de connoître la figure d'une Machine, pour ſavoir quel effet peut produire une puiſſance qui lui eſt appliquée, lorſqu'on connoît celui qu'elle pourroit produire ſans Machine : ſuppoſons, par exemple, qu'un homme ſoit capable d'exercer un effort continuel de 25 ℔, en ſe mouvant continuellement lui-même avec une vîteſſe de trois pieds par ſeconde ; cela poſé, lorſqu'on l'appliquera à une Machine, le moment d'activité $F\,V\,t$ qu'exercera cet homme, ſera (XXXII) 25 ℔ 3 pi. t, c'eſt-à-dire qu'on aura $F\,V\,t =$ 25 ℔ 3 pi. t, t exprimant le nombre des ſecondes ; donc, à cauſe de $F\,V\,t = q$, on aura $q =$ 25 ℔ 3 pi. t, quelle que puiſſe to regard the effort produced by the moving force as being due to the weight & the height by which it has been raised - such that PH is what one naturally understands by the effect produced [work done]. But, on the other hand, this quantity PH is precisely what we have called the moment of activity exerted by the resisting force P. Therefore the moment of activity (in other words, q) is what one naturally means, in this case, by the effect produced.

However, in other cases, it is evident that q is always a quantity analogous to the one at issue. This is why I will often in what follows call this quantity q, the *effect produced* or the *work done*. Thus, by *work done*, I will understand the moment of activity exerted by the resisting forces. Thus by virtue of the equation $Q = q$, we can establish as a general rule that the *work done in a given time by any system of moving forces is equal to the moment of activity consumed at the same time by all these forces*.

LIV. We see by the equation $FVt = q$, found in the preceding article, that it is unnecessary to know the specific form of a Machine in order to know what effect a force applied to it can produce, when one [already] knows what effect it can produce without the Machine. Let us suppose, for example, that a man is capable of exerting a continual effort of 25 lb * while he himself is moving at a speed of three feet per second. That assumed, when one applies to a Machine the moment of activity Fvt that this man will exert, it will have the value 25lb $\times$ 3 ft$\times$ t (by XXXII), that is to say, we will have $FVt = 25\text{lb} \times 3\text{ft} \times t$, where t expresses the number of seconds. Thus, as $FVt = q$, we will have $q = 25\text{lb} \times 3\text{ft} \times t$ whatever

*The symbol used in the original is ℔, the French 'livre tournois', a unit of currency.

être la Machine; donc, l'effet q eſt abſolument indépendant de la figure de cette Machine, & ne peut jamais ſurpaſſer celui que la puiſſance eſt en état de produire naturellement & ſans Machine.

Ainſi, par exemple, ſi cet homme avec ſon effort de 25 ℔, & ſa vîteſſe de trois pieds par ſeconde, eſt en état avec une Machine donnée, ou ſans Machine, d'élever dans un temps donné, un poids p à une hauteur H, on ne peut inventer aucune Machine par laquelle il ſoit poſſible, avec le même travail, (c'eſt-à-dire la même force & la même vîteſſe que dans le premier cas), d'élever dans le temps donné, le même poids à une plus grande hauteur, ou un poids plus grand à la même hauteur, ou enfin le même poids à la même hauteur dans un temps plus court: ce qui eſt évident, puiſqu'alors q étant (XXXII) égal à $P\,H$, on a par l'article précédent, $P\,H = 25$ ℔ 3 $p^{i}.$ $t.$

L V. L'avantage que procurent les Machines, n'eſt donc pas de produire de grands effets avec de petits moyens, mais de donner à choiſir entre différents moyens qu'on peut appeller égaux, celui qui convient le mieux à la circonſtance préſente. Pour forcer un poids P à monter à une hauteur propoſée, un reſſort à ſe fermer d'une quantité donnée, un corps à prendre par degrés inſenſibles un mouvement donné, ou enfin tel autre agent que ce ſoit, à produire un moment quelconque donné d'activité, il faut que les forces mouvantes qui y ſont deſtinées, conſomment elles-mêmes un moment d'activité, égal au premier; aucune Machine ne peut en diſpenſer; mais comme ce moment réſulte de pluſieurs termes ou facteurs, on peut les faire varier à volonté, en diminuant la the Machine. Therefore the effect q is absolutely independent of the form of this Machine, & can never exceed that which the force is capable of producing naturally & without the Machine.

Thus, for example, if this man with his effort of 25 lb & his speed of three feet per second, is capable with a given Machine, or without the Machine, of raising in a given time a weight p to a height H, we cannot invent any Machine by which it is possible, with the same work (that is to say, the same force & the same speed as in the first case), of raising in the given time the same weight to a greater height, or a greater weight to the same height, or finally the same weight to the same height in a shorter time. This is obvious, as q being equal to PH ([by] XXXII), we have by the preceding article, $PH = 25\text{lb} \times 3\text{ft} \times t$.

LV. The advantage that Machines give is therefore not to produce a greater effect with smaller means, but to allow us to choose between different means that one can call equal, and to select the one that is the most appropriate in the given circumstances. To force a weight P to climb to a proposed height, a spring to close up by a given quantity, a body to take by insensible degrees a given motion, or finally any other agent whatever to produce some given moment of activity, it is necessary that the proposed moving forces themselves consume a moment of activity equal to the first. No Machine can avoid this; but as the moment [of activity] results from several terms or factors, one can vary them by choice, in diminishing the

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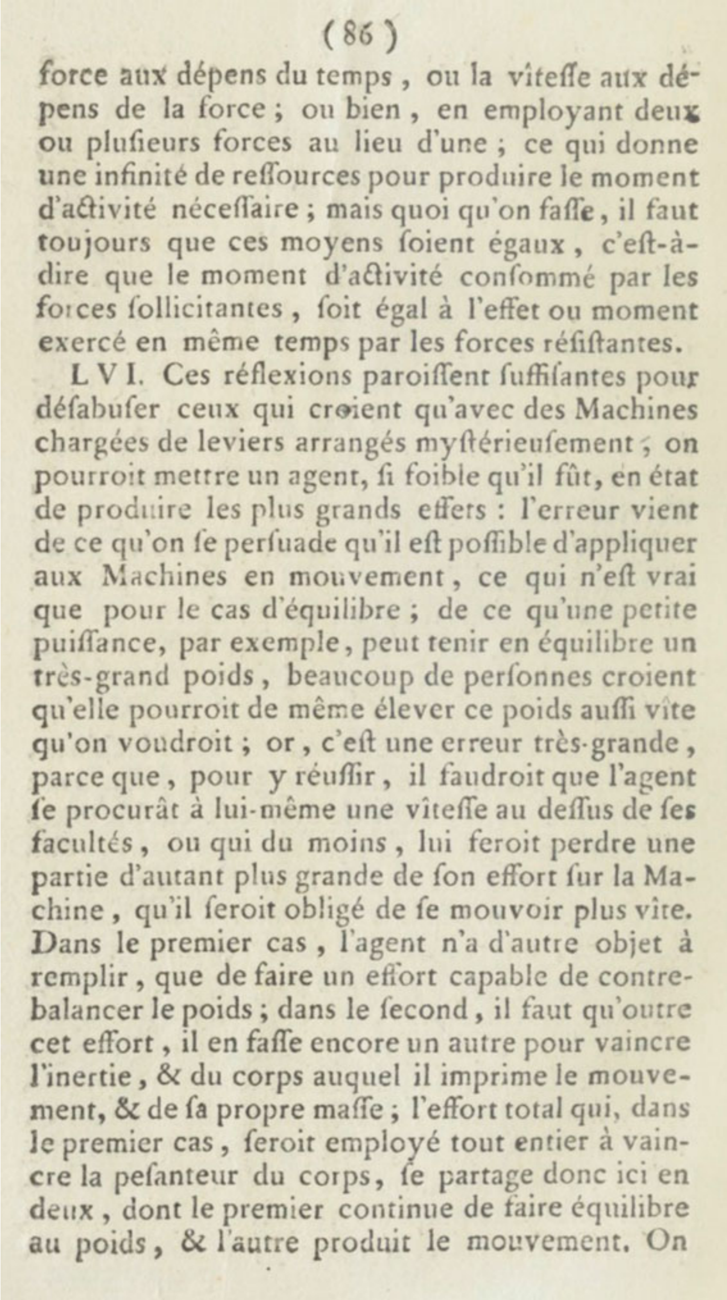
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force aux dépens du temps, ou la vîteſſe aux dépens de la force; ou bien, en employant deux ou pluſieurs forces au lieu d'une; ce qui donne une infinité de reſſources pour produire le moment d'activité néceſſaire; mais quoi qu'on faſſe, il faut toujours que ces moyens ſoient égaux, c'eſt-à-dire que le moment d'activité conſommé par les forces ſollicitantes, ſoit égal à l'effet ou moment exercé en même temps par les forces réſiſtantes.

LVI. Ces réflexions paroiſſent ſuffiſantes pour déſabuſer ceux qui croient qu'avec des Machines chargées de leviers arrangés myſtérieuſement, on pourroit mettre un agent, ſi foible qu'il fût, en état de produire les plus grands effets: l'erreur vient de ce qu'on ſe perſuade qu'il eſt poſſible d'appliquer aux Machines en mouvement, ce qui n'eſt vrai que pour le cas d'équilibre; de ce qu'une petite puiſſance, par exemple, peut tenir en équilibre un très-grand poids, beaucoup de perſonnes croient qu'elle pourroit de même élever ce poids auſſi vîte qu'on voudroit; or, c'eſt une erreur très-grande, parce que, pour y réuſſir, il faudroit que l'agent ſe procurât à lui-même une vîteſſe au deſſus de ſes facultés, ou qui du moins, lui feroit perdre une partie d'autant plus grande de ſon effort ſur la Machine, qu'il ſeroit obligé de ſe mouvoir plus vîte. Dans le premier cas, l'agent n'a d'autre objet à remplir, que de faire un effort capable de contre-balancer le poids; dans le ſecond, il faut qu'outre cet effort, il en faſſe encore un autre pour vaincre l'inertie, & du corps auquel il imprime le mouvement, & de ſa propre maſſe; l'effort total qui, dans le premier cas, ſeroit employé tout entier à vaincre la peſanteur du corps, ſe partage donc ici en deux, dont le premier continue de faire équilibre au poids, & l'autre produit le mouvement. On force at the expense of time, or the speed at the expense of force, or better, in employing two or several forces in place of one. This gives an infinity of means for producing the necessary moment of activity; but whatever is done, it is always necessary that the means are equal, that is to say, the moment of activity consumed by the soliciting forces is equal to the effect of moment [(the moment of activity)] exerted at the same time by the resisting forces.

LVI. These reflections appear sufficient to disabuse those who believe that employing Machines filled with levers arranged in some mysterious fashion, one could put an agent, as weak as could be, and still produce the greatest of efforts. The error arises in persuading oneself that it is possible to apply to Machines in motion what is true only for the case of equilibrium, that a small force, for example, can hold a very great weight in equilibrium (many people even believe that the weight could be raised as quickly as desired). But this is a very great error, for to succeed, it would be necessary that the agent gives to itself a speed above its powers, or at least, would make it lose the greater part of its effort, which would oblige it to move more quickly. In the first case, the agent has no other purpose to fulfill than to make an effort capable of counter-balancing the weight; in the second, it is necessary that outside of this effort, it [the agent] makes an extra one [effort] in order to overcome inertia, both [the inertia] of the body to which it imparts motion, & [the inertia] of its own mass. The total effort, which in the first case would be employed entirely to overcome the gravitational force on the body, is thus shared into two parts, the first of which continues to balance the weight, & the other to produce motion. One ne peut donc augmenter l'un de ces efforts, qu'aux dépens de l'autre ; & voilà pourquoi l'effet des Machines en mouvement, eſt toujours tellement limité, qu'il ne peut jamais ſurpaſſer le moment d'activité exercé par l'agent qui le produit.

C'eſt ſans doute faute de faire une attention ſuffiſante à ces différents effets d'une même Machine conſidérée tantôt en repos, & tantôt en mouvement, que des perſonnes auxquelles la ſaine théorie n'eſt point inconnue, s'abandonnent quelquefois aux idées les plus chimériques, tandis qu'on voit de ſimples ouvriers, faire valoir, par une eſpece d'inſtinct, les propriétés réelles des Machines, & juger très-bien de leurs effets. *Archimede* ne demandoit qu'un levier & un point fixe pour ſoulever le globe de la terre ; comment donc ſe peut-il faire, dit-on, qu'un homme auſſi fort qu'*Archimede*, ne puiſſe pas, quand même il ſeroit muni de la plus belle Machine du monde, élever un poids de cent livres, en une heure de temps, à une hauteur médiocre donnée? C'eſt que l'effet d'une Machine en repos, & celui d'une Machine en mouvement, ſont deux choſes fort différentes, & en quelque choſe hétérogenes : dans le premier cas, il s'agit de détruire, d'empêcher le mouvement ; dans le ſecond, l'objet eſt de le faire naître & de l'entretenir ; or, il eſt clair que ce dernier cas exige une conſidération de plus que le premier ; ſavoir : la vîteſſe réelle de chaque point du ſyſtême ; mais on pourra ſentir mieux la raiſon de cette différence, par la remarque ſuivante.

