

Nicholas Rescher

Knowledge at the Boundaries

Logic, Epistemology, and the Unity of Science

Volume 48

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This series provides a venue where philosophers and logicians can apply specific technical insights to fundamental philosophical problems. While the series is open to a wide variety of perspectives, including the study and analysis of argumentation and the critical discussion of the relationship between logic and the philosophy of science, the aim is to provide an integrated picture of the scientific enterprise in all its diversity.

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Knowledge at the Boundaries

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For Robert Audi

*Exemplary Philosopher, Scholar,
and Colleague Extraordinary*

Preface

I am grateful to my friend and collaborator Patrick Grim for permitting inclusion of a joint paper in the volume. And I also grateful to Estelle Burris for her ever-competent assistance in putting this material into publishable form and to James Baron for editing the manuscript.

Pittsburgh, PA, USA
January 2020

Nicholas Rescher

Introduction

The book provides reflection on the nature, scope, and limits of knowledge has been at the focus of my philosophical concerns over many decades. The essays collected in this volume expound and extend these efforts in exploring the outer fringes of understanding: the outer boundaries of conceivability, the limits of cognition, and the ramifications of ineffability and paradox. They join in exploring the lay of the land at the boundaries of knowledge.

Fundamental to the deliberations of the book is the distinction between *any* and *every*, between *each* and *all*—between saying that any entrant can win the race and that every entrant can do so. This simple fact has significant philosophical ramifications, in particular in relating to the “Principle of Sufficient Reason.” For consider the idea that the real is rational—that the facts about reality all admit of explanation. This may well be true of any *given* fact—any fact that has been established and put on the agenda of consideration. But the Kantian idea of question propagation still looms—the idea that every answer we achieve raises further questions. Each does not ensure all: The circumstance that any member of the sequence of integers can be arrived at in counting does not mean that they can all be reached in this way. The problem of totalization affects not only counting but also knowing.

Against this background, the present deliberations are variations on the theme of cognitive limitedness. No doubt there are no questions about the real that are philosophically off limits—that we should not address and try to answer. But nevertheless, inquiry in this domain has its limitations. In the life of the mind, as in the life of action, perfection is not in prospect. On both sides alike, we are well advised to do the best we can in the full realization that the best we can manage will fall short of idealized aspirations.

In the pursuit of an aspiration, as in the winning of a race, there are two measures of progress: How far we have come from the start and how near we have come to the finish. When the race is of finite scope, these two come to the same thing: Each step further from the start brings one a step closer to the finish. But when the course is of indefinite—let alone infinite—extent, this comfortable situation drops away. Progress can only be measured by doing more and better in relation to the past. We

can aspire to an ongoing improvement. But in the cognitive domain, or the moral life, there is no confident assurance of goal realization. We can indeed always do better but perfection is beyond our grasp. In the development of knowledge and understanding, we are always brought to the boundaries.

Overall, the book consists of five parts, as follows. Part I (Chaps. 1–4) address issue of conception and specification; Part II (Chaps. 5–8) address issues of truth and knowledge; Part III (Chaps. 9–16) address issues of paradox and cognitive incompleteness; and finally Part IV (Chaps. 17–20) addresses issues of philosophizing itself. Overall the thematic studies of the book moves from the concrete and particular to the abstract and general—from the specifics and particular to the general and total. And, the overall lesson is that such transit cannot be smooth owing to the problems and paradoxes that we eventually encountered along the way. Notwithstanding this acknowledgment of limits and limitations, the overall perspective is not skeptical and nihilistic. Instead, it insists that here as elsewhere in life we must do the best we can in the expectation of success within our own terms of reference. In philosophy as in medicine, we cannot expect to find a single diet or regime that will suit everyone. But there is no reason to expect that we cannot find one that is suitable (actually and objectively) for ourselves.

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Part I
Issues of Specification and Conception

Chapter 1

Default Reasoning



1.1 Default Inference

The topic of default reasoning affords some instructive insights into the nature of the relationship between theoretical and practical reasoning. A *default* in logic is a fall-back position in point of conclusion drawing—one to which we can appropriately take resort when things go wrong. But, of course, things ought not to go wrong in logic. So what is going on here? Default reasoning is a matter of presumption—of how we can take matters to stand in the absence of good reasons to the contrary.

Orthodox inferential reasoning proceeds via *logically valid* inference processes which, as such, do—and must—lead to true conclusions when the premisses are true. By contrast, default reasoning—which involves an information gap between premisses and conclusion—is such that premisses that are true will lead to plausible (though possibly false) conclusions.

The logical validity of inference rules in standard (truth-functional) logic is determined on an input-output basis, a valid rule being one that will invariably yield true outputs (conclusions) from true premisses. All such inference rules will faithfully and unfailingly transmit the truth of premisses to the conclusions. By contrast, the inference processes of default logic are such that the truth of the premisses does not assure that of the conclusion but will at most establish that conclusion as plausible. Such inferences are *ampliative*: the conclusion can go beyond what the premisses provide, thanks to a shortfall of information. And this means that such reasonings are fallible and can—and occasionally will—lead from true premisses to false conclusions.

We shall represent *logically valid deducibility* (in its classical construction) by $\vdash$ and, by contrast, use $\Vdash$ to represent the *plausible inferability* at issue with default reasoning. A reasonable approximation to the nature of the latter is available via the principle that:

$$p \Vdash q \text{ iff } (\exists r)(\Pi r \ \& \ (p \ \& \ r) \vdash q)$$

where Π is the modality of plausibility, so then Πr indicates r to be an inherently plausible contention.¹

Some examples of inference processes in default logic are as follows:

- (1) p is highly likely $\Vdash p$
- (2) p is very likely, and q is very likely $\Vdash (p \ \& \ q)$ is very likely
- (3) there is strong evidence in favor of p and no more than weak evidence against it $\Vdash p$
- (4) p has obtained in all past instances, $\Vdash p$ will obtain in the next instance.

If all we are told of some number is that it is a prime, we would, plausibly enough, conclude that it is not an even integer—even though we are aware that this conclusion will prove false once out of an infinity of cases (viz. that of the number two).

As these examples indicate, the inference processes of default logic can all be assimilated to a deductive pattern of the following structure (which does clearly obtain as valid in traditional logic):

- In all ordinary (normal, standard, commonplace) cases, whenever P , then Q
 - P obtains in the case presently at hand
 - <The present case is an ordinary (normal, standard, etc.) one>
- $\therefore Q$ obtains in the present case

But while this reasoning as such is valid, its conclusion can readily go amiss, since that third premiss—with its affirmation—can readily fail to hold, being itself a merely plausible proposition.

Yet here that third, usually tacit and thereby enthymematic, premiss plays a pivotal role. And it is, in general, able to do so not because we have secured it as a certified truth, but simply because it is a plausible (albeit defeasible) presumption that is strongly supported by the available evidence though not, of course, guaranteed. Default reasoning accordingly rests on arguments which would be valid if all of their premisses—explicit and tacit alike—were authentic truths, which they are not since at least one of the critical premisses of the argument is no more than a mere presumption.

The key point is that we are virtually entitled to undertake those default-grounding presumptions—as per “The present case is a normal one”—in situations where there are no case-specific indications to the contrary.