Les points fixes & obſtacles quelconques, ſont des forces purement paſſives, qui peuvent abſorber un mouvement, ſi grand qu'il ſoit, mais qui ne peuvent jamais en faire naître un, ſi petit qu'on can thus only augment one of these efforts at the expense of the other, & that is why the effect of Machines in motion is always so limited, that it can never exceed the moment of activity exerted by the agent which produced it.

It is doubtless by not giving sufficient attention to the different effects of the same Machine considered sometimes at rest, & sometimes in motion, that people to whom the sound theory is unknown sometimes abandon themselves to the most chimerical ideas, while on the other hand we see simple workers assert, by a kind of instinct, the real properties of Machines, & judge their effects very well. *Archimedes* asked only for a lever & a fixed point to raise the globe of the world. How can it be, then – one may well ask – that a man as strong as *Archimedes*[52] cannot, even equipped with the most beautiful Machine in the world, raise a weight of a hundred pounds in an hour of time to some given middling height? It is because the effect [work capabilities] of a Machine at rest, & that of a Machine in motion, are two very different things &, in certain respects, having different natures. In the first case, it is a matter of destroying or impeding the motion; in the second case, the object is to create & maintain motion. However, it is clear that this last case requires more consideration than the first; one must know the real speed of each point of the system. But one will be better able to perceive the reason for the difference after by the following remark.

The fixed points & barriers are purely passive forces that can absorb a motion, however large it is, but can never give rise to one, as small as

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veuille l'imaginer, dans un corps en repos : or, c'eſt improprement que dans le cas d'équilibre, on dit d'une petite puiſſance, qu'elle en détruit une grande : ce n'eſt pas par la petite puiſſance, que la grande eſt détruite ; c'eſt par la réſiſtance des points fixes : la petite puiſſance ne détruit réellement qu'une petite partie de la grande, & les obſtacles font le reſte. Si *Archimede* avoit eu ce qu'il demandoit, ce n'eſt pas lui qui auroit ſoutenu le globe de la terre, c'eſt ſon point fixe ; tout ſon art auroit conſiſté, non à redoubler d'effort pour luter contre la maſſe de ce globe, mais à mettre en oppoſition les deux grandes forces, l'une active, l'autre paſſive, qu'il auroit eues à ſa diſpoſition : ſi au contraire il eût été queſtion de faire naître un mouvement effectif, alors *Archimede* auroit été obligé de le tirer tout entier de ſon propre fond ; auſſi n'auroit-il pu être que très-petit, même après pluſieurs années : n'attribuons donc point aux forces actives, ce qui n'eſt dû qu'à la réſiſtance des obſtacles, & l'effet ne paroîtra pas plus diſproportionné à la cauſe, dans les Machines en repos, que dans les Machines en mouvement.

LVII. Quel eſt donc enfin le véritable objet des Machines en mouvement ? Nous l'avons déjà dit ; c'eſt de procurer la faculté de faire varier à volonté, les termes de la quantité Q, ou *momentum* d'activité, qui doit être exercé par les forces mouvantes. Si le temps eſt précieux, que l'effet doive être produit dans un temps très-court, & qu'on n'ait cependant qu'une force capable de peu de viteſſe, mais d'un grand effort, on pourra trouver une Machine pour ſuppléer la viteſſe néceſſaire par la force : s'il faut au contraire élever un poids très-conſidérable, & qu'on n'ait qu'une foible puiſſance, mais capable d'une grande vîwe want to imagine, in a body at rest: however, it is improper that, in the case of equilibrium, one says of a small force that it cancels out a great one. It is not by the small 'puissance' that the larger one is destroyed; [rather,] it is by the resistance of fixed points. The small force really cancels only a small part of the large one [force], & the barriers [cancel out] the remainder. If *Archimedes* had been given what he asked for [that is, a lever], it is not he who would have supported the globe of the earth, it is the fixed point. All his art would have consisted, not in the redoubling of the effort in the struggle against the mass of the earth, but (using all the forces at his disposal) and placing in opposition the two great [categories of] force, one active, the other passive. If, on the contrary, it had been a question of giving rise to some motion (a motion that had the capability of doing work), then *Archimedes* would have been obliged to draw it in its entirety out of his own reserves. Also, these reserves would have been very small, even after several years. Let us, therefore, not attribute to active forces that which is due only to the resistive-force of barriers, & the disproportion between cause & effect will not seem more extreme in Machines at rest than in Machines in motion.

LVII. What is then the true purpose of Machines in motion? We have already said, it is to procure the ability to vary at will the terms of the quantity Q, or *momentum* of activity, that must be exerted by the moving forces. If time is precious, so that the effect must be produced in a very short time, but one only has a force capable of little speed but great effort, one can find a Machine to supply the necessary speed by the force. If, on the contrary, one must raise a very considerable weight but one has only a weak force capable of great speed,

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teſſe, on pourra imaginer une Machine avec laquelle l'agent ſera en état de compenſer par ſa vîteſſe, la force qui lui manque : enfin, ſi la puiſſance n'eſt capable ni d'un grand effort, ni d'une grande vîteſſe, on pourra encore, avec une Machine convenable, lui faire produire l'effet deſiré; mais alors on ne pourra ſe diſpenſer d'employer beaucoup de temps ; & c'eſt en cela que conſiſte ce principe ſi connu, que *dans les Machines en mouvement, on perd toujours en temps ou en vîteſſe ce qu'on gagne en force.*

Les Machines ſont donc très-utiles, non en augmentant l'effet dont les puiſſances ſont naturellement capables, mais en modifiant cet effet : on ne parviendra jamais par elles, il eſt vrai, à diminuer la dépenſe ou *momentum* d'activité, néceſſaire pour produire un effet propoſé ; mais elles pourront aider à faire de cette quantité une répartition convenable au deſſein qu'on a en vue : c'eſt par leur ſecours qu'on réuſſira à déterminer, ſinon le mouvement abſolu de chaque partie du ſyſtême, du moins à établir entre ces différents mouvements particuliers, les rapports qui conviendront le mieux ; c'eſt par elles enfin qu'on donnera aux forces mouvantes, les ſituations & directions les plus commodes, les moins fatigantes, les plus propres à employer leurs facultés de la maniere la plus avantageuſe.

L V I I I. Ceci nous conduit naturellement à cette queſtion intéreſſante : quelle eſt la meilleure maniere d'employer des puiſſances données, & dont l'effet naturel eſt connu, en les appliquant aux Machines en mouvement ? C'eſt-à-dire, quel eſt le moyen de leur faire produire le plus grand effet poſſible ?

La ſolution de ce problême dépend des cirone can imagine [deploying] a Machine whereby the [active] agent can compensate by its speed what it lacks in force. Finally, if the 'puissance' is capable neither of great effort nor of great speed one can again, with a suitable Machine, produce the desired effect, but only by employing lots of time; & it is for this reason that the well-known principle arise [:] that *in Machines in motion, we always lose in time or in speed what we gain in force.*

Machines are thus very useful, not in augmenting the effect of which the forces are naturally capable, but in modifying this effect. One will never succeed, it is true, in diminishing the expenditure or *momentum* of activity necessary to produce a proposed effect, but they [(Machines)] can help to make a repartition of this quantity suitable to the design one has in view. It is with their help that one succeeds in determining, if not the absolute motion of each part of the system, at least in establishing the relations between different specific motions which are more suitable. It is finally by them [the relations] that we will [be able to] match the moving forces to those cases where the situations & directions are the most convenient, the least tiring, the most proper to employ their abilities in the most advantageous manner.

LVIII. This naturally leads us to the interesting question: what is the best way of employing given forces, & of known natural effect, in applying them to Machines in motion? That is, what is the way for them to produce the greatest possible effect?

The solution of this problem depends on

conſtances particulieres ; mais on peut faire là-deſſus des obſervations générales & applicables à tous les cas : en voici quelques-unes des plus eſſentielles.

L'effet produit étant la même choſe (LIII) que le moment d'activité exercé par les forces réſiſtantes , la condition générale , eſt que q ſoit un *maximum ;* or , q ne pouvant jamais ſurpaſſer Q , il faut, 1°. que la quantité Q ſoit elle-même la plus grande poſſible ; 2°. que tout ce moment Q ſoit employé uniquement à produire l'effet propoſé.

Pour faire que Q ſoit un *maximum* , il faut conſidérer qu'elle dépend de quatre choſes, ſavoir ; de la quantité de force exercée par l'agent qui doit produire l'effet q , de ſa vîteſſe , de ſa direction , & du temps pendant lequel il agit. Or, 1°. quant à ce qui regarde la direction de la force, il eſt évident que cette puiſſance doit être, toutes choſes égales d'ailleurs , dirigée dans le même ſens que ſa vîteſſe ; car le moment d'activité qu'exerce pendant dt une puiſſance F dont la vîteſſe eſt V, & l'angle compris entre F & V, Z, étant (XXXII) $F\,V\,dt \cos Z$, il eſt clair que ce produit ne ſera jamais plus grand que lorſque coſ Z ſera égal au ſinus total , c'eſt-à-dire lorſque la force & ſa vîteſſe ſeront dirigées dans le même ſens ; 2°. quant à ce qui regarde l'intenſité de la force exercée , ſa vîteſſe , & le temps pendant lequel elle eſt exercée ; on ne ne doit point déterminer ces choſes d'une maniere abſolue , mais ſeulement mettre entr'elles les rapports que l'expérience aura fait connoître pour les plus avantageux : par exemple , on a reconnu , je ſuppoſe , qu'un homme attaché pendant huit heures par jour à une manivelle d'un pied de specific circumstances; but we can offer [on this problem] some observations that are general & applicable in all cases. Here are some of the more essential.

The effect produced being the same thing (LIII) as the moment of activity exerted by the resisting forces, the general condition is that q should be a *maximum*. However, q not being able exceed Q, it is necessary. 1st that the quantity Q should itself be the greatest possible; 2nd that all the moment Q should be employed only to produce the proposed effect.

To *maximize* Q, it is necessary to consider that it depends on four things, these are the quantity of force exerted by the agent that must produce the effect q, its speed, its direction, & the time during which it acts. However: 1st, (concerning the direction of the force) it is evident that this force must, other things being equal, be in the same direction as the speed because for a force F whose speed is V acting for a duration dt & the angle between F & V being Z, the moment of activity ([by] XXXII) is $FVdtcosZ$. It is clear that this product will never be greater than when $cosZ$ is equal to the whole sine, that is to say, when the force & the speed are directed in the same direction; 2nd. As to that which regards the intensity of the exercised force, its speed, & the time during which it is exerted, we can not determine these things in an absolute manner but only give them the relationships that experience has shown us to be most advantageous. For example, we have recognized, I suppose, that a man hooked up to a crank of a foot in radius, for eight hours a day, rayon, peut faire continuellement un effort de 25 ₶, en faiſant un tour en deux ſecondes, ce qui fait à peu-près la vîteſſe de trois pieds par ſeconde; mais ſi l'on forçoit cet homme à aller beaucoup plus vîte, croyant par là avancer la beſogne, on la retarderoit, parce qu'il ne ſeroit plus en état de faire un effort de 25 ₶, ou ne pourroit plus ſoutenir un travail de huit heures par jour. Si au contraire, on diminuoit la vîteſſe, la force augmenteroit, mais dans un moindre rapport; & le moment d'activité diminueroit encore: ainſi, ſuivant l'expérience, pour que ce moment ſoit un *maximum*, il faut proportionner la Machine, de maniere à conſerver à la puiſſance la vîteſſe de trois pieds par ſeconde, & ne le faire travailler qu'environ huit heures par jour. On ſent bien que chaque eſpece d'agent a, eu égard à ſa nature ou conſtitution phyſique, un *maximum* analogue à celui dont on vient de parler, & que ce *maximum* ne peut en général ſe trouver que par expérience.

LIX. Cette premiere condition étant remplie, il ne reſtera donc plus, pour faire produire à une Machine donnée, le plus grand effet poſſible, qu'à faire enſorte que toute la quantité Q ſoit employée à produire cet effet; car ſi cela eſt ainſi, on aura $q = Q$; & c'eſt tout ce qu'on peut prétendre, puiſque jamais Q ne peut être moindre que q.

Or, pour remplir cette condition, je dis premiérement, qu'on doit éviter tout choc ou changement bruſque quelconque; car il eſt facile d'appliquer à tous les cas imaginables, le raiſonnement qui a été fait (XLVII) ſur les Machines à poids; d'où il ſuit que toutes les fois qu'il y a choc, il y a en même temps perte de moment can make a continual effort of 25 lb, in making one turn in two seconds, which makes a speed of roughly three feet per second; but if one forces the man to go much faster, believing that this will speed up the task, one will slow it down, because he will not be in a state to make an effort of 25 lb, or will not be able to sustain the work for eight hours a day. If, on the contrary, one would reduce the speed, the force would increase, but in a lesser relation – & the moment of activity is again diminished. Thus, in the light of experience, for the moment to be a *maximum*, it is necessary to proportion the Machine in such a way as to keep the force at a speed of three feet per second, & to make him work for only around eight hours per day. One perceives that each type of agent has, having regard to its nature or physical constitution, a *maximum* analogous to that of which we have just spoken, & that this *maximum* can in general be found only by experience.

LIX. This first condition being fulfilled, it will remain only, in order to make a given Machine produce the greatest effort possible, to make it so that the whole of the quantity Q may be employed to produce this effect; because if this is so, one will have $q = Q$, & this is all that one can ever lay claim to, as Q can never be less than q.

Now, to fulfill this condition, I say firstly, that one must avoid all shock or sudden change; because it is easy to apply to all imaginable cases the reasoning that has been applied (XLVII) to weight Machines. From this it follows that any time there is a shock, there is at the same time a loss of moment d'activité de la part des forces ſollicitantes ; perte ſi réelle, que l'effet en eſt néceſſairement diminué, comme nous l'avons fait voir par les Machines à poids, dans l'article qui vient d'être cité : c'eſt donc avec raiſon que nous avons avancé (LI), que pour faire produire aux Machines le plus grand effet poſſible, il faut néceſſairement qu'elles ne changent jamais de mouvement, que par degrés inſenſibles ; il en faut ſeulement excepter celles qui, par leur nature même, ſont ſujettes à éprouver différentes percuſſions, comme ſont la plupart des moulins ; mais dans ce cas-là même, il eſt clair qu'on doit éviter tout changement ſubit, qui ne ſeroit pas eſſentiel à la conſtitution de la Machine

LX. On peut conclure de là, par exemple, que le moyen de faire produire le plus grand effet poſſible à une Machine hydraulique, mue par un courant d'eau, n'eſt pas d'y adapter une roue dont les aîles reçoivent le choc du fluide. En effet, deux raiſons empêchent qu'on ne produiſe ainſi le plus grand effet : la premiere eſt celle que nous venons de dire, ſavoir ; qu'il eſt eſſentiel d'éviter toute percuſſion quelconque ; la ſeconde eſt, qu'après le choc du fluide, il a encore une vîteſſe qui lui reſte en pure perte, puiſqu'on pourroit employer ce reſte à produire encore un nouvel effet qui s'ajouteroit au premier. Pour faire la Machine hydraulique la plus parfaite, c'eſt-à-dire capable de produire le plus grand effet poſſible, le vrai nœud de la difficulté conſiſteroit donc, 1°. à faire enſorte que le fluide perdît abſolument tout ſon mouvement par ſon action ſur la Machine, ou que du moins il ne lui en reſtât préciſément que la quantité néceſſaire pour s'échapper après ſon action ; 2°. à ce qu'il perdît tout ce mouveof activity on the part of the soliciting forces - a loss so noticeable in reality that the effect is necessarily diminished, as we have shown for weight Machines in the article cited. It is therefore with reason that we have advanced (LI), that to make Machines produce the greatest effect possible, it is necessary that they never change their motion other than by insensible degrees. We must just make one exception in the case of those [Machines] which, by their very nature, are subject to experiencing different percussions - such as most mills - but even in this case it is clear that one must avoid all sudden changes that are not essential to the constitution of the Machine.

LX. One may conclude from this, for example, that the means of producing the greatest possible effect in a hydraulic Machine, moved by a current of water, is not to adapt a wheel whose blades would receive the shock of the fluid. As a consequence there are two reasons that prevent one from producing the greatest effect in this way: the first is that of which we are going to speak, that is, that it is essential to avoid any percussion whatever; the second is that, after the shock of the fluid, there is still a speed which remains as a pure loss, since one could employ this residual [speed] to produce a new effect which would add to the first. To make the most perfect hydraulic Machine, that is to say, capable of producing the greatest effect possible, the true nub of the difficulty would thus consist of, 1st, to arrange things such that the fluid loses absolutely all its motion by its action on the Machine, or that at least only such quantity remains as is precisely what is needed so that the fluid can escape [exit the Machine] after its action; 2nd, that it loses all its motion ment par degrés inſenſibles, & ſans qu'il y eût aucune percuſſion, ni de la part du fluide, ni de la part des parties ſolides entr'elles : peu importeroit d'ailleurs quelle fût la forme de la Machine, car une Machine hydraulique qui remplira ces deux conditions, produira toujours le plus grand effet poſſible ; mais ce problême eſt très-difficile à réſoudre en général, pour ne pas dire impoſſible ; peut-être même que dans l'état phyſique des choſes, & eu égard à la ſimplicité, il n'y a rien de mieux que les roues mues par le choc ; & dans ce cas, comme il eſt impoſſible de remplir à la fois les deux conditions deſirables, que plus on voudra faire perdre au fluide de ſon mouvement pour approcher de la premiere condition, plus le choc ſera fort ; que plus au contraire on voudra modérer le choc pour approcher de la ſeconde, moins le fluide perdra de ſon mouvement : on ſent qu'il y a un milieu à prendre, au moyen duquel on déterminera, ſinon d'une maniere abſolue, au moins eu égard à la nature de la Machine, celle qui ſera capable du plus grand effet.

LXI. Une autre condition générale qui n'eſt pas moins importante, lorſqu'on veut que les Machines produiſent le plus grand effet poſſible, c'eſt de faire enſorte que les forces ſollicitantes ne faſſent naître aucun mouvement inutile à l'objet qu'on ſe propoſe : ſi mon but, par exemple, eſt d'élever à une hauteur donnée la plus grande quantité d'eau poſſible, ſoit avec une pompe ou autrement, je dois faire enſorte que l'eau, en arrivant dans le réſervoir ſupérieur, n'ait préciſément qu'autant de vîteſſe qu'il lui en faut pour s'y rendre, car toute celle qu'elle auroit au delà, conſommeroit inutilement l'effort de la puiſſance motrice. Il eſt clair en effet (XLV) que dans ce by insensible degrees & without any percussion, either on the part of the fluid, or on the solid parts between them. In other respects, little of the [exact specification] of the Machine is important as a hydraulic Machine which fulfills these two conditions will always produce the greatest effect possible. But this problem is very difficult to solve in general, not to say impossible. Perhaps, accepting the physical state of things, & having regard to simplicity, there is [still] nothing better than the wheels being moved by shock; & in this case, it is impossible to simultaneously fulfill both desirable conditions (the more one would wish to make the fluid lose its motion in order to approach the first condition, the greater will be the shock; & on the contrary, the more one wishes to moderate the shock to approach the second condition, the less the fluid will lose its motion). One feels that there is a middle road to take and by this means one will determine, if not in an absolute way, at least having regard to the nature of the Machine, that which will be capable of yielding the greatest effect.

LXI. One other general condition which is not less important – when one wants Machines that can produce the greatest possible effect – is to make it so that the soliciting forces give rise to no motion useless to the object that is proposed. If my goal, for example, is to raise to a given height the greatest possible quantity of water, whether with a pump or otherwise, I must do it so that the water, in arriving at the higher reservoir, has only precisely that speed which was required [just] in order to convey it [shift it from one place to another], because anything beyond that will uselessly consume the effort of the driving force. It is clear (XLV) that in this

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cas cette puiſſance auroit à conſommer un moment d'activité inutile, & qui ſeroit égal à la moitié de la force vive avec laquelle l'eau ſeroit arrivée dans le réſervoir.

Il n'eſt pas moins évident que pour faire produire aux Machines le plus grand effet poſſible, on doit éviter ou diminuer, du moins autant que faire ſe peut, les forces paſſives, telles que le frottement, la roideur des cordes, la réſiſtance de l'air, leſquelles ſont toujours, dans quelque ſens que ſe meuve la Machine, au nombre des forces que j'ai nommées reſiſtantes (1).

Enfin, il eſt aiſé d'étendre ces remarques particulieres; & mon objet n'eſt pas d'entrer là-deſſus dans un plus grand détail.

LXII. On peut conclure de ce que nous venons de dire au ſujet du frottement & autres forces paſſives, que le mouvement perpétuel eſt une choſe abſolument impoſſible, en n'employant, pour le produire, que des corps qui ne ſeroient ſollicités par aucune force motrice, & même des corps peſants; car ces forces paſſives auxquelles on ne peut ſe ſouſtraire, étant toujours réſiſtan-

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(1) On parle ſouvent des forces paſſives; mais qu'eſt-ce qu'une force paſſive; qu'eſt-ce qui la différencie d'une force active? Je crois qu'on n'a pas encore répondu à cette queſtion, & même qu'on ne ſe l'eſt jamais faite. Or, il me ſemble que le caractere diſtinctif des forces paſſives, conſiſte en ce qu'elles ne peuvent jamais devenir ſollicitantes, quel que ſoit ou puiſſe être le mouvement de la Machine, au lieu que les forces actives peuvent agir, tantôt en qualité de forces ſollicitantes, & tantôt en qualité de forces réſiſtantes. Sur ce pied, les obſtacles & points fixes ſont évidemment des forces paſſives, puiſqu'ils ne peuvent agir ni comme forces ſollicitantes, ni comme forces réſiſtantes (XXXI).

case the force will consume a useless moment of activity, equal to half the live force with which the water will arrive in the reservoir.

It is no less evident that in order to make Machines produce the greatest possible effect, one must (at least as much as possible) avoid or diminish passive forces, such as that of friction, the stiffness of cords, the resistance of the air, that are always, in whatever way the Machine moves, among the number of those I have named resisting (I).

Finally, it is easy to extend these specific remarks; & my object is not to go into the above in greater detail.

LXII. One may conclude from what we are coming to say on the subject of friction & other passive forces, that perpetual [53] motion is absolutely impossible, as it employs – for the production of perpetual motion – only bodies that are not solicited by any motive force, or even by [the descent of] heavy bodies; because these passive forces which we cannot eliminate, being always resisting,

(I) One often speaks of passive forces, but what is a passive force, what is the difference between it and an active force? I believe that we have not yet answered this question, & even that no one hs ever done so. But, it seems to me, the distinctive characteristic of passive forces consists in the fact that they can never become soliciting, whatever is or could be the motion of the Machine, whereas the active [non-passive] forces can act, sometimes as soliciting forces & sometimes as resisting forces. On this view, barriers & fixed points are evidently passive forces - as they can act neither as soliciting forces, nor as resisting forces (XXXI).

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tes, il eſt évident que le mouvement doit ſe ralentir continuellement : & d'après ce que nous avons dit (XLV), on voit que ſi les corps ne ſont ſollicités par aucune force motrice, la ſomme des forces vives ſera réduite à rien ; c'eſt-à-dire que la Machine ſera réduite au repos, lorſque le moment d'activité, produit par le frottement depuis le commencement du mouvement, ſera devenu égal à la demi-ſomme des forces vives initiales : & ſi les corps ſont peſants, le mouvement finira, lorſque le moment produit par les frottements, ſera égal à la demi-ſomme des forces vives initiales, plus la moitié de la force vive qui auroit lieu, ſi tous les points du ſyſtême avoient une vîteſſe commune, égale à celle qui eſt due à la hauteur du point où etoit le centre de gravité dans le premier inſtant du mouvement, au deſſus du point le plus bas où il puiſſe deſcendre ; ce qui eſt évident par l'article (XLII).

Il eſt aiſé d'appliquer les mêmes raiſonnements au cas où il y a desreſſorts, & en général, à tous ceux où, abſtraction faite du frottement, les forces ſollicitantes ſont obligées, pour faire paſſer la Machine d'une poſition à une autre, d'exercer un moment d'activité auſſi grand que celui qui eſt produit par les forces réſiſtantes, lorſque la Machine revient de cette derniere poſition à la premiere.

Le mouvement finiroit bien plus vîte encore, s'il arrivoit quelque percuſſion, puiſque la ſomme des forces vives, diminue toujours en pareil cas (XXIII).

Il eſt donc évident qu'on doit déſeſpérer abſolument de produire ce qu'on appelle un mouvement perpétuel, s'il eſt vrai que toutes les forces motrices qui exiſtent dans la nature, ne it is evident that the motion must continually be retarded. [&] After what we have said (XLV), we see that if bodies are not solicited by any motive force, the sum of the living force will reduce to nothing. That is to say, the Machine will come to rest when the moment of activity, produced by friction since the start of the motion, will become equal to half the sum of the original live force. & If the bodies are heavy, the motion will stop when the moment produced by the forces of friction equals half the sum of the initial live force, plus half the live force which would have applied if all points of the system had a common speed, this speed being determined by the change in height of the centre of gravity from its starting height at the first instant of motion, to its lowest point; this is evident from article (XLII).

It is easy to apply the same reasoning to the case where there are springs, & in general, to all those cases where, ignoring friction, in order to move the Machine from one position to another, the soliciting forces must exert a moment of activity as great as that which is produced by the resisting forces, when the Machine returns from its last position to its first.

The motion would finish even more quickly if there was some percussion, as the sum of the living forces always diminishes in this case (XXIII).