Such a *defeasible presumption* is emphatically not to be regarded as an established truth but merely something that can be taken to hold only provisionally, as long as no counter-indicatively conflicting information comes to light. Against this background, the procedure that is definitively characteristic of default reasoning is:

To treat what is generally (normally, standardly, generally, usually, etc.) the case *as if* it were the case always and everywhere, and therefore as applicable in the present instance.

Here, in effect, ignorance is bliss: where there is no good reason to see the case at hand as being out of the ordinary, we simply presume it to be an ordinary one in the absence of visible counter-indications. Such reliance on a *principle of presumption*—to the effect that what generally holds, also holds here in the case presently at hand—elucidates the *modus operandi* of default reasoning.²

1.2 Facing the Prospect of Error

Of course, such a plausible presumption can go awry. For it may well happen that the situation at hand fails to be standard and representative as the enthymematic comportment of the argument requires. This is brought out vividly in John Godfrey Saxe's poem *The Blind Men and the Elephant*, which tells the story of certain blind sages, those "six men of Indostan/To learning much inclined/Who went to see the elephant/(Though all of them were blind)." One sage touched the elephant's "broad and sturdy side" and declared the beast to be "very like a wall." The second, who had felt its tusk, announced the elephant to resemble a spear. The third, who took the elephant's squirming trunk in his hands, compared it to a snake; while the fourth, who put his arm around the elephant's knee, was sure that the animal resembled a tree. A flapping ear convinced another that the elephant had the form of a fan; while the sixth blind man thought that it had the form of a rope, since he had taken hold of the tail:

And so these men of Indostan,
Disputed loud and long;
Each in his own opinion,
Exceeding stiff and strong:
Though each was partly in the right,
And all were in the wrong.

None of those blind sages was altogether in error, it is just that the facts at their disposal were nontypical and unrepresentative in a way that gave them a biased and misleading picture of reality. It is not that they did not know truth, but rather that an altogether plausible inference from the truth they knew propelled them into error.

But since such a policy of typicality presumption may well lead us down the primrose path into error, how is it ever to be justified? The answer here lies precisely in the consideration that what is at issue is not a truth-claim but a policy or procedure. And such policies of procedure are not justified in the theoretical (i.e., factual) order but in the practical or pragmatic order of deliberation. The validation at issue runs roughly as follows:

1. We have questions to which we need a (satisfactory) answer and in the face of this we take the stance that—
2. We are rationally entitled to use a premiss that holds good promise of finding one (i.e., is effective or more effective than the other available alternatives) even though it may occasionally fail.

On this basis we proceed subject to the idea that if and when things go wrong, this is a bridge we can cross when we get there, invoking “explanations” and excuses to indicate the unusual (anormal, extraordinary) circumstances of the case.

Even as in real life we cannot manage our affairs sensibly without running risks, so in the cognitive life one must, on occasion, take the risk of error in one’s stride, since the inevitable result of a radical nothing-risk policy is the nothing-have of radical skepticism. And this situation is particularly prominent in inductive contexts.

1.3 Induction as Default Reasoning

The term “induction” is derived from the Latin rendering of Aristotle’s *epagôgê*—the process for moving to a generalization from its specific instances.³ Gradually extended over an increasingly wide range, induction can be seen as a question-answering device encompassing virtually the whole range of nondeductive reasoning. Thus, consider a typical inductive argument—that from “All the magnets we have examined attract iron filings” to “All magnets attract iron filings.” It would be deeply problematic to regard this as a deductive argument that rests on the (obviously false) premiss: “What is the case in all examined instances is universally the case.” Rather, what we have here is a plausible presumption that takes the cases in hand to be typical and generally representative in the absence of concrete counter-indications—that is, an instance of default reasoning.

Induction, so regarded, is accordingly not so much a process of *inference* as one of presumption-based *truth-estimation*. We clearly want to accomplish our explanatory gap-filling in the least risky, minimally problematic way, as determined by plausibilistic considerations. This is illustrated by such examples as:

- There is smoke yonder
- Usually, where(ever) there’s smoke, there’s fire
- <The present situation fits the usual run>
 ∴ There is fire yonder

or again:

- Two thirds of the items in the sample are defective
- <The sample is representative of the whole>
 ∴ Two thirds of the items in the whole population are defective

(Here, the enthymematically tacit premisses needed to make the argument deductively cogent have been indicated.)

Its reliance on a presumption of typicality, normalcy, or the like, means that any inductive process is inherently chancy. Induction rests on presumption-geared default reasoning and its conclusions are thus always at risk to further or better data, since what looks to be typical or representative may in due course turn out not to be so.

1.4 Default Reasoning as Nonmonotonic

Because default reasonings rest on a presumption of normality, typicality, or the like, it may well transpire that while a group of premisses $\parallel$ implies a certain conclusion, nevertheless the conjunction of these premisses with some further propositions may fail to do so. Such implications are called nonmonotonic because while “If p then q ” obtains, nevertheless it can happen that q sometimes fails to obtain in certain circumstances where p holds, so that:

$$p \Rightarrow q \text{ need not yield } (p \& r) \Rightarrow q$$

Additional information can destabilize default implications.

Clearly, the reason why the monotonicity-characterizing principle of:

- Whenever $p \vdash q$, then $(p \& r) \vdash q$

works in deductive context, is that there is then no normality linkage here between p and q which requires the addition of further material that may or may not be forthcoming—as with the stipulation of normalcy or of “all things equal” in matters of inductive reasoning. The reliance of default reasoning on a presumption of normality, typicality, or the like, means that throughout this domain new information can undo earlier findings.

Thus, consider the claim:

- If you are in America, then you might be in New York.

This is, of course, perfectly correct. But it will not do to “strengthen” the antecedent as per:

- If you are in America and you are in Texas, then you might be in New York.

The conclusions we arrive at with nonmonotonic implication relations are no more than presumption. For in making the inference, we have to presume that the situation is not one where some as yet unseen conclusion-averting circumstance comes into operation.

This state of affairs also means that with nonmonotonic implications *modus ponens* fails: the combination of p and $p \Rightarrow q$ need not *demonstrate* that q obtains but may do no more than to establish a *presumption* to that effect.

Nonmonotonicity is thus a standard feature of default inference as is illustrated by contrasting:

- If I had put sugar in the tea, then it would have tasted fine

with:

- If I had put sugar and cayenne pepper in the tea, then it would have tasted fine.

Or again, contrast:

- If you greet him, he will answer politely

with:

- If you greet him with an insult, he will answer politely.

After all, that first implication effectively (but tacitly) comes to:

- If you greet him *in the usual and ordinary way*, he will answer politely

and the antecedent of the second implication violates that initial condition.

With default inferences, we have to do with what is, from the standpoint of standard logic, a decidedly eccentric mode of reasoning. For no qualification additional to the antecedent as such can abrogate what a valid monotonic implication implies: the antecedent will, in and of itself, suffice to guarantee the consequent. But whenever that “inevitably, invariably, unavoidably, etc.” becomes weakened to “generally, usually, probably, possibly, etc.,” the monotonicity that is requisite for authentic implication is lost. To obtain a conclusion we must now suppose that nothing untoward is hidden from our sight—that nothing unmentioned intervenes. And this always brings the factor of presumption upon the scene.