It is therefore evident that one must absolutely despair of producing what one calls perpetual motion, if it is true that all the motive forces which exist in nature, ſoient autre choſe que des attractions, & que cette force ait pour propriété générale, comme il le paroît, d'être toujours la même à diſtances egales, entre des corps donnés, c'eſt-à-dire d'être une fonction qui ne varie que dans le cas où la diſtance de ces corps varie elle-même.

LXIV. Une obſervation générale qui réſulte de tout ce qui vient d'être dit, c'eſt que cette eſpece de quantité, à laquelle j'ai donné le nom de *moment d'activité*, joue un très-grand rôle dans la théorie des Machines en mouvement : car c'eſt en général cette quantité qu'il faut économiſer le plus qu'il eſt poſſible, pour tirer d'un agent tout l'effet dont il eſt capable.

S'agit-il d'élever un poids, de l'eau, par exemple, à une hauteur donnée ; vous en éleverez d'autant plus dans un temps donné, non que vous aurez conſommé une plus grande quantité de force, mais que vous aurez exercé un plus grand moment d'activité (XLIV).

Qu'il ſoit queſtion de faire tourner la meule d'un moulin, ſoit par le choc de l'eau, ſoit par le vent, ſoit par la force des animaux, ce n'eſt pas à faire que le choc de l'eau, de l'air, ou l'effort de l'animal ſoit le plus grand que vous devez vous attacher, mais à faire conſommer à ces agents le plus grand moment d'activité poſſible.

Veut-on faire un vuide quelconque dans l'air, de quelque maniere qu'on s'y prenne, il faudra, pour y parvenir, conſommer un *moment d'activité* auſſi grand que celui qui ſeroit néceſſaire pour élever à trente-deux pieds de hauteur, un volume d'eau égal au vuide qu'on veut occaſionner.

Eſt-ce un vuide dans une maſſe d'eau indéfinie comme la mer ; il faudra conſommer pour cela le are only attractions, & that [attractive forces have] as a general property, or so it seems, to be always the same at equal distances between given bodies, that is to say, to be a function which varies only in the case where the distance between bodies itself varies.

LXIV. A general observation which results from all that has just been said, is that this type of quantity, to which we have given the name *moment of activity*, plays a very great role in the theory of Machines in motion - because it is in general *this* quantity which must be reduced as much as possible, in order to extract from an agent all the effort of which it is capable.

If it is a question of raise a weight – water, for example – to a given height, you will raise all the more in the given time, not by consuming a greater quantity of force, but by exerting a greater moment of activity (XLIV).

If it is a question of making the millstone of a mill turn – whether by the shock of water, wind, or the force of animals – it is not in maximizing the shock of water, of air, or the effort of the animal that you have to apply yourself, but to make these agents consume the greatest moment of activity possible.

If one wants to make any sort of vacuum in the air, however we do it, it will be necessary in order to achieve this, to consume a moment of activity as great as that which would be necessary to raise to a height of thirty-two feet, a volume of water equal to the the vacuum that one wants to produce.

If this vacuum was in an indefinite mass of water like the sea, it would be necessary to consume in order to achieve this le même *moment d'activité* que ſi la mer étoit un vuide, le vuide qu'on veut faire un volume d'eau de mer, & qu'il fallût élever ce volume à la hauteur du niveau de la mer.

Eſt-ce dans un vaſe de figure donnée, qu'il faut produire un vuide? On ne peut viſiblement y parvenir, ſans faire monter le centre de gravité de la maſſe totale du fluide d'une quantité déterminée par la figure du vaſe; il faudra donc conſommer un *moment d'activité* égal à celui qui ſeroit néceſſaire pour élever toute l'eau du vaſe d'une quantité égale à celle dont il faut que monte le centre de gravité du fluide.

Dans une Machine en repos, où il n'y a d'autre force à vaincre que l'inertie des corps, voulez-vous y faire naître un mouvement quelconque, par degrés inſenſibles, le *moment d'activité* que vous aurez à conſommer, ſera égal à la demi-ſomme des forces vives que vous y ferez naître; & s'il eſt ſeulement queſtion de changer le mouvement qu'elle a déjà, le *moment d'activité* à produire, ſera ſeulement la quantité dont cette demi-ſomme augmentera par le changement (XLV).

Enfin, ſuppoſons qu'on ait un ſyſtême quelconque de corps, que ces corps s'attirent les uns les autres, en raiſon d'une fonction quelconque de leurs diſtances; ſuppoſons même, ſi l'on veut, que cette loi ne ſoit pas la même pour toutes les parties du ſyſtême, c'eſt-à-dire que cette attraction ſuive quelle loi on voudra, (pourvu qu'entre deux corps donnés, elle ne varie que lorſque la diſtance de ces corps varie elle-même), & qu'il ſoit queſtion de faire paſſer le ſyſtême d'une poſition quelconque donnée à une autre: cela poſé, quelle que ſoit la route qu'on fera prendre à chacun des corps, pour remplir cet same *moment of activity* as if the sea were a vacuum [&] the vacuum that one wishes to make [was] a volume of sea water, & then this volume was raised to sea-level height.

Is it that one wants to produce a vacuum in a vessel of given shape? One obviously cannot achieve this without making the centre of gravity of the total mass of fluid rise by an amount determined by the shape of the vessel. It will be necessary, therefore, to consume a *moment of activity* equal to that needed to raise all the water in the vessel by an amount equal to that by which the fluid's centre of gravity must rise.

In a Machine at rest, where there is no other force to overcome except the inertia of the bodies, do you want to give rise to some sort of motion by insensible degrees [then] the *moment of activity* that you will have to consume will be equal to half the sum of the live forces that will be generated. & if it is only a question of changing the motion which it [the Machine] already has, the moment of activity needed will be only the amount by which this half-sum will be increased [by the changing motion] (XLV).

Finally, let us suppose that one has any system of bodies, that these bodies attract each other according to some function of their distances; let us suppose even, if one wants, that this law will not be the same for all parts of the system, that is to say, this attraction follows whatever law one wishes (provided that between two given bodies, it varies only when the distance of these bodies itself varies), & that it is a question of making the system pass from one given position to another. This assumed, whatever the route that one makes each of these bodies take, in order to fulfill this

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objet, qu'on mette tous ces corps en mouvement à la fois, ou les uns après les autres, qu'on les conduise d'une place à l'autre, par un mouvement rectiligne ou curviligne, & varié d'une maniere quelconque, (pourvu qu'il n'arrive aucun choc ni changement brusque); qu'on emploie enfin quelles Machines on voudra, même à ressort, pourvu que dans ce cas, on remette à la fin les ressorts au même état de tension où on les a pris au premier instant; le *moment d'activité* qu'auront à consommer, pour produire cet effet, les agents extérieurs employés à mouvoir ce systême, sera toujours le même, en supposant que le systême soit en repos au premier instant du mouvement, & au dernier.

Et si outre cela, il s'agit de faire naître dans le systême un mouvement quelconque, ou qu'il soit déjà en mouvement au premier instant, & qu'il s'agisse de modifier ou changer ce mouvement, le *moment d'activité* qu'auront à consommer les agents extérieurs, sera égal à celui qu'il faudroit consommer, s'il s'agissoit seulement de changer la position du systême, sans lui imprimer de mouvement, (c'est-à-dire considéré comme en repos au premier instant & au dernier); plus, la moitié de la quantité dont il faudra augmenter la somme des forces vives.

Il importe donc fort peu, quant à la dépense ou *momentum* d'activité à consommer, que les forces employées soient grandes ou petites, qu'elles emploient telles ou telles Machines, qu'elles agissent simultanement ou non; ce moment d'activité est toujours égal au produit d'une certaine force, par une vitesse & par un temps, ou la somme de plusieurs produits de cette nature; & cette somme doit être toujours la même, de quelque object, and whether one puts all the bodies in motion at the same time or one after the other, or leads them from one place to another by a rectilinear or curvilinear motion, & varying in whatever way (provided no shock or sudden motion happens); finally that one employs whatever Machines one wants, even with springs – provided in this case that one returns the springs at the end to the same state of tension they had in the first instant – [then] the *moment of activity* that one will have to consume in order to produce this effect (the exterior agents employed to move this system always being the same) is such that if the system is at rest in the first instant of motion then it will be at rest at the last instant.

And if, besides this, it is a question of giving rise to any motion in the system, or the system already is in motion at the first instant & it is a question of modifying or changing this motion, the *moment of activity* which the exterior agents will have to consume will be equal to that consumption necessary solely to change the position of the system, without imparting motion (that is to say, [when the system is] considered to be at rest at the first & last instants) and in addition, half the necessary increase in the sum of the live forces.

It thus matters very little as to expenditure or *momentum* of activity to be consumed, whether the forces employed are large or small, whether this or that Machine is employed, whether it acts simultaneously or not. This moment of activity is always equal to the product of a certain force, speed, & time, or the sum of several products of this nature; & this sum must always be the same in whatever maniere qu'on s'y prenne : les agents ne gagneront donc jamais rien d'un côté, qu'ils ne le perdent de l'autre.

Pour concluſion ; qu'en général on ait un ſyſtême quelconque de corps animés, de forces motrices quelconques, & que pluſieurs agents extérieurs, comme des hommes ou des animaux, ſoient employés à mouvoir ce ſyſtême en différentes manieres quelconques, ſoit par eux-mêmes, ſoit par des Machines : cela poſé ;

Quel que ſoit le changement occaſionné dans le ſyſtême, le moment d'activité, conſommé pendant un temps quelconque par les puiſſances extérieures, ſera toujours égal à la moitié de la quantité dont la ſomme des forces vives aura augmenté pendant ce temps, dans le ſyſtême des corps auxquels elles ſont appliquées : moins la moitié de la quantité dont auroit augmenté cette même ſomme de forces vives, ſi chacun des corps s'étoit mu librement ſur la courbe qu'il a décrite, en ſuppoſant qu'alors il eût éprouvé à chaque point de cette courbe, la même force motrice, que celle qu'il y éprouve réellement : pourvu, toujours, que le mouvement change par degrés inſenſibles, & que ſi l'on emploie des Machines à reſſorts, on laiſſe ces reſſorts dans le même état de tenſion où on les a pris.

LXV. Ces remarques ſur le moment d'activité, me font naître l'idée d'un principe d'équilibre particulier au cas où les forces exercées dans le ſyſtême, ſont des attractions ; j'ai cru que le Lecteur ne ſeroit pas fâché de le trouver ici ; voici en quoi il conſiſte :

Pluſieurs corps ſoumis aux loix d'une attraction exercée en raiſon d'une fonction quelconque des diſtances, ſoit par ces corps même les uns ſur les autres, ſoit par différents points fixes, étant appliqués à une manner it is taken. The agents never gain anything on one side, that is not lost on the other.

In conclusion; in general one has any system of moving bodies, with any motive forces, & several exterior agents, such as men or animals, employed to move this system in different ways, whether by themselves or by Machines. With these assumptions:

Whatever change is occasioned in the system, the moment of activity consumed during any time whatever by the external forces will always be equal to half the increase in the sum of live forces during this time in the system of bodies to which they are applied, less half the increase in the sum of live forces if each of the bodies followed their described path freely, assuming that at each point on the curve, a body experiences equal motive and actual forces - provided, as always, that the motion changes by insensible degrees, & that if one employs Machines with springs, one leaves the springs in the same state of tension as when one took them.

LXV. These remarks on the moment of activity gave me an idea for a principle of equilibrium specific to the case where the forces exerted in the system are attractions. I thought the Reader would not be annoyed to find it here; this is what it consists in:

Several bodies subject to laws of attraction exerted according to any function of distance, whether by the same bodies on each other, or *by different fixed points,* [or] *being applied to*

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Machine quelconque ; ſi l'on fait paſſer cette Machine d'une poſition quelconque donnée, à celle de l'équilibre, le moment d'activité conſommé dans ce paſſage par les forces attractives dont ces corps ſeront animés pendant ce mouvement, ſera un maximum.

C'eſt-à-dire que ce moment ſera toujours plus grand qu'il ne l'auroit été, ſi, au lieu de faire paſſer ce ſyſtême à la poſition d'équilibre, on l'eût contraint de prendre une route différente, & de paſſer dans une autre ſituation quelconque.

Par exemple, s'il s'agit de la gravité, qu'on peut regarder comme une attraction exercée vers un point infiniment éloigné, les forces attractives ſeront les poids appliqués à la Machine ; le moment d'activité qui ſera exercé par ces forces, lorſqu'on fera changer de ſituation à cette Machine, ſera donc égal au poids total du ſyſtême multiplié par la hauteur dont aura monté ou deſcendu le centre de gravité pendant ce changement de poſition (XXXII) : or, la ſituation d'équilibre eſt celle où le centre de gravité eſt au point le plus haut ou le plus bas poſſible ; donc, la hauteur à laquelle doit monter le centre de gravité, ou dont il doit deſcendre pour paſſer d'une ſituation quelconque donnée à celle de l'équilibre, eſt plus grande que pour paſſer à toute autre ſituation : donc, le moment d'activité conſommé dans le paſſage, par les forces motrices, eſt auſſi plus grand dans le premier cas que dans tout autre.

Si l'attraction étoit toujours conſtante comme la gravité ordinaire, mais qu'elle fût dirigée vers un point fixe, placé à une diſtance finie, on concluroit aiſément du principe précédent, que dans le cas d'équilibre, la ſomme des moments des corps du ſyſtême, relativement à ce point fixe, *any manner of Machine; if one makes this Machine pass from some given position to that of equilibrium,* [then] *the moment of activity consumed in this passage by the attractive forces driving the bodies throughout this motion, will be a maximum.*

That is to say, this moment will always be greater than it otherwise would have been if, instead of making the system pass to a position of equilibrium, one had constrained it to take a different path, & pass into some other situation.

For example, if the force under consideration is gravity which one may regard as an attraction exerted towards a point infinitely far away, the attractive forces will be the weights applied to the Machine. The moment of activity exerted by these forces when the Machine has changed its state will therefore be equal to the total weight of the system multiplied by the height through which the centre of gravity will have risen or descended during the change of position (XXXII). However, the situation of equilibrium is where the centre of gravity is at the highest or lowest possible point. Therefore, the height to which the centre of gravity must climb or descend in order to pass from any given situation to that of equilibrium, is greater than for passing to all other situations. Thus, the moment of activity consumed in this passage by the motive forces is greater in the first case than in all other cases.

If the attraction is always constant – as for ordinary gravity – but is directed towards a fixed point placed at a finite distance, one will easily conclude from the preceding principle that, in the case of equilibrium, the sum of the moments of the system of bodies, relative to this fixed point, eſt un *maximum*, c'eſt-à-dire que la ſomme des produits de chaque maſſe, par ſa diſtance au point fixe, eſt moindre lorſqu'il y a équilibre, que ſi le ſyſtême ſe trouvoit dans une autre ſituation quelconque.

Si l'attraction vers le point fixe, au lieu d'être conſtante, étoit proportionnelle aux diſtances de ce corps, à ce point fixe, on concluroit de même que la ſomme des produits de chaque maſſe par le quarré de la diſtance à ce point fixe, eſt un *maximum*.

On ſait que la ſomme des produits de chaque maſſe, par le quarré de ſa diſtance à un point fixe quelconque, eſt égale à la ſomme des produits de chaque maſſe, par le quarré de ſa diſtance au centre de gravité ; plus, au produit de la maſſe totale, par le quarré de la diſtance du centre de gravité à ce point fixe : (c'eſt une propoſition de géométrie fort connue, & dont il eſt facile de trouver la preuve) ; ainſi dans le cas d'attraction que nous examinons, la ſomme de ces deux quantités, doit, dans le cas d'équilibre, être un *maximum*, c'eſt-à-dire que ſa différentielle eſt égale à zéro. Suppoſons donc, par exemple, que toutes les parties du ſyſtême ſoient liées entr'elles, de maniere qu'elles ne faſſent qu'un même corps, & que ce corps ſoit ſuſpendu par ſon centre de gravité, tellement que ce point ſoit fixe ; il eſt clair que chacune des quantités dont on vient de parler, ſera conſtante, c'eſt-à-dire reſtera la même, quelque ſituation qu'on donne à ce corps, & que la différentielle de leur ſomme, ſera, par conſéquent, nulle ; donc, il y aura équilibre : c'eſt-à-dire que ſi toutes les parties d'un corps, ſont attirées vers un point fixe, proportionelle-ment à leurs diſtances à ce point, & qu'on ſuſis a *maximum*, that is, the sum of the products of each mass and the distance to the fixed point, is less when there is equilibrium than if the system is found in any other situation.

If the attraction towards the fixed point, instead of being constant, was proportional to the distances of the bodies to this fixed point, one would draw the same conclusion – that the sum of the products of each mass by the square of the distance to the fixed point is a *maximum*.

We know that the sum of the products of each mass and the square of its distance to any fixed point, is equal to the sum of the products of each mass and the square of the distance to the centre of gravity, and moreover, to the product of the total mass and the square of the distance between the centre of gravity & the fixed point (this is a well-known proposition of geometry, & one for which it is easy to find the proof). Thus, in the case of attraction which we are examining, the sum of these two quantities must, in the case of equilibrium, be a maximum, that is to say, the differential is equal to zero. Let us therefore suppose, for example, that all parts of the system are bound to each other, [and] in such a way that they are [may be treated as] one and the same body, & that this body is suspended by its centre of gravity, such that this point is fixed. It is clear that each of these quantities (of which we just spoke) will be constant, that is, will stay the same, whatever situation is given to the body, & that the differential of their sum will be, in consequence, null – therefore, there will be equilibrium. That is to say, if all the parts of a body are attracted towards a fixed point, [with a strength] proportional to their distance to this point, & one pende ce corps par ſon centre de gravité, il reſtera en équilibre préciſément comme dans le cas de la peſanteur ordinaire. Il ne faut cependant pas conclure de là, que dans une Machine à laquelle ſeroient appliqués pluſieurs corps attirés vers un point fixe, en raiſon des diſtances, la poſition d'équilibre fût celle où le centre de gravité du ſyſtême ſeroit au point le plus bas, c'eſt-à-dire le plus proche poſſible du point fixe; car cela n'arrive que dans le cas où toutes les parties du ſyſtême tiennent enſemble & ne font qu'un ſeul corps; au lieu que dans le cas de la gravité naturelle, il n'eſt pas néceſſaire, pour que le centre de gravité ſoit au point le plus bas, que les parties du ſyſtême ſoient liées les unes aux autres.

Si les corps étoient attirés vers le point fixe, en raiſon inverſe de leurs diſtances à ce point, le principe allégué ci-deſſus feroit voir que la ſituation d'équilibre eſt alors celle où la ſomme des produits de chaque maſſe, par le logarithme de ſa diſtance au point fixe, eſt un *maximum*.

En général, ſi les corps m du ſyſtême ſont attirés en raiſon d'une puiſſance n, de leurs diſtances x, à ce point, la ſituation d'équilibre ſera celle où la quantité $\int m x^{n+1}$ ſera un *maximum*, ou plus grande que dans toute autre ſituation; c'eſt à-dire où la différence de cette quantité à ce qu'elle ſeroit, ſi le ſyſtême étoit dans une ſituation infiniment voiſine, eſt égale à zéro.

S'il y a dans le ſyſtême pluſieurs points fixes, vers chacun deſquels les corps m ſoient attirés en raiſon d'une puiſſance donnée de leurs diſtances à ce point, de ſorte que x, y, z, &c. étant les diſtances de m à ces différents points fixes, Ax^n, By^p, Cz^q, &c. ſoient les forces centrales suspends this body by its centre of gravity, it will stay in equilibrium exactly as in the case of ordinary gravity. One must not conclude from this, however, that in a Machine to which is applied several bodies attracted towards a fixed point and by an attraction depending only on distances, the position of equilibrium would be where the centre of gravity of the system is at its lowest point, that is to say, the closest possible to the fixed point, because this only happens in the case where all parts of the system hold together as *one* single body. [However] in the [exceptional] case of natural gravity, for the centre of gravity to be at the lowest point it is not essential that the parts of the system are bound to each other.

If the bodies are attracted towards a fixed point [with a strength] in an inverse ratio to their distances to this point, the principle claimed above will show that the situation of equilibrium is achieved when the sum of the products of each mass and the logarithm of its distance to the fixed point, is a *maximum*.

In general, if the bodies m of the system are attracted because of a power n, of their distances x, to this point, the equilibrium 'situation' will arise where the quantity $\sum mx^{n+1}$ is a *maximum*, or greater than in any other situation - that is to say, where the difference between this quantity and what it would be if the system was in a neighbouring 'situation' infinitesimally close by, is equal to zero.

If there are, in the system, several fixed points, towards each of which the bodies m are attracted by a power depending on their distance to this point, such that x, y, z &c. are the distances of m to these different fixed points, Ax^n, By^p, Cz^q &c [etc.] are the central forces de m vers ces différents foyers, ce ſera la quantité $\frac{A}{n+1}\int m x^{n+1} + \frac{B}{p+1}\int m y^{p+1} + \frac{C}{q+1}\int m z^{q+1} + \&c.$ qui ſera un *maximum* dans la poſition de l'équilibre.

Et ſi outre cela, les corps s'attirent les uns les autres, en raiſon d'une puiſſance quelconque donnée des diſtances, de ſorte que X exprimant la diſtance de la molécule m à chacune des autres molécules du ſyſtême, $F X^r$ ſoit la force motrice, attractive de m vers cette autre molécule, la ſituation d'équilibre, ſera celle où la quantité $\frac{F}{2r+2}\int m X_r{}^{+1} + \frac{A}{n+1}\int m x^{n+1} + \frac{B}{p+1}\int m y^{p+1} + \frac{C}{q+1}\int m z^{\;+1} +$, &c. eſt un *maximum*; c'eſt-à-dire plus grande que dans toute autre ſituation.

Il ſeroit aiſé d'étendre encore ces conſéquences à d'autres hypotheſes d'attraction; mais la choſe paroît inutile : ainſi je me bornerai à remarquer qu'on peut, par un principe général à celui qu'on vient de voir, établir que,

Quelle que ſoit la nature des puiſſances motrices appliquées à une Machine, ſi on la fait mouvoir de maniere qu'elle paſſe par la poſition d'équilibre, l'inſtant où elle arrivera dans cette ſituation, ſera celui où le moment d'activité conſommé pendant le mouvement, par ces puiſſances motrices, ſera le plus grand.

C'eſt-à-dire que le moment d'activité que les puiſſances propoſées conſomment pendant le mouvement, va toujours en augmentant, jusqu'à ce que la Machine ait atteint la poſition d'équilibre; après quoi, ce moment va en diminant, à me-of m towards these different centres, this will be the quantity

$\frac{A}{n+1}\sum mx^{n+1} + \frac{B}{p+1}\sum my^{p+1} + \frac{C}{q+1}\sum mz^{q+1} +$ &c. which will be a *maximum* in the position of equilbrium.

And if besides this, the bodies attract each other, with whatever given power of the distances, so that if X expresses the distance of the molecules m to each of the other molecules in the system, FX^r is the attractive motive force towards this other molecule, the situation of equilibrium will be that where the quantity $\frac{F}{2r+2}\sum mX^{r+1} + \frac{A}{n+1}\sum mx^{n+1} + \frac{B}{p+1}\sum my^{p+1} + \frac{C}{q+1}\sum mz^{+1} +$ &c. is a *maximum* - that is to say, greater than in all other situations.*

It will be easy again to extend these consequences to other hypotheses of attraction; but the thing seems to be of no utility. Thus I will confine myself to remarking that we can, by a general principle analogous to that which we saw before, establish that:

Whatever kind ofmotive forces are applied to a Machine, if one makes the Machine move in such a way that it passes through the position of equilibrium, then the instant when it arrives at this situation will be that when the moment of activity consumed during the motion, by the motive forces, will be greatest.

That is to say, the moment of activity that the proposed motive forces consume during the motion always increases, right up to stage when the Machine attains the position of equilibrium; after which, this moment [of activity] will keep on diminishing, [and to an extent dependent on how far]

*Clearly the exponent in the last term is incorrect. It should read $\sum mz^{q+1}$.

ſure que le ſyſtême s'éloigne de cette poſition ; lorſqu'il l'a dépaſſée ; quelle que ſoit d'ailleurs la route qu'on ait fait prendre àcette Machine, pour l'amener à cette ſituation.

Suppoſons, par exemple, que chacune des puiſſances appliquées à la Machine, ſoit donnée de grandeur, & qu'on connoiſſe de plus un des points de la direction qu'elle doit avoir, pour qu'il y ait équilibre ; je dis que cette ſituation d'équilibre eſt celle où la ſomme des produits de chacune de ces puiſſances données par la diſtance du point de la Machine où on l'a ſuppoſe appliquée, au point fixe donné ſur ſa direction, eſt la moindre poſſible (1) ; ce qui ſe tire aiſément du principe précédent.

Toutes ces choſes ſont ſi faciles à prouver, après ce qui a été dit dans le cours de cette ſeconde partie, qu'il paroît inutile de s'y arrêter. Je finirai donc cet opuſcule par quelques réflexions ſur les loix fondamentales dont je ſuis parti pour établir la théorie qu'il contient.

Réflexions ſur les loix fondamentales de l'équilibre & du mouvement.

Parmi les Philoſophes qui s'occupent de la re-

(1) Il eſt à remarquer que dans tout ce qui vient d'être dit au ſujet d'une Machine conſidérée dans différentes poſitions, & de ſon paſſage de l'une à l'autre ; il eſt, dis-je, à remarquer que ces poſitions ſont toujours ſuppoſées telles, qu'on paſſe de l'une à l'autre par un mouvement qui ſoit à chaque inſtant de ceux que j'ai appellés *géométriques* ; autrement toutes ces propoſitions ſeroient ſujettes aux mêmes défauts que nous avons cru (V) pouvoir reprocher au principe de *Deſcartes*, & à pluſieurs autres.

the system moves away from this position, once it has gone beyond it, whatever route among others that one makes the Machine take, to bring it to this situation.