1.5 Some Comforting Considerations

But what if those normality presumptions should prove unjustified? How are we to proceed in the context of conclusions arrived at by reasoning that we see as potentially misleading? The short answer is: Cautiously! But a somewhat more informative response lies in the important prospect of *blurring* that conclusion—making it less specified and detailed. As stated at the outset, default reasoning calls for the possibility of resort to a fall-back position. And in managing our cognitive risks we can always fall back upon *vagueness* and its inherent qualifications.

With default reasoning in general and induction in particular, we run the risk that our conclusions may go awry thanks to our reliance on (generally tacit) suppositions of normality or typicality that may fail in the circumstances at hand. To offset the risk error, we can resort to the introduction of decreasing definiteness for the sake of increasing security. Thus, instead of reasoning:

- q is highly likely wherever p
- In the present case, p obtains
- The present case looks to be a typical, majority-conforming one
<Looks are not deceiving here>
 $\therefore$ In the present case, q obtains

we would instead reason to:

- In the present case, q probably obtains.

Thereby, we are taking a sensible step in the direction of safety. But, of course, likelihoods do not answer a “yes/no” question and where such a question confronts us, we have little choice but to resort to (circumstances permitting) chance the risks of the presumption of typicality/normality that characterizes default reasoning. There are, however, some promising precautions here.

After all, a fundamental feature of inquiry is represented by the following observation:

THEESIS 1: *Insofar as our thinking is vague, truth is accessible even in the face of error.*

Consider the situation where you correctly accept P -or- Q . But—so let it be supposed—the truth of this disjunction roots entirely in that of P while Q is quite false. However, you accept P -or- Q only because you are convinced of the truth of Q ; it so happens that P is something you actually disbelieve. Yet despite your error, your belief is entirely true.⁴ Consider a concrete instance. You believe that Mr. Kim Ho is Korean because you believe him to be a North Korean. However, he is in fact a South Korean, something you would flatly reject. Nevertheless, your belief that he is Korean is unquestionably correct. Thanks to the indefiniteness of that disjunctive belief at issue, the error in which you are involved, although real, is not so grave as to destabilize the truth of your belief.

This example illustrates a more far-reaching point:

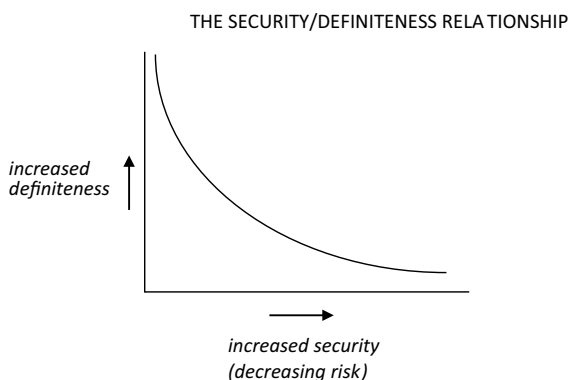
THEESIS 2: *There is, in general, an inverse relationship between the precision or definiteness of a judgment and its security: detail and probability stand in a competing relationship.*

It is a basic principle of epistemology that increased confidence in the correctness of our estimates can always be purchased at the price of decreased accuracy. We estimate the height of the tree at around 25 feet. We are *quite sure* that the tree is 25 ± 5 feet. We are *virtually certain* that its height is 25 ± 10 feet. But we are *completely and absolutely sure* that its height is between 1 inch and 100 yards. Of this we are completely sure, in the sense that we deem it absolutely certain, secure beyond the shadow of a doubt, as certain as we can be of anything in the world, so sure that we

would be willing to stake our life on it, and the like. With any sort of estimate, there is always a characteristic trade-off relationship between the evidential *security* of the estimate on the one hand (as determinable on the basis of its probability or degree of acceptability), and the informative *definiteness* (exactness, detail, precision, etc.) of its asserted content on the other. Vaguer and looser statements are for that very reason more secure because they embody larger margins of error. This relationship between security and definiteness is graphically characterized by a curve of the general form of an equilateral hyperbola (see Display 1). And this sort of relationship holds just as well for our *truth* estimates as of others.

This state of affairs has far-reaching consequences. It means, in particular, that no secure statement about objective reality can say exactly and in complete detail how matters stand universally, always and everywhere. To capture the full complexity of the truth of the matter of things by means of language we must often proceed by way of “warranted approximation.” In general, we can be sure of how things “usually” are and how they

Display 1



Note: The overall quality of the information provided by a claim hinges on combining its security with its definiteness. Given suitable ways of measuring security (s) and definiteness (d), the curve at issue can be supposed to be an equilateral hyperbola obtained with $s \times d$ as constant.

“roughly” are, but not how they always and exactly are. And this impels our reasoning in the direction of presuppositions of normalcy, typicality, and the like, which are characteristic of default argumentation.

But be this as it may, the present considerations indicate that “inductive inference,” as traditionally conceived, affords a paradigm instance of default reasoning, which

itself emerges in their light as an exercise in standard deductive inference subject to a recourse to the potentially defeasible presumption of typicality.

Yet how is the adoption of a potentially defeasible thesis to qualify as rationally appropriate? The answer, as noted above, lies in the general principle of risk management. For what is at issue with presumption is at bottom less an endorsement of the truth than the implementation of a policy. And rationality here—as elsewhere in matters of practical procedure—pivots on the principle of a favorable balance of potential benefit over potential loss. In many situations, default reasoning affords our best available pathway to our ultimately very practical need for information—for answering in a cogent and epistemically responsible way a question that we need to resolve. For in truth-estimation, as in so much of life, we have to rest content with doing the best we can actually manage to achieve in the circumstances.

Notes

1. On issues of this nature, see the author's *Conditionals* (Cambridge, MA: MIT Press, 2007) as well as his *Plausible Reasoning* (Assen: Van Gorcum, 1976).
2. Regarding default reasoning and its ramifications, see "Common-Sense Reasoning" in *The Routledge Encyclopedia of Philosophy* (London: Routledge, 2000); William L. Harper, "A Sketch of Some Recent Developments in the Theory of Conditionals," in W. L. Harper, L. G. Pearson, and Robert Stalnaker (eds.), *IFS: Conditionals, Belief, Decision, Chance and Time* (Dordrecht: D. Reidel, 1981); Henry E. Kyburg, Jr. and Chou Man Teng, *Uncertain Inference* (Cambridge: Cambridge University Press, 2001); Hans Rott, *Choice and Inference: A Study of Belief Revision and Nonmonotonic Reasoning* (Oxford: Clarendon Press, 2001); Alexander Bochman, *A Logical Theory of Nonmonotonic Inference and Belief Change* (Berlin: Springer, 2001); J. L. Pollock, "A Theory of Defeasible Reasoning," *International Journal of Intelligent Systems*, vol. 6 (1991), pp. 33–54; R. B. Reiter, "Nonmonotonic Reasoning," *Annual Review of Cognitive Science*, vol. 2 (1987), pp. 147–86; and Nicholas Rescher, *Induction* (Oxford: Basil Blackwell, 1980).
3. See W. D. Ross, *Aristotle's Prime and Posterior Analytics* (Oxford: Clarendon Press, 1949), pp. 47–51.
4. Examples of this sort indicate why philosophers are unwilling to identify knowledge with true belief.