Let us suppose, for example, that each of the forces applied to the Machine have a given size, & that one knows in addition one of the points of direction that it must have in order for there to be equilibrium. I declare that the situation of equilibrium is that where the sum of the products of each of these given forces and the distance from the supposed point of application to the given fixed point along its [given] direction, is the least possible (I). This is easily obtained from the preceding principle.

All these things are so easy to prove after what has been said during the course of this second part, that it seems pointless to linger here. I will therefore finish this little work with some reflections on the fundamental laws from which I started to establish the theory contained herein.

Reflections on the fundamental laws of equilibrium & motion.

Among the Philosophers who occupy themselves with re-

(I) It is to be remarked that in all that has been said on the subject of a Machine considered in different positions, & its passage from one to the other; there is, I say, [a further need] to remark that these positions are always supposed such that one passes from one to the other by a motion that is at each instant what I call *geometric*; otherwise all these propositions would be subject to the same flaws that we believe (V) can be found in the principle of *Descartes*, & in several others.

cherche des loix du mouvement, les uns font de la Méchanique, une ſcience expérimentale, les autres, une ſcience purement rationnelle; c'eſt-à-dire que les premiers comparant les phénomenes de la nature, les décompoſent, pour ainſi dire, pour connoîrre ce qu'ils ont de commun, & les reduire ainſi à un petit nombre de faits principaux, qui ſervent enſuite à expliquer tous les autres, & à prévoir ce qui doit arriver dans chaque circonſtance; les autres commencent par des hypotheſes, puis raiſonnant conſéquemment à leurs ſuppoſitions, parviennent à découvrir les loix que ſuivroient les corps dans leurs mouvements, ſi leurs hypotheſes étoient conformes à la nature, puis comparant leurs réſultats ave les phénomenes, & trouvant qu'ils s'accordent, en concluent que leur hypotheſe eſt exacte, c'eſt-à-dire que les corps ſuivent en effet les loix qu'ils n'avoient fait d'abord que ſuppoſer.

Les premiers de ces deux claſſes de Philoſophes, partent donc dans leurs recherches, des notions primitives que la nature a imprimées en nous, & des expériences qu'elle nous offre continuellement; les autres partent de définitions & d'hypotheſes: pour les premiers, les noms de corps, de puiſſances, d'équilibre, de mouvement, répondent à des idées premieres; ils ne peuvent ni ne doivent les définir; les autres au contraire ayant tout à tirer de leur propre fond, ſont obligés de définir ces termes avec exactitude, & d'expliquer clairement toutes leurs ſuppoſitions; mais ſi cette méthode paroît plus élégante, elle eſt auſſi bien plus difficile que l'autre; car il n'y a rien de ſi embarraſſant dans la plupart des ſciences rationnelles, & ſur-tout dans celle-ci, que de poſer d'abord d'exactes définisearching the laws of motion, some are in Mechanics, an experimental science, the others, purely rational. That is to say, the first compare the phenomena of nature, decompose them, so to speak, in order to understand what they have in common & so reduce them to a small number of principal facts – which then serve to explain the others & to predict what will happen in any circumstance; the others start from hypotheses, then reason from their suppositions, succeed in discovering the laws that the bodies would follow in their motions if their hypotheses conformed to nature, then, comparing their results with phenomena & finding that they agree, conclude that their hypotheses are accurate, that is to say that the bodies in effect do follow the laws which they had at first only assumed.

The first of these two classes of Philosophers therefore follow in their researches the primitive ideas that nature has imposed on us & the experiences that she continually offers us. The others start from definitions & hypotheses for the first, [they assign] the names of bodies, forces, equilibrium, motion, that correspond to primitive ideas; they neither can nor must define them; the others, on the contrary, having to draw everything from their own depths, are obliged to define these terms with precision, & explain clearly all their suppositions. But if this [second] method appears more elegant, it is also more difficult than the other one; as there is nothing more perplexing in most of the rational sciences, & above all in this one, than to first pose exact definitions

tions sur lesquelles il ne reste aucune ambiguité : ce seroit me jeter dans des discussions métaphysiques, bien au dessus de mes forces, que de vouloir approfondir toutes celles qu'on a proposées jusqu'ici : je me contenterai d'examiner la premiere & la plus simple.

Qu'est-ce qu'un corps ? C'est, disent la plupart, une étendue impénétrable, c'est-à-dire qui ne peut en aucune maniere être réduite à un espace moindre : mais cette propriété n'est-elle pas commune au corps & à l'espace vuide ; un pied cube de vuide peut-il occuper un espace moindre ? Il est clair que non. Supposons qu'un pied cube d'eau, par exemple, soit enfermé dans un vase capable de contenir deux pieds cubes, & fermé de tout côté ; qu'on agite, qu'on boulverse ce vase tant qu'on voudra, il restera toujours un pied cube d'eau & un pied cube de vuide : voilà deux espaces d'une nature différente, à la vérité, mais tout aussi irréductibles l'un que l'autre : ce n'est donc pas en cela que consiste la propriété caractéristique des corps. D'autres disent que cette propriété consiste dans la mobilité ; l'espace indéfini & vuide, disent-ils, est immobile, tandis que les corps peuvent se transporter d'un lieu de cet espace à un autre : mais lorsque le corps *A* passe en *B*, par exemple, l'espace vuide qui étoit en *B*, n'a-t-il pas passé en *A* ? Il n'y a, ce me semble, pas plus de raison d'attribuer le mouvement au plein qui étoit en *A*, qu'au vuide qui étoit en *B* ; le mouvement consiste en ce que l'un de ces espaces a remplacé l'autre ; & ce remplacement étant réciproque, la mobilité est une propriété qui n'appartient pas plus à l'un qu'à l'autre. Sans sortir de notre premiere supposition, lorsque j'agite le vase moitié in which no ambiguity remains - it would throw me into metaphysical difficulties, well beyond my powers, to want to explore all those proposed up till now. I will content myself with [examining] the first & most simple [definition].

What is a body? It is, most say, an extended impenetrable object, that is to say, it cannot in any manner be reduced to a smaller space: but this property, is it not common to bodies & to empty space can a cubic foot of the void occupy less space? It is clear that it cannot. Let us suppose that a cubic foot of water, for example, is enclosed in a vessel capable of containing two cubic feet, & closed on all sides; shake it, or overturn it as long as one wants, there always remains a cubic foot of water & a cubic foot of void here are two spaces truly different, but one as irreducible as the other. Therefore it is not in this that the characteristic property [of a body] consists. Others say that this property consists in mobility. Indefinite & empty space, they say, is immobile, while bodies can be transported from one place in space to another: but when the body *A* passes into *B*, for example, has not the empty space that was in *B* passed into *A*? There is not, it seems to me, more reason to attribute the motion to the occupied space which was in *A*, than to the void that was in *B*; the motion consists in one of these spaces replacing the other, & this replacement being reciprocal, mobility is no more a property of one than the other. Without leaving our first supposition, when I shake the vessel [that is both]

(107)

vuide & moitié plein, le vuide n'eſt-il pas mu tout auſſi bien que le fluide? Je plonge une boule de métal, creuſe, dans une bouteille; la boule va au fond; ne voilà-t-il pas un vuide qui ſe meut dans un plein, tout de même que les corps ſe meuvent dans le vuide? L'eſpace plein ne differe donc de l'eſpace vuide, ni par la mobilité, ni par l'irréductibilité; l'impénétrabilité qui diſtingue le premier du ſecond, n'eſt donc pas la même choſe que cette irréductibilité; c'eſt un je ne ſais quoi qu'on ne peut définir, parce que c'eſt une idée premiere.

Les deux loix fondamentales dont je ſuis parti (XI), ſont donc des vérités purement expérimentales; & je les ai propoſées comme telles. Une explication détaillée de ces principes n'entroit pas dans le plan de cet ouvrage, & n'auroit peut-être ſervi qu'à embrouiller les choſes: les ſciences ſont comme un beau fleuve, dont le cours eſt facile à ſuivre, lorſqu'il a acquis une certaine régularité; mais ſi l'on veut remonter à la ſource, on ne la trouve nulle part, parce qu'elle eſt par-tout; elle eſt répandue en quelque ſorte ſur toute la ſurface de la terre: de même ſi l'on veut remonter à l'origine des ſciences, on ne trouve qu'obſcurité, idées vagues, cercles vicieux; & l'on ſe perd dans les idées primitives.

FIN.

half void & half full, is not the void moved just as much as the fluid? I plunge a hollow metal ball into a bottle; the ball goes to the bottom; is this not [a demonstration of] a void which moves in an occupied space, the same as a body moving in a void? The occupied space does not differ, then, from empty space, either by its mobility, or by its irreducibility. The impenetrability which distinguishes the first from the second is then not the same thing as irreducibility: it is an I–know–not–what that one cannot define because it is a primitive idea.

The two fundamental laws from which I started (XI) are therefore purely experimental truths & I have proposed them as such. A detailed explanation of these principles did not enter into the plan of this work, & would perhaps have only served to obscure things: the sciences are like a beautiful river, whose course is easy to follow once it has acquired a certain regularity. But if one wants to go back to the source, one cannot find it anywhere, because it is everywhere; it is spread in some way over the whole surface of the earth. Similarly, if one wants to go back to the origin of the sciences, one will find only obscurity, vague ideas, vicious circles; & one is lost in primitive ideas.

END.

J'AI lu par ordre de Monſeigneur le Garde des Sceaux, un Manuſcrit intitulé *Eſſai ſur les Machines en général.* Cet Ouvrage m'a paru joindre au mérite des choſes, celui de la clarté & de la préciſion ; & je penſe que l'impreſſion en ſera utile aux progrès de la Méchanique. A Dijon, ce 6 Janvier 1782,

MARET, Cenſeur royal.

I have read by order of Monseigneur le Garde des Sceaux, a Manuscript entitled *Essay on Machines in General*. This work, appears to me to add to the merit of things, the merits of clarity & precision; & I think that the printing [of it] will be useful to the progress of Mechanics. At Dijon, this 6th January 1782,

MARET, Royal Censor

ERRATA.

fautes indiquées dans cet Errata, doivent être corrigées avant lire l'Ouvrage, parce qu'elles sont pour la plupart essentielles.

vj, *ligne* 5, *au lieu de* exactement, *lisez* exacts.

22, *lignes* 1 *&* 2, *au lieu de* est la force résultante, *lisez* est tante.

31, *ligne* 26, *au lieu de* si on les suppose, *lisez* donc si on les

38, *ligne* 2 *de la note*, *au lieu de* successivement, *lisez* exclu- t.

42, *ligne* 7, *au lieu de* cos$y$, *lisez* V cos$y$.

46, *ligne* 1 *&* 2, *au lieu de* par la vitesse, *lisez* par le carré de e.

49, *ligne* 16, *au lieu de* XXI *lisez* XIX.

53, *ligne* 1 *de la note*, *au lieu de* (1), *lisez* (2).

77, *ligne* 3, *au lieu de* note c, *lisez* note 2.

103, *ligne* 19, *au lieu de* général à celui, *lisez* général analo- celui.

ERRATA.

faults indicated in this Errata, must be corrected before reading the Work, because they are for the most part essential.

[Page] vj, line 5, in place of exactly read exact.

[Page] 22, lines 1 & 2, in place of is the resultant force, read is the resultant.

[Page] 31, line 26, in place of one supposes them read therefore if one supposes.

[Page] 38 line 2 of the note, in place of successively read exclusively.

[Page] 42, line 7 in place of cos y read V cos y.

[Page] 46,line 1 & 2, in place of by the speed, read by the square of the speed.

[Page] 49, line 16 in place of XXI read XIX.

[Page] 63 line 1 of the note, in place of (1) read (2).

[Page] 77 line 3 in place of note c read note 2.

[Page] 103 line 19 in place of general to that read general analogue to that.

Chapter 2
End Notes

1. Lazare Carnot uses the terms force and puissance interchangeably and without additional specification. As proposed above, we will use the modern term, force. Moment–of–Activity is one of the crucial terms in Carnot's theory and is nowadays called (mechanical) work. The Moment–of–Activity is, at first glance, fundamental for calculating input against output in machine processes. We also note that Carnot's conceptualisation of Moment–of–Activity is linked to time rather than to space. For details see Gillispie and Pisano (2014, Chaps. 2, 7, and 11). See also the Italian translation by Drago and Manno (1994). Hereafter, Gillispie and Pisano's book is the source book for all references for the reader who would like to read more on the subject.
2. Taking into account also Lazare Carnot's lost–quantity–of–motion, where the geometrical form is abstracted and irrelevant in the *Essay on Machines in General*, equivalence between hard bodies and mechanical plastic bodies is reasonable—noting that in the collision of hard bodies there is no force of restitution generating recoil. In other parts, for example in Lazare Carnot's second fundamental law (see below), the conceptualisation is more formal as in the following: hard bodies ('corps durs') or perfectly hard ('parfaitement durs & sans ressort') similar to d'Alembert's useage (d'Alembert [1743] 1758, Lemme XI, pp. 144–145). The definition of bodies depends on their elasticity (Carnot 1786, pp. 22–23; see also 1803a, pp. 8–10). We also remark that Carnot did

R. Pisano et al., *Essay* on Machines in General *(1786)*, Logic, Epistemology, and the Unity of Science 47, https://doi.org/10.1007/978-3-030-44385-6_2

not deal with elastic bodies or those defined as a kind of limit–case of hard bodies. The elastic bodies were considered as composed of an infinity of hard bodies linked by elastic springs (i.e., see Carnot 1786, p. 23; see also 'corps solides' in Carnot 1803a, p. 8). As argued persuasively in Gillispie and Pisano (2014), we speculate that Lazare Carnot adopted a model of hard bodies for reasons of mathematical simplicity (as plastic bodies are surely not exactly equivalent to hard bodies). For details, see Gillispie and Pisano (2014, Chap. 11).

3. This is evidence of Carnot's preoccupation with the efficiency of machines.
4. Pisano and Capecchi (2015).
5. 'direct' means 'analytically and without *ad absurdum* proofs'.
6. In Ancient Greece, Archimedes studied three machines: lever, pulley and screw. Heron of Alexandria (see *Mechanica*, Heron 1899–1914, vol. II) extended the classification to five machines: winch, lever, pulley, wedge, and screw. Guidobaldo del Monte in *Mecanicorum Liber* (Del Monte 1577) brought in an advance by considering the role played by gravity. In this context, he remarked upon the limits of the ancient approach (i.e., see: Aristotle 1955a, b, pp. 328–414). In Galileo's Le Mecaniche (Galilei 1599) the inclined plane was added. The number of simple machines became six. For details see Pisano and Bussotti (2014, 2015b, 2020a, b); Pisano and Capecchi (2015); Pisano and Gatto, pre-print.
7. Carnot is alluding to the principle of virtual work, see: Pisano (2017); Pisano and Capecchi (2013); Pisano and Gillispie (2014a, b, Chaps. 7, 11).
8. *A posteriori*, we could refer to the Science of Weights, also *Scientia de ponderibus*. In the early 16th century mechanics was concerned mainly with what is now called statics and was referred to as the *Scientia de ponderibus*, generally pursued by two very different approaches. The first was usually referred to as Aristotelian, where the equilibrium of bodies was set as a balance of opposite tendencies to motion. The second, usually referred to as Archimedean, identified statics with centrobarica, the theory of centres of gravity based on symmetry considerations. In between

the two traditions the Italian scholar Niccolo Fontana, better known as Tartaglia (1500?–1557), wrote the treatise *Quesiti et inventioni diverse* (Tartaglia 1546, 1554; Pisano and Capecchi 2015; Pisano 2020).

9. This is to be understood as force-weight.
10. Possibly Carnot is referring to the principle of virtual work outside of a Newtonian context.
11. For weight understand force-weight, here and in all other instances on this page.
12. Note that the speed is not constant, in other words, the centre of gravity accelerates.
13. The equilibrium is both among masses and force-weights.
14. The first use of the principle of virtual work applied to a machine (see also Carnot 1803a). Torricelli wrote: Praemittimus. Duo gravia simul coniuncta ex se moveri non-posse, nisi centrum commune gravitatis ipsorum discenda. (Two heavy bodies linked together cannot move by themselves unless their common centre of gravity descends); Torricelli (1644), *Liber primus De motu gravium naturaliter descendentium*, 99, line 4. see Pisano (2017) and related references; Coopersmith (2017), Gillispie and Pisano (2014).
15. The reader should pay attention to the explanation of this equilibrium law (the principle of reciprocal velocities) because of the role played by infinitesimal–and–infinite in his mathematical description with respect to previous, i.e., physical *Descartes*' principle.
16. Note that the conservation law is only true if the motion has been communicated smoothly—not by impact or collision (Gillispie and Pisano 2014, Chap. 2).
17. We remark how Carnot—in his early days—discusses a Science of Machines as something distinct from Mechanics (Newtonian science).
18. We remark on the logical structure of Carnot's scientific procedure. For a detailed enquiry, see Gillispie and Pisano, Chaps. 6–7.
19. The word "action" is far from Newton's "action of a force". Hereafter, Carnot uses this word in the context of a lost quantity of motion. An interesting example is Carnot's use of "action" in the collision of bodies (Gillispie and Pisano, Chap. 2).

20. In Carnot, the Newtonian first law of inertia arises from Carnot's law of action-reaction (which corresponds to Newton's Third Law). See: Pisano and Bussotti (2016a, b, 2017a, b, c).
21. Carnot means hard in the sense of being plastic, that is, non-elastic.
22. Following Carnot's trigonometrical reasoning, the two equations are strictly linked. In fact, $F = -F''$ so their cosines are opposite signs from each other. It follows that, for an isolated system, $V' = V''$. In other words the Equation (B) = Equation (A)F'. This is a step towards the birth of vector algebra. See Gillispie and Pisano (2014); Pisano (2017).
23. In summary: Carnot proposes to systematically consider the actions of bodies on each other, then the action of particles on a single particle. Carnot's long speech ends with summing over all particles, from which his fundamental equation (E), see below, is obtained. Let us note that the small incompressible rods and small inextensible strings are conceptualised as local–action. See Pisano (2017).
24. This is speed à la d'Alembert, at the first instant of time.
25. Taking into account our previous remark, in these passages we used the summation operator instead of the integral operator. We also note below the redundant (added/deleted) use of $\sum V F \cos q$.
26. This equation becomes null for plastic bodies; that is a conservation of quantity of motion is applicable. This equation also appears in a calculation in the *Principes fondamentaux de l'équilibre et du mouvement* (Carnot 1803a, §§129–131, pp. 103–104) as special case concerning collision among bodies. It is exactly the inverse procedure applied to the *Essay on machines in general*. This intricate reasoning in the two books could be justified due to a research for conservation energy law. However, we have no historical evidences on that (Pisano 2017; Gillispie and Pisano 2014).
27. Based on Lazare Carnot's discussions (Carnot 1786, pp. 28–30) a geometric motion is a non–mechanical interaction. Particularly, Lazare Carnot also dealt with the principle of virtual work and then, by means of geometric motions (nowadays called virtual velocities), canonically formulated the principle of virtual velocities starting from a fundamental theorem (see below Carnot 1786, §XXXIV, pp. 68–69). In effect, since in his theory geometric

motions coincide with velocities and not with displacements, this allowed Lazare Carnot to avoid, in the formulation of the principle of virtual work infinitesimal displacements, which could have produced some scientific embarrassment with respect to his assumptions (Carnot 1813). Furthermore, for the principle of virtual velocity related with any (general) mechanical machine, one can claim that the weights (strictly speaking, force–weights) that balance each other are reciprocal to their virtual velocities. Incidentally, the two conceptually different formulations can be made mathematically equivalent using the concept of virtual motion. Lazare Carnot also defined these motions as invertible: a motion assigned to a physical system of interacting bodies is geometric if the opposite motion is also possible. The result is a possible motion, but it is not always invertible (e.g. the motion of a sliding ring on a rotating rod). Therefore, one should add the hypothesis of invertibility for obtaining the concept of geometric motion. Conversely, a geometric motion, when integrated, gives an invertible motion. At this point, for constraints independent of time, a geometric displacement is equivalent to a virtual invertible displacement (but not vice versa). On the contrary, a possible displacement, only if it is invertible, produces, after its derivative, a geometric motion. In this sense, we note that, initially, the geometric motion is a kind of uniform motion applied to the whole physical system—noting the equivalence of the state of rest and the state of uniform motion. See above the Introduction; Cfr. Gillispie and Pisano, Chap. 11.

28. In other words the geometric motion produces no change in the system interaction (see real cases suggested by Carnot).
29. Carnot intends to generalise his previous Equation (E). See above. The objective is to obtain the Equation (F) as below. In order to do this it is necessary to introduce an arbitrary geometric motion u. The latter substitutes V in the previous equation and so (F) equation is determined. But Carnot wrote an inaccurate procedure. In fact, he concludes (see below) that V is the lost speed along a supposed interaction. Without doubt, u is the final speed. (Gillispie and Pisano 2014, Chaps. 2, 11).

30. In Carnot's wording "[...] as if each body had been free [...]" means the case without a geometric motion. Further, the well-known principle refers to the conservation of quantity of motion. Carnot is very clever to pay attention to the particular cases. In fact, in the equation (F) the (*a priori* indeterminate) u_i, if, adequately specified, offers *ad hoc* relationships between causes (i.e., forces) and speeds produced by these causes. Carnot's objective is to find the law of motions for each particular case, provided that an ad hoc geometric motion is determined. In order to do that, for the case of an isolated system, Carnot allocates the same values to u_i, (but $\neq 0$). Thus, he obtains a geometric motion for a uniform translation of the system. In detail, we have $\sum m_i \mathbf{U}_i \cdot \mathbf{u}_i = 0$ (**U** and **u** are vectors) where m_i = mass of the i–th body; $\mathbf{U}_i$ = velocity lost (by that body) during the collision; $\mathbf{u}_i$ = velocity called 'mouvement géométrique'. This is clearly an extension of the principle of virtual velocities (Pisano 2017) to the collision of several bodies using u_i = const. We can write: $\sum m_i \mathbf{U}_i \cdot \mathbf{u}_i = 0$ and $\mathbf{u}_i \cdot \sum m_i \mathbf{U}_i = 0$. Since **u** is arbitrary, we have: $\sum m_i \mathbf{U}_i = 0$ where $\mathbf{U}_i = \mathbf{W}_i - \mathbf{V}_i$. So, it follows that (in modern terms): $\sum m_i \mathbf{W}_i = \sum m_i \mathbf{V}_i$. The latter is the conservation of quantity of motion for a system of bodies (*Ibidem*; Gillispie and Pisano 2014; Pisano 2017). In short we can mainly summarize (Pisano 2017) as in the following: (a) A theory of interacting bodies by means of collisions; (b) A collision is a basic phenomenon. In particular, continuously accelerated motion is obtained as a limiting case of a system driven by a series of pulses; (c) Newton's second law is replaced by Lazare Carnot's second fundamental equation for a system of n–bodies; (d) Due to the arbitrariness of u_i, it can be assumed constant, that is to say, the same translation of geometric uniform motions of all bodies is adopted. This method could be considered an early beginning of the use of symmetries in the history of theoretical physics (Cfr. Drago and Manno 1994).
31. By considering another ad hoc geometric motion, $\mathbf{u}_i = \omega \times \mathbf{r}_i$, e.g., the rotation of the system with angular velocity around a fixed axis, and using the properties of the triple product and the arbitrariness of ω. He (taking into account our reasoning in the

previous endnote), he obtained the following (in modern terms) laws of conservation as invariants of motion: $\sum m_i \mathbf{W}_i = \sum m_i \mathbf{V}_i$ (Law of conservation of the quantity–of–motion) $\sum m_i \mathbf{r}_i \times \mathbf{W}_i = \sum m_i \mathbf{r}_i \times \mathbf{V}_i$ (Law of conservation of the angular–momentum). Where $\mathbf{W}_i$, $\mathbf{V}_i$, ω, $\mathbf{r}_i$ are vectors.

32. In order to obtain idem equations, Carnot wants to generalise his method assigning other values to u.
33. Here we note that Carnot claims his first lines concerning the application of synthetic method to his theory. Generally speaking the synthetic method (Carnot 1786, pp. 33–35, p. 85 Carnot 1813, pp. 12–21, 189, 200, 242–243, 217–253; Gillipie and Pisano, Chap. 11) and the analytic method have their historical origins in the ancient period with Pappus from Alexandria (290–350 b.C.; Pappus 1588), and later assumed different meanings even for a single author, e.g. Descartes (1897–1913; on Descartes see the remarkable works by John Schuster). With regard to Lazare Carnot, the synthetic method also explains the nature and the frame of mind behind his research against the metaphysical conception—which prevailed at the time—of infinitesimals, as he declares at the beginning of his famous book, *Réflexions sur la métaphysique du calcul infinitésimal*. In other books by him: “It is my object to ascertain in what the true spirit of the Infinitesimal Analysis consists” (Carnot 1832, p. 1. [“Je cherche à savoir en quoi consiste le véritable esprit de l’Analyse infinitesimal” (see also, 1813, p. 1, line 1). Let us note that the English translation of the 1832 version is differently organized in comparison to the original 1813 version. E.g., the number of paragraphs in the two versions does not correspond (Cfr. Gillispie and Pisano 2014, Chap. 11).
34. Carnot will not deal with these examples of motions. He only mentions them as a supposition.
35. Concerning the concept of mass, Carnot—as for other scientists at that time—yet had no very clear idea of the analytical role played by this quantity in the theory. For example, Carnot still—seventeen years after the second edition of the Essay on machines en general (1786)—demonstrated an ambiguity between the Cartesian and Newtonian approaches as shown in: “L’espace apparent qu’occupe

un corps, s'appelle son volume; l'espace effectif qu'occupe ce même corps ou sa quantité réelle de matière, se nomme sa masse." In English: "The visible space which a body occupies is called its volume; the effective space which this same body occupies or its real quantity of matter is named its mass" (Carnot 1803a, p. 6; Cfr. Gillispie and Pisano 2014; Pisano's works on the subjects).