Chapter 2

Vagueness: A Variant Approach



2.1 The Sorites Paradox and Its Problems

Vagueness is a prime source of paradox. For vague terms have a more or less well-defined central core of application, surrounded by a large penumbra of indefiniteness and uncertainty. And since a term, T , that is vague will automatically have a complement, non- T , that is so as well, there will inevitably be a nebulous region of ambivalent overlap between T situations and non- T situations. Here, matters seem to stand both ways, so that a paradoxical inconsistency arises.

The most familiar ways of addressing the well-known paradoxes of vagueness call for the use of very heavy machinery, requiring either a nonstandard mode of reasoning (adopting a multi-valued logic, abandoning the Law of Excluded Middle) or a nonstandard semantics (abandoning the Principle of Bivalence, accepting truth-value gaps), or both. By contrast, the presently contemplated approach to vagueness proposes to leave the machinery of classical logic and of standard semantics pretty much intact, and to let the burden of paradox resolution be carried by strictly epistemological considerations. Unavailable information rather than deficient theorizing is here asked to bear the brunt.

The problem of vagueness has a long history. Among the ancient Greeks, Eubulides of Megara (b. ca. 400 BC) was the most prominent and influential member of the Megarian school of dialecticians as whose head he succeeded its founder, Euclid of Megara, a pupil of Socrates.¹ Eubulides did more to promote concern for the paradoxicality of vagueness than any other single thinker in the history of the subject. He is credited with seven important paradoxes: *The Liar* (*pseudomenos*), *The Overlooked Man* (*dialanthanôn*), *Electra and her Brother*, *The Masked Man* (*egkekalumenos*), *The Heap* (*sôritês*), *The Horns* (*keratinês*), and *The Bald Man* (*phalakros*). All of them pivot on issues of vagueness or equivocation.

What particularly concerns us here among these ancient puzzles is the “Paradox of the Heap”—the *Sorites Paradox* (from the Greek *sôros* = heap). It is posed in the following account:

A single grain of sand is certainly not a heap. Nor is the addition of a single grain of sand enough to transform a non-heap into a heap: when we have a collection of grains of sand that is not a heap, then adding but one single grain will not create a heap. And so, by adding successive grains, moving from 1 to 2 to 3 and so on, we will *never* arrive at a heap. And yet we know full well that a collection of 1,000,000 grains of sand is a heap, even if not an enormous one.²

Throughout the ages, theorists have diagnosed the problem at issue here through locating its difficulty in vagueness, thus affiliating it to a vast panoply of similar puzzles. (Example: a newly sharpened bread knife is not dull, and cutting a single additional slice of bread with a knife that is not dull will not dull it. Yet when the knife has cut a million slices, it will be dull. Or again: if you are still on time for an appointment, the delay of a nanosecond will not make you late, and yet a great multitude of such delays engenders lateness.) The guiding idea is that in all such cases the pivotal concept—be it “a heap” or “dull” or “late”—is vague in that there is no sharp and definite cut-off point between the IN and OUT of its application. The “borderline” at issue is not exactly that, but rather a blurred band that is imprecise, nebulous, indefinite, inexact, or some such. And just this is seen as the source of difficulty.

To come to grips with the core to the problem, let $H(n)$ abbreviate the thesis that “A unified collection of n grains of sands is a heap.” We can then formalize the premisses of the Sorites paradox as follows:

- (1) $\sim H(2)$ (“Two grains do not form a heap.”)
- (2) $(\forall n)[\sim H(n) \rightarrow \sim H(n + 1)]$ (“If n grains are insufficient to form a heap, then adding just one will not mend matters.”)
- (3) $H(1,000,000)$ (“A million grains will form a heap.”)

Starting out from premiss (1), repeated application of (2) will yield the negation of (3). So those three premisses are inconsistent. And yet individually considered they all look to be plausible. Hence the paradox. How is it to be resolved? Clearly, at least one of those premisses must go.

Since premisses (1) and (3) are uncontestable, it is clearly premiss (2) that will have to bear the burden of doubt. But in rejecting (2) we will, by classical logic’s Law of Excluded Middle, be saddled with its negation, namely:

- (4) $(\exists n)[\sim H(n) \& H(n + 1)]$

But now if *this* is accepted, grave problems seem to follow, for by in the widely endorsed Substitutional Construal of Existential Quantification we will have the principle:

- (S) If $(\exists x)Fx$, then there must be a particular value x_0 of the variable x for which Fx_0 obtains.

And if this is so, then there will be an identifiable transition point—a particular and specific integer N for which not-(2) holds good. And so, we have:

- (5) For some particular, specific integer N there obtains:

$$\sim H(N) \& H(N + 1)$$

This upshot appears to be altogether counterintuitive and unacceptable. But nevertheless, we seem to have a natural and inevitable transit by standard logic from the rejection of (2) to an acceptance of (4) and thence via (S) to (5). Where does this unpalatable result leave us?

To block this course of reasoning most theorists have proposed to embargo the move from not-(2) to (4) by some maneuver or other. Mathematical intuitionists propose to accomplish this by prohibiting the move from the refutation of a universal claim to the maintenance of an existential one. Supporters of a “fuzzy” logic propose to abandon the classical laws of excluded middle and *tertium non datur*.

Contrary to such approaches, however, the present discussion maintains the availability of another, logically far less radical, alternative—an alternative to which, so it appears, one must in any case resort on other grounds. This alternative approach pivots on bringing the idea of vagrant predication into operation.

2.2 Vagueness as Vagrancy

An important, albeit eccentric, mode of reference occurs when an item is referred to obliquely in such a way that, as a matter of principle, any and all prospect of its specific identification is precluded. This phenomenon is illustrated by claims to the existence of:

- a thing whose identity will never be known
- an idea that never has or will occur to anybody
- a person whom everyone has utterly forgotten
- an occurrence that no-one ever mentions
- an integer that is never individually specified.

Although such items unquestionably exist, they all remain *referentially inaccessible*: to indicate them concretely and specifically as bearers of the predicate at issue is straightaway to unravel them as so-characterized items.³ Yet one cannot but acknowledge that there are such items, notwithstanding the infeasibility of identifying them.

The concept of an applicable but nevertheless noninstantiable characterization comes to view at this point. The realizations of such a predicate F will be unavoidably unexemplified. For while it holds in the abstract that this property at issue is indeed exemplified—so that $(\exists u)Fu$ will be true—nevertheless, the very manner of its specification renders it impossible to specify any particular individual u_0 such that Fu_0 obtains. Such predicates are “vagrant” in the sense of *having no known address or fixed abode*. Despite their having applications, these cannot be specifically instanced—they cannot be pinned down and located in a particular spot. So, on this basis we may define:

F is a *vagrant* predicate if $(\exists u)Fu$ is true, while nevertheless
 Fu_0 is false for each and every specifically identified u_0 .

Predicates of this sort will be such that, while general principles show that there indeed are items to which they apply, nevertheless it lies in their very nature that such instances can never be specifically identified.⁴ It lies in the very make-up of their specification that when F is vagrant, then Fx_0 is a contradiction in terms where x is a specifically identified item—an incoherent, meaningless contention. And this is a very real phenomenon, seeing that such predicates as:

- being a person who has passed into total oblivion
- being a never-formulated question
- being an idea no-one any longer mentions.

illustrate this phenomenon. Throughout such cases, specifically identified instantiation stands in direct logical conflict with the characterization at issue. To identify an item instantiating such a predicate is thereby to contradict its very characterization.⁵

It is this conception of predicative vagrancy that will provide the key to the presently contemplated approach to vagueness.⁶

2.3 Vagrancy Roots in Limited Cognition

With vagrant predicates, the *existence* of exemplifications may be an *ontological* fact, but this is offset by the no less firm *epistemological* fact that the *identification* of such exemplifying instance is simply impossible. The impossibility lies not in “being an F ” as such but in “being a concretely instantiated F .” The problem is not with the indefinite “*something* is an F ” but with the specific “*this* is an F .” Difficulty lies not with F -hood as such but with its specific application—not with the ontology of there being an F but with the epistemology of its apprehension in individual cases. Accordingly, vagrant predicates mark a cognitive divide between reality and our knowledge of it.

Now, in the abstract and formalistic reasonings of logic or mathematics—where predicates are cast in the language of abstraction—cognitive operators of the sort at issue in predicative vagrancy simply have no place. Here, one will never encounter vagrant predicates. For in such contexts, matters of cognition are never invoked: we affirm *what* we know but never claim *that* we know. However, with matters of empirical fact, the situation can be very different.

For in those matters of vagrancy that now concern us, cognitive inaccessibility is built into the specification at issue. Here, being instantiated stands in direct logical conflict with the characterization at issue, just as with:

- being a sand grain of which no-one ever has or will take note
- being a person who has passed into total oblivion
- being a mosquito of which no-one ever takes note
- being a never-formulated question
- being an idea no-one any longer mentions.

To identify an item of this sort is thereby to unravel its specifying characterization.

The difference between predicate vagrancy and its contrary mirrors the contrast between:

- *generic knowledge*: It is known *that something* has F : $K(\exists x)Fx$
and:
- *specific knowledge*: *something that* has F is known about, that is, one knows *of* something in specific that *it* has F : $(\exists x)KFx$

Here, K can be read either as the impersonal “It is known that” or alternatively as the egocentric “I know that.”

In the former case, it is merely known that F has application; in the latter case, one is in a position to identify a specific example of F -application—to adduce a *known instance* of F . From the logical standpoint, then, the issue comes down to the relative placement of the existential quantifier and the cognitive operator.

2.4 A Vagrancy Approach to Vagueness

And now back to vagueness. Wherever it functions, there is no viable way of separating the INs from the OUTs. But, here, one can take either an ontological or an epistemic approach. The former effectively says “there is no definite boundary,” whereas the latter says “there indeed is a definite boundary but there is no practicable way of locating it, no feasible way of noting where it lies.” The one denies the existence of boundaries, the other their identifiability.

In the case of the Heap Paradox, these opposites afford two possibilities. One consists in flat-out denying the thesis:

$$(\exists n)[\sim H(n) \& H(n + 1)]$$

But yet another alternative approach proceeds by retaining this contention but blocking the move from it to:

There is a particular, determinable value N of the variable n for which the preceding contention holds.

In effect we now bring the concept of vagrant predicates to bear. For by treating vagueness as vagrancy, we effectively block the Heap Paradox and its congeners. For once that pivotal predicate which characterizes a transition from non-heap to heap is seen as vagrant, the whole idea of locating that problematic transition value vanishes from the scene. The two conceptions—vagueness and vagrancy—can thus be seen as functionally symbiotic.

To be sure, an approach to vagueness along these lines involves a nonstandard handling of the issue of a transition point between the INs and the OUTs. For the traditional approach to such boundaries is that of the ontological contention that they do not exist as such (i.e., as actual boundaries), but are to be replaced by penumbral

regions (whose boundaries themselves are penumbral in turn—all the way through). And this means that there will fail to be a “fact of the matter” in regard to being IN or being OUT, so that the logical principle of *tertium non datur* has to be abandoned.

With vagueness, there will be a region of indeterminacy as between the INs and the OUTs, but that this region is, as it were, penumbral. It will not itself have sharp, razor-edged boundaries but must be nebulous, with the boundary between IN and INDETERMINATE (and again between INDETERMINATE and OUT), and will itself be comparably indeterminate (penumbral, “fuzzy”) once more. The absence of clear transitional borders will hold “all the way through,” so to speak. For this reason, a three valued logic of TRUE—FALSE—INDETERMINATE will not do the job that is needed here. Any “fuzzy logic” adequate to the taste of accommodating vagueness must be infinite-valued, with never-ending room for shades and gradations. Pretty complex logical machinery needs to be brought to bear.

By contrast, our present vagrancy-based approach takes an *epistemological* line. It does not call for denying that there is such a thing as a (classically conceived) boundary. And it does not deny that any given item is either IN or not. In sum, it does not conflict with the idea that *facts* are at issue here. But what it does insist upon is that these facts are *in principle undeterminable*. For the predicate:

- being the boundary between IN and OUT

is now classed as vagrant. The correlative shift from ontology to epistemology leaves traditional logic pretty much intact.

The vagrancy-based approach to vagueness pivots on the critical distinction between the located and the locatable. As it views the matters, there indeed is (ontologically, so to speak) a sharp and clear boundary between the INs and the OUTs, but that there is (epistemically, so to speak) no possible way of locating it. In taking this line, the recourse to predicative vagueness shifts the burden from the ontological to the epistemological side of things. The advantage of such a strategy is that it makes it possible to keep in place a classically binary logic and foregoes abandoning the classical principles of excluded middle and *tertium non datur*. The only innovation needed—and one that will be required in any case—is to accept the prospect of vagrant predication.

What we have here is the anomaly of a *boundary* (as between being a heap and a non-heap, a sharp knife and a dull knife, a same color patch and a different color patch) representing an IN/OUT demarcation that is inherently invisible. Such a boundary exists—so it is held—but remains inherently unidentifiable. Viewed from this perspective, vagueness emerges as a product of insufficient cognition. The indefiniteness at issue is now ascribed not to reality’s indecisiveness, but rather to that of our epistemically problematic concepts—as reflected in the indefiniteness of vagrant predicates.

And so, while the standard view of vagueness sees the separation of vaguely bounded regions as a matter of unlimitedness—the result of definite boundaries—the present nonstandard approach combines an insistence on the *existence* of boundaries with an insistence on their (epistemic) inability to be located with precision.

The positions are very different but their net effect is, in one respect, the same: no specifiable boundaries.

2.5 Further Perspectives

There is a multitude of examples of objects that are real but unidentifiable. As regards the future, the person who will win the 2028 US presidential election is certainly currently alive and active among us, but cannot yet possibly be identified. And as regards abstractions, there must exist an unprovable arithmetical theorem whose Gödel number is the lowest, although this too cannot possibly be identified. A large range of cases is at issue here.

After all, one must avoid equating nonspecificability with nonexistence. For as we have seen time and again, vagrant predicates, though uninstantiable by us, need not in themselves be uninstantiated. There will certainly be (some) totally forgotten people, though none of us can possibly provide an example. And analogously, it could be held that there indeed is a sharp boundary between heaps and non-heaps (of sand grains of a given size) even though it is in principle impossible ever to say just where this boundary lies. It is concealed in a cognitive blindspot, as it were.⁷ For while from such a perspective, there indeed is a transition and even a transition point, nevertheless this is not something that can possibly be fixed upon and identified.

Consider, for example, a color strip of distinct compartments as per:

$$C_1 \quad C_2 \quad C_3 \quad C_4$$

where adjacent compartments are visually indistinguishable in point of phenomenal color:

$$(\forall i)[P(c_i) = P(c_i + 1)]$$

Nevertheless, the situation is such that there will be notable differences among sufficiently remote compartments. Thus, we will have:

$$P(c_1)^1 \neq (c_{100})$$

But where is one to place the transition between $P(c_1)$ and $P(c_{100})$? Where does $P(c_1)$ end and where does $P(c_{100})$ begin? Here, we have exactly the same problem as with heaps. And exactly the same sort of solution looms before us with a resort to predicative vagrancy able to do the needed work.⁸

Now, on the present epistemic perspective, the crux of vagueness is that while one knows that there is a transition point between IN and OUT, nevertheless one cannot possibly manage to locate it. And just this represents a fundamental aspect of vagueness in general: there just is no way of saying at what point predicate-applicative

begins and where it ends. We know that a crossover is eventually reached, but cannot possibly say just where it lies.

2.6 The Epistemological Turn

Such treatment of vagueness takes the line that there indeed is a boundary between the INs and the OUTs in matters of vagueness, so that one can maintain:

(I) $(\exists B)$ [B marks the boundary between IN and OUT]

Nevertheless, there is no way of *fixing* this boundary, no way of determining just exactly where it lies. There is no prospect of identifying a particular value of B_0 of the variable B such that:

(II) B_0 marks the boundary between IN and OUT.

From the ontological/existential point of view, the existence of a boundary is acknowledged as per (I). But from an epistemological/cognitive point of view, any and all possibility of locating this boundary—of determining or specifying it—is precluded. We know *that* it is, without knowing *where* it is.

Exactly this is the characteristic situation of predicative vagrancy. And in viewing the matter in this light the existence of a boundary point is conceded, but any and all prospect of its specifiability is denied.

So viewed, the ultimate responsibility for the indefiniteness of vagueness thus lies not with what is at issue in our discourse, but rather in the imperfection of our knowledge: “the fault is not in our stars, but in ourselves” in that our very vocabulary precludes exact knowledge by being indefiniteness-friendly.

The crux of such an approach to vagueness is that the descriptive qualifier “is a transition point between IN and OUT” is to be seen as a vagrant predicate—it applies someplace, but we know not where: items may well fall into the indeterminate “just can’t say” region. (The boundaries of that indeterminate region will themselves be specified by vagrant predicates.) In principle, undecidable propositions occur not just in mathematics but in the factual domain as well.

But just what is the pay-off difference between saying that there just is no boundary and saying that there is one but it is altogether unidentifiable? Simply and exactly the difference between the epistemic and the existential. It is one thing to say that there is nothing in the box and quite another to say that there is no way for anyone to know what it contains. (Think of the magic box—impenetrable to external scanning—whose content is annihilated by opening the lid.)

2.7 Ramifications

Our claims regarding reality generally fall short in point of accuracy and detail for reasons ultimately rooted in our human condition as beings whose knowledge is mediated by language. A descriptive term is *equivocal* when its application invites the question: “In what sense?” (Example: *gay* or *crooked*.) A descriptive term is *vague* when its application invites the question: “Of what sort or kind?” (Example: *vehicle* or *metal*.) A descriptive term is *ambiguous* when its application invites the question: “In what mode, respect, or manner?” (Example: *instructive* or *incompetent*.) A term is *inexact* or *imprecise* when its application invites the question: “In what degree or to what extent?” (Example: *large* or *old*.) Moreover, a descriptive term is *figurative* when it is in some respect metaphorical or analogical, so as to invite the question: “Just how is this so?” As such cases indicate, human communication is replete with unclarity and inexactness, ever admitting further questions about the purport of what has been said. While reality itself is interrogatively complete, our thought and discourse about it certainly is not: we are constantly constrained to use loose terminology and fill our discourse with expressions on the order of “roughly,” “approximately,” “something like,” “in the neighborhood of,” “in his 70 s,” “some six feet tall,” and so on. This prominence in our discussions of indecisiveness—of vagueness, equivocation, and the rest—has larger ramifications.

Consider, for example, dealing with an inscription that reads:

R■T

Here, we just cannot make that middle letter out. On the basis of the general principles of its English-language setting, we can maintain:

- (1) ■ must be a vowel
- (2) Only A, O, and U are real possibilities

However, if we had some additional context we could go further, as per:

- He was bitten by a R■T
- He left it in the street to R■T
- He got stuck in a R■T

Context often will, or at least can, pave a way to determination here. But in the absence of a context, all we can say is:

- (1) We know that the missing letter is one of A, O, or U:
 $K(\blacksquare = A \vee \blacksquare = O \vee \blacksquare = U)$
- (2) But we do not know which of them in particular it is:
 $\sim K(\blacksquare = A) \ \& \ \sim K(\blacksquare = O) \ \& \ \sim K(\blacksquare = U)$

In sum, what we have here is the typical vagrancy situation of:

$$K(\exists x)(\blacksquare = x) \ \& \ \sim(\exists x)K(\blacksquare = x)$$

We know *that* ■ is one of A, O, or U, but we have no clue as to which of this trio our problematic ■ actually is. It is clear on this basis the predicate:

- being the letter represented by ■

is (contextually) indeterminate.⁹

And so, with equivocation too we have a situation that can be analyzed in terms of predicative indeterminacy. And the situation of vagueness can be seen as simply a more radical version of this same phenomena. In effect, both vagueness and equivocation can be seen as yet another mode of unknowability.

2.8 Why Vagueness Pays

The fact of it is that reality is so vastly complex in its mode of operation that a shortfall of detail in our description of it is an inevitable reality. In characterizing the real in man's natural language, the indecisiveness of vagueness is not a failing but an inevitability. And so, one reason for our tolerance of congeners lies in our having little choice about it.

All the same, its vagueness does not stop a statement from being true. If we could not describe the grass of our experience as vaguely green or indeed even merely greenish, but only had the choice of a myriad exact shades of green, color communication would virtually grind to a halt. If we had to decide when "rock" leaves off and "boulder" begins, we would be in difficulty. Despite its manifest problems, vagueness is immensely useful simply because precision is too hard to come by and deploy.

And so, in the final analysis we tolerate vagueness because we have no choice and we do so gladly, not just because it is convenient but also because greater detail is generally not needed in the relevant contexts of operation. (We do not need to know whether the approaching storm will bring 1 or 1.5 inches of rain for deciding whether or not to take an umbrella.)¹⁰

Notes

1. Pretty well all that is known about Eubulides derives from Diogenes Laertius, *Lives of the Philosophers*, Bk. II, Sects. 106–20. See Zeller, *Philosophie der Griechen*, vol. II/1, pp. 246.
2. On this paradox and its ramifications, see Chapter 2 of R. M. Sainsbury, *Paradoxes*, 2nd ed. (Cambridge: Cambridge University Press, 1995), pp. 23–51. Originally, the paradox also had a somewhat different form, as follows: Clearly, 1 is a small number. And if n is a small number, so is $n + 1$. But this leads straightway to having to say that an obviously large number (say, a zillion billion) is a small number. (See Prantl, *Geschichte der Logik im Abendlande*, vol. I, p. 54.) Note that the paradox could equally well be developed regressively (i.e., from heapness by substantive regression) as progressively from non-heapness by additive progression. The former regressive style of reasoning is called Galenic after Galen (129 AD–c. 210), who wrote prolifically on logic;

the latter progressive style is called Goclenic after Randolph Goclenius (1547–1628), who discussed the matter in his *Introduction to Aristotle's Organon. Isagoge in Organon Aristotelis* (Frankfurt, 1598).

3. We can, of course, refer to such individuals and even to some extent describe them. But what we cannot do is to *identify* them.
4. A uniquely characterizing description on the order of “the tallest person in the room” will single out a particular individual without specifically identifying him.
5. To be sure, one could (truthfully) say something like “The individual who prepared Caesar’s breakfast on the fatal Ides of March is now totally unknown.” But the person at issue here goes altogether unknown, that is, he or she is alluded to but not specified—individuated but not concretely identified. So, I cannot appropriately claim to know *who* the individual at issue is but only at best *that* a certain individual is at issue.
6. For Further details regarding such vagrancy, see the author’s *Epistemic Logic* (Pittsburgh: University of Pittsburgh Press, 2005).
7. For further, different cases of this general sort, see Roy E. Sorensen, *Blindspots* (Oxford: Clarendon Press, 1988).
8. This shows that transitional continuity is not the core of the problem: the selfsame situation can confront us in the discrete case.
9. It is not, however, vagrant on logically inevitable grounds.
10. Further information on paradoxes can be found in the author’s *Paradoxes* (Chicago: Open Court, 2001). An extensive literature is cited there, including: J. C. Beall (ed.), *Liars and Heaps: New Essays on Paradox* (Oxford: Clarendon Press, 2003); L. Burns, *Vagueness: An Investigation into Natural Languages and the Sorites Paradox* (Dordrecht: Reidel, 1991); V. McGee, *Truth, Vagueness, and Paradox* (Indianapolis: Hackett, 1990); R. M. Sainsbury, *Paradoxes*, 2nd ed. (Cambridge: Cambridge University Press, 1995) [see especially Chapter 2, “Vagueness: The Paradox of the Heap”].

Chapter 3

Conceivability



3.1 Conceiving Facts

For the most part we do not make facts: generally, they are just “out there,” beyond our reach and control. All that we can do is to think about them. But alike in making and in thinking about them, we must have a conception of the facts.

The human mind has two principal cognitive powers: to image possibilities and to adjudge realities, enabling it to deal with fact and fiction alike. In a way, possibility management is the more fundamental. After all, if it's not possible then it can't possibly be real, and if it's not conceivable by us then we can't possibly accept it as actual. (All this is not, however, to say that if we cannot conceive of it that it can't be actual—reality and possibility alike can hold very big surprises for us.)

Conceivability is a matter of the possibilities that people are in a position to contemplate given the concepts and beliefs at their disposal. It relates to both facts and fictions. A four-sided triangle is inconceivable, one that is small and red is not. The concepts and beliefs at our disposal set our conceptual horizons. They delimit the range of our cognitive domain beyond which there lies what is, for us, mere *terra incognita*.

Epistemologists have focused on our knowledge of the real and have pretty well left possibility to the logicians. But the logicians have left the epistemology of possibility to others, and have attended to what actually *is* possible, omitting concern for how we conduct the business of learning about it. The present discussion will offer some comments on this rather neglected theme.

Conceivability calls for being available as an object of meaningful thought. It is not a matter of imagining or picturing. One cannot picture or imagine a thousand-sided polygon but one can certainly conceive in describing it and supposing its possibility. Conceivability is sometimes mis-equated to understandability based on a sufficient knowledge. (“I cannot conceive how she came to realize that he disliked her—he was such a good actor.” A misconception is at work here: you can certainly conceive of its being so; it is just that you don't know how it came to be.)

Some conceptions have to be formed systematically—they ramify out into related issues whose co-understanding they presuppose. To have a proper conception of a propeller, one needs some understanding of the technology of early airplanes; to have a proper conception of an electron, one needs some understanding of subatomic physics.

As construed here, conceivability is the prospect of entertaining something as a meaningful possibility. Two sorts of items can be inconceivable to a person: things and facts. A thing is effectively inconceivable to someone if its definitive features are wholly outside that person's experience. (A Polynesian cannot conceive of solid water (i.e., ice), Aristotle could not conceive of X-rays.) A fact is inconceivable to someone when they have totally unshakable belief in its contrary. (Pigs that can fly like bats or bees are inconceivable to most of us.) For individuals, the personally inconceivable is either (or both) foreign to established experience or contrary to absolutely certain conviction.

The truths we contemplate may well not actually *characterize* reality, but rather be related to its constitution in more complex and indirect ways. For example, we can have:

- negative truths ("No cats talk.")
- vague truths ("He looked thirtyish.")
- inexact truths ("It looks something like this.")
- approximate truths ("The table is roughly 32 inches wide.")
- indefinite truths ("She looked pleased.")
- possibilistic truths ("It might rain.")
- impressionistic truths ("They were lucky.")
- metaphorical truths ("It was a veritable bonanza.")

No doubt such truths will be so in virtue of what the facts are. But they certainly do not *characterize* the real facts. Thus, truths can be indefinite. But reality cannot; it must be concrete (rather than an abstract), definite (rather than vague, approximate, etc.), and positive (rather than negative), whereas truths need not be any of these. Thus, truths do not *correspond* to what the realities are, although their being truths is (loosely) dependent upon that. All truths have their "truth-makers" in reality—that is, there is (and must be) a "basis in concrete fact" for every truth, an aspect of reality in virtue of which that truth is true.

To *characterize* reality—to "*agree*" with it—would be to give an accurate representation of it that is correct and complete in all relevant detail. Thus, only a detailed (precise, exact, accurate) account of something can actually correspond to the reality of it. And this is something which our language-framed statements about the real—however true—almost invariably fail to achieve. An account that is vague, imprecise, approximate, fuzzy, or the like may well be *true* but nevertheless not be accurately consonant with it. The truth in general falls well short of the detailed accuracy that would be required here. No doubt the truth is *grounded* in reality, and concurs with it. But it certainly need not and often will not *correspond* to it.

Seeing that our true contentions regarding reality are generally indefinite (vague, ambiguous, metaphorical, etc.), whereas reality itself is always definite (precise,

detailed, concrete), it follows that those truths of ours do not—cannot—give an adequate (faithful, accurate, precisely correct) account or representation of reality. It is a merciful fact of life in human communication that truth can be told without the determinative detail of precision, accuracy, and the like required for an accurate representation of the facts. Reality's detail involves more than we can generally manage. We can achieve the truth and nothing but the truth, but the whole truth about something is always beyond our grasp.

There was a time when it was fashionable for English Hegelian philosophers such as Bernard Bosanquet to say that only the accurate truth is the real truth and that the real truth of things must be altogether exact and fully detailed. But this contention would involve us in critical errors of omission regarding reality. Thus, we would not be able to declare the truth that grass is green or the sky is blue. And moreover, we would lose the crucial principles that the logico-conceptual consequences of the truth must also be true, seeing that the inference from "There are 48 people in the room" to its vague logical consequence "There are several dozen people in the room" would now not qualify as correct, since the latter would not qualify as a truth. The truth is one thing, but the *precise* truth or the *exact* truth quite another.¹ Our truths need surely not convey the detailed nature of the realities that make them so. But in the end, we cannot come to cognitive grips with reality save via our true acceptance about it.

3.2 Inconceivability

Certain considerations may be inconceivable to someone owing to having mistaken ideas on the subject or because certain matters do not fall within the range of their experience. This sort of subjective (person-relative) inconceivability is not at issue here. Here, we are concerned only with inconceivability relative to meaningful conceptions and correct convictions, matters inconceivable on the basis of correct and adequate information.

And there is also the impersonal or generic conceivability characteristic of the typical and representative members of the group. Generic inconceivability is not a matter of what a particular individual can manage in thought but of what can be managed in rational thought as such. Items that are inconceivable include a greatest integer, a fastest motion, a largest circle—a thing whose very identification includes a contradiction in terms. But either way, personal or generic, conceivability requires experiential access and consonant belief.

We must, however, distinguish between subjective person-relative conceivability, which is a function of a particular individual's knowledge, and objective or culture-relative conceivability, which is a function of language and cognitive state of the art. Both alike set limits but these differ in that the former are personal and the latter societal. From the theoretical point of view, it is the latter that are paramount, and our focus will be on groups rather than individuals, and principles rather than people.

There are three principal levels of inconceivability/conceivability:

- I. *Grammatical*. Meaningless gibberish: having not informative sense. (“Twas brillig...”) Violation: Meaninglessness.
- II. *Logico-Conceptual*. At odds with what is to be seen as absolutely necessary. (“A day without hours; a four-sided triangle; a sphere without a center.”) Violation: Incoherence.
- III. *Factual*. Inconsistent with what is seen as a patent and necessary fact. (“A talking tree; a brass banana.”) Violation: Unacceptability.

Rather different modes of necessity/possibility are at issue with II and III. Absolute or logico-conceptual necessity/possibility ($\Box$ and $\Diamond$) is dealt with in II. This is the way in which it is necessary for triangles to have vertices or a bird to have wings. By contrast, III is the way in which it is necessary for animals to secure nourishment in order to survive, or for fires to have oxygen in order to burn. These envision the sort of necessity involved in accommodating to the workings of the actual world ($\Box$ and $\Diamond$). In this sense of the term, the basic laws of nature provide the basis for necessity.

One cannot, of course, give an illustrative example of something that is in principle inconceivable because presenting it defeats the very purpose.

And inconceivable theses cannot sensibly be maintained as informative truths; they can only be maintained, if at all, as suppositions or hypotheses. In failing to make tenable assertions and convey a meaningful message, they fail to fall under the descriptivity of correct/incorrect (and similarly probable, plausible, and the like). Its unintelligibility precludes it from qualifying for those evaluative assessments which—like the preceding—are applicable only to propositions that make a coherent claim of some sort. After all, only meaningful propositions (claims) can have a truth-status—be it actually or even by assumption or supposition. Incomprehensible (and thereby meaningless) discourse cannot even be assured to be true or false. Its lack of truth-status is unconditional and unavoidable.

In actual fact, claims to the realization of something inconceivable are always untenable and false. However, here as elsewhere, error is possible. Someone ill-informed can certainly think (mistakenly) that something inconceivable is real. It is clear that something can be acceptable to one person and not to another. Thus, when one person is better informed than another, they can differ in regard to conceivability—either way. If x does not realize that squaring the circle is impossible, he mistakenly conceives of someone (perhaps himself) having solved the problem. On the other hand, if x does not realize that black swans are possible, he may mistakenly regard the prospect of a black-swan dinner as inconceivable.

3.3 Meaninglessness

Logic deals with the truth relationships among propositions. But before there is truth there must be meaning. And the *bête noir* in this regard is meaninglessness.

Meaninglessness is a malfunction of communication, something that results when our apparatus of communication does not manage to do its intended job.

There are several importantly distinct ways in which a statement can be meaningless, although all of them are alike in basing what is said on a presupposition that is simply false.

One mode of meaninglessness results from asserting absolute gibberish: “The number three ate yellow.” We cannot even begin to make sense of this. This is *assertoric* meaninglessness, the failure to make any intelligible contention whatsoever. The mistaken presupposition here is that meaninglessness can be achieved simply by stringing words together grammatically. The senselessness of such gibberish that one can make neither heads nor tails of is the most drastic mode of meaninglessness. All of its other modes are at least minimally intelligible in that what is being said is sufficiently intelligible that one can comprehend the senselessness of it.

A prime form of such meaninglessness is *categorical* in nature and consists in ascribing to something a certain type of feature that items of its category simply cannot have; for example, assigning a physical location to numbers (one cannot position the number three at the North Pole), or ascribing a color to obligations (one cannot have a yellow duty toward one’s children).

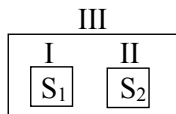
A further form of meaninglessness is *conceptual* in making statements that conflict with the established meaning of words. Thus, consider such statements as “John’s spouse is unmarried” or “Two’s double is an odd number.”

Then, too, meaninglessness obtains when any attempt to class a statement either as true or false results in failure because a contradiction results either way. This is *alethic* meaninglessness, the failure to have any determinate truth-status. The classically paradoxical self-contradictory thesis “This statement is false” is an example.

Yet another mode of meaninglessness is the *delusional* which presupposes as existent something that just is not there. Examples are such statements as “The present king of France is bald” or “Noplace is the capital of Antarctica.” And a further pathway to meaninglessness is by purporting the existence of something that not only does not but also cannot exist. “The prime number between five and seven” and “The product of three multiplied by an even divisor of seven” are examples. This is *referential* meaninglessness, rooted in the here unavailable something for the statements to characterize.

Why is it that meaninglessness statements can and should be dismissed from serious consideration without much further ado? We do so for reasons of *cognitive economy*. We thereby spare ourselves from any further fruitless effort to deal with the matter.

Are self-contradictory statements meaningless? It all depends. Individually self-contradictory statements are indeed meaningless. There is nothing we can do with such a statement as “The pair of them consisted of three items.” But by way of contrast, consider the example of the three boxes I, II, and III:



Now, let it be that that S₁ is:

The statement in Box II is true, but some statement in Box III is false.

Suppose S₁ is true. Then so (according to S₁ itself) is S₂. But with S₁ and S₂ both true, all the statements in Box III are true, so S₁ is false. Since S₁-true entails S₁-false, S₁ is self-contradictory and thus false.

The falsity of S₁ means that not-S₁ will be true. But by the content of S₁ we have:

$$\sim S_1 \text{ iff } \sim S_2 \vee [S_1 \& S_2]$$

Since S₁ & S₂ is inescapably false by the reasoning indicated above, we have: $\sim S_1$ iff $\sim S_2$. So, the falsity of S₁ constrains that of S₂. That is, S₂ will be false *irrespective of what it is that S₂ asserts*. But this is absurd and we can make no stable sense of this paradoxical situation.

It must be stressed, however, that those individual sentences S₁ and S₂ are not really meaningless. After all, their meaning is essential to the derivation of the paradox. Moreover, if you change one of them, the other can become perfectly meaningful. What is meaningless here is the whole complex