36. We note that, by means of a dimensional analysis, the names given by Carnot are not, by today's standards, accurate (Cfr. Pisano 2010; Drago and Manno 1994). In fact the dimensions are, for energy, $[ML^2T^{-2}]$, and not those for the quantity of motion, $[MLT^{-1}]$. At that time, Jean-Baptiste Joseph Fourier (1768–1830) introduced physical dimensions as a *modus operandi* in his *Théorie analytique de la chaleur* (Fourier 1822; see also Pisano and Capecchi 2009a, b).
37. We note that here Carnot claims his new idea (with respect to the works of previous scholars) of a physical quantity which is conserved along collisions (Gillispie and Pisano, Chaps. 2, 11).
38. We note that Descartes, Maupertius and d'Alembert are the unique mechanicists cited by Lazare Carnot in his *Essay on Machines in General*. There are embryonic hints of Maupertius' principle also in the previous Corollary I.
39. On the subject see recently: Pisano (2017).
40. Here Lazare Carnot intends summations. Due to editorial obligations we retain his original wording. But the reader should pay attention step by step Lazare's wording and our related comments.
41. Here Lazare Carnot intends summations. Due to editorial obligations we retain his original wording. But the reader should pay attention step by step Lazare's wording and our related comments.
42. By means of Corollary II, we remark that in the traditional mechanical theory of hard bodies, Carnot's principle of virtual work formally defines the condition of equilibrium of forces that act on bodies in order to produce work. Lazare Carnot takes into account, (1) continuous changes of motion (degrés insensibles), (2) the concept of motrice force (nowadays, acceleration, see below), and (3) the mathematical formula for the principle of virtual work (Gillispie and Pisano 2014, Chaps. 10–11; Pisano 2017) and studies the the-

oretical conditions that translate the practical conditions of equilibrium and also obtains his invariants with regard to the efficiency and reversibility of mechanical machines. His first Equation (E) (Carnot 1786, p. 27) and second equations (F) (*Ivi*, p. 32) generalized for multi–body discrete systems, too. We also remark his crucial concept, the lost speeds, in this case, added to the accelerations, p. Since he considers the instantaneous change of motion ($\mathrm{d}V$) along the collisions, then the accelerations p gives any connection to the collision. In short, we can summarize as follows: (a) The mass of the parts of a machine; (b) Global magnitudes, abstracting from the mass of the mechanism; (c) Kinematics first, then dynamics, and statics is a special case of dynamics; (d) A theory of machines concerns a theory of the communication of motions; (e) A machine is a connected system of (hard) bodies; (f) The connections between the bodies constrain the communication of motion of the bodies; (g) The theory of interaction–collisions by means of insensible degrees (e.g., see also Carnot 1803a, §293, pp. 261–262; see Introduction above) as the result of a sequence of infinitesimally small percussions.

43. In agreement with recent accredited literature (Gillispie and Pisano 2014, Chaps. 2, 11; Pisano 2017) Carnot clarifies the physical vector definition of impelling forces and resisting forces. The moment of activity is—in practice—the modern conceptualisation of work ($\mathbf{F}d\mathbf{s}$). In addition, no metaphysical causes of variation of motion are suggested. On the contrary, he considered the (work) moment–of–activity "q", operated by resisting forces, as the effect produced by impelling forces. Instead, he considered the (work) moment–of–activity "Q", consumed by impelling forces at a given t–time (Carnot 1786, §§LII–LIII, pp. 83–84, §§LXIII–LXIV, pp. 95–99) in order to investigate (the efficiency and agency of) machines in a general way. His son, Sadi did the same for heat machines (Gillispie and Pisano 2014; see also numerous works by Pisano on the subject). Just to also mention that by the word "work" (travail) Coriolis meant mechanical power (puissance mécanique), that is, the quantity of action, also called dynamical effect (Coriolis 1829, p. III; see also 1844, 1830–1837; Gillipsie and Pisano 2014, Chap. 4).

44. This quantity, $\sum Pcoszudt$, is force x distance—that is, 'work' in modern usage.
45. This theorem is exactly a generalisation of the modern principle of virtual work (Pisano 2017).
46. Here and hereafter, 'agents' means 'working substance'.
47. On the concept of heavy body, the reader can surf the recent book on Science of Weights and Tartaglia (Pisano and Capecchi 2015).
48. That is, mathematically and without the use of infinitesimals.
49. On the centre of gravity and its relationship with the principle of virtual work, see: Capecchi and Pisano (2010a, b, c), Pisano (2017), Capecchi and Pisano (2007, 2010b, b); et al. Pisano's works on the subject).
50. This fifth Corollary can be considered as the basic (and modern) conceptualization for running machines, taking into account the relationship between soliciting and resisting forces.
51. Carnot returns to his crucial concept of work. See our previous endnote above.
52. On the subject of machines, see Pisano and Capecchi (2015), Pisano and Bussotti (2014, 2015a, b), Capecchi and Pisano (2010b).
53. On the subject applied to both Lazare and Sadi Carnot, see Gillispie and Pisano (2014).

Chapter 3
Lazare Carnot's Manuscripts and Documents*

Lazare Carnot: 3 portraits on paper (20,5x28) cm; (19,5x27,5) cm; (25,5x34,5) cm	[Raffaele Pisano : plates from orginal (y. 2010) in .jpeg by *Académie des Sciences, Institut de France*, Paris]
Dissertation sur la théorie de l'infini mathématique, ouvrage destine it concourir au prix qu'a propose L'Académie Royale des Sciences, arts et belles–lettres de Berlin, pour l'année 1786. The manuscript is dated from Arras 8 September 1785. It is conserved in the Archives of the *Deutsche Akademie der Wissenschaften zu Berlin*, and consists of 100 paragraphs in 90 folios.	[It is reproduced in its entirety in Gillispie (1971), Appendix A, pp. 171–267, *op. cit.*; and microfilm copies of the complete manuscript are also deposited in the Firestone Library of Princeton University.]
Carnot L (1778) *Mémoire sur la théorie des machines pour concourir au prix de 1779 propose par l'Académie Royale des Sciences de Paris.* The manuscript is dated 28 March 1778. It is conserved in the Archives de l'Académie des sciences, Institut de France. Ms. (18,5x23,5) cm consists of: 85 sections in 63 folios (31 folios r/v)	[Raffaele Pisano : plates from original (y. 2010) in .jpeg by *Académie des Sciences, Institut de France*, Paris] [Sections 101–160 are reproduced In: Gillispie (1971), Appendix C, pp. 299–343, *op. cit.*; and microfilm copies of the complete manuscript are also deposited in the Firestone Library of Princeton University]

*Cfr. Pisano's works; Gillispie and Pisano 2014

R. Pisano et al., *Essay* on Machines in General *(1786)*, Logic, Epistemology, and the Unity of Science 47, https://doi.org/10.1007/978-3-030-44385-6_3

Carnot L (1780) Mémoire sur la théorie des machines pour concourir au prix que l'Académie Royale des Sciences de Paris doit adjuger en 1781. The manuscript is dated from Béthune 15 July 1780. It is conserved in the *Archives de l'Académie des sciences, Institut de France.* Ms. (20,5x32) cm consists of 191 sections in 106 folios. (47 folios r/v + 2 planches de dessins de même dimensions).	[Raffaele Pisano : plates from original (y. 2010) in .jpeg by *Académie des Sciences, Institut de France*, Paris] [Sections 101–160 are reproduced In: Gillispie (1971), Appendix C, pp. 299–343, *op. cit.*; and microfilm copies of the complete manuscript are also deposited in the Firestone Library of Princeton University]
Lazare Carnot's portraits	[Raffaele Pisano: many plates from the original kindly authorized by Monsieur Gaetan Carnot of *Carnot's family*. The original are conserved at *Académie François Bourdon*. Other Lazare and Sadi Carnot's primary documents are conserved at the *Archives at the Collections archives de la bibliothèque de l'Ècole polytechnique de Palaiseau* (Essonne, France)].

Lazare Carnot

Carnot L (1798) Réponse de L.–N.–M. Carnot, citoyen français, l'un des fondateurs de la République, et membre constitutionnel du Directoire exécutif, au rapport fait sur la conjuration du 18 fructidor au Conseil des Cinq-Cents par f.-Ch. Bailleul au nom d?une commission spéciale, s. 1., 8 floréal an VI.

Carnot H (1861–1863) Mémoires sur Carnot par son fils, 2 vols. Pagnerre, Paris.

Carnot L ([1783] 1786) Essai sur les machines en général. Defay, Dijon.

Carnot L (1778) Mémoire sur la théorie des machines pour concourir au prix de 1779 propose par l'Académie Royale des Sciences de Paris. The manuscript is conserved in the Archives de l'Académie des sciences, Institut de France, and consists of 85 sections in 63 folios. Sections 27–60 are reproduced. In: Gillispie (1971), Appendix B, pp 271–296.

Carnot L (1780) Mémoire sur la théorie des machines pour concourir au prix que l'Académie Royale des Sciences de Paris doit adjuger en 1781. The manuscript is dated from Béthune 15 July 1780. It is conserved in the Archives de l'Académie des sciences, Institut de France, and consists of 191 sections in 106 folios. Sections 101–160 are reproduced. In: Gillispie (1971), Appendix C, pp 299–343.

Carnot L (1784) Éloge de M. le maréchal de Vauban, discours qui a remporté le prix de1'Académie des sciences, arts et belles-lettres de Dijon en 1784, Dijon.

Carnot L (1784) Lettre sur les aérostats. A manuscript memoir addressed to the Académie Royale des Sciences on 17 Jan 1784 [document lost]

Carnot L (1785) Dissertation sur la théorie de l'infini mathématique, ouvrage destine it concourir au prix qu'a propose l'Académie Royale des Sciences, arts et belles–lettres de Berlin, pour l'année 1786 [The manuscript is dated by Arras 8 Sept 1785].

Carnot L (1786) Observations sur la lettre de M. Choderlos de Laclos à Messieurs de l'Académie françoise, concernant l'Éloge de Monsieur le Maréchal de Vauban. Arras, Imp. Vve Michel Nicolas.

Carnot L (1787a) LETTRE de M. Carnot, Capitaine en premier au Corps Royal du Génie, à M. le Marquis de Montalembert. In: Réponse au Mémoire sur la fortification perpendiculaire, Par plusieurs Officiers du Corps Royal du Génie, Présenté à l'Académie Royale des Sciences, Ouvrage enrichi de plusieurs grandes Planches, avec une Planche de Supplément relative aux Affûts dits Affûts à aiguille. Par M. le Marquis de Montalembert, Maréchal des Camps & Armées du Roij Lieutenant-Général des Provinces de Saintonge & Angoumois, de l'Académie Royale des Sciences & de l'Académie– Impériale de Pétersbourg. A Paris, Pierres. Chez Didot, Fils aîné, Libraire du Roi pour l'Artillerie & le Génie, rue Dauphine, N* 116, p. 16; pp 17–18.

Carnot L (1787b) Maximilien Robespierre Le droit et l'état des bâtards. Lazare Carnot Le Pouvoir de l'habitude. In: Berthe L-N et De Langre M (eds). Discours inédits prononcés devant l'Académie d'Arras les 27 Avril 1786 et 25 Mai 1787. Académie des Sciences, Lettres et Arts, Arras.

Carnot L (1789a) Mémoire présenté au Conseil de la Guerre, Au sujet des Places fortes qui doivent être démolies ou abandonnées; ou examen de cette question : Est-il avantageux au Roi de France qu'il y ait des Places fortes sur les frontières de ses États? Paris, Barrois.

Carnot L (1789b) Réclamation adressée à l'Assemblée nationale contre le régime oppressif sous lequel est gouverné le Corps royal du Génie, en ce qu'il s'oppose aux prowès de l'art et au bien qu'l serait possible de faire par M. Carnot, Capitaine dans ce même Corps, 28 septembre 1789, Paris, Imp. vve Delaguette.

Carnot L (1792) Sur les citadelles, Carnot l'aîné, député du département du Pas–de–Calais à ses collègues, 5 janvier, l'an IV de la Liberté, Paris, Imp. nat.

Carnot L (1793a) Rapport fait à la Convention Nationale par ses commissares Carnot, Garrau et Lamarque, envoyés par elle aux frontières des Pyrénées. 12 janvier 1793, Paris, Imp. nat.

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R. Pisano et al., *Essay* on Machines in General *(1786)*, Logic, Epistemology, and the Unity of Science 47, https://doi.org/10.1007/978-3-030-44385-6

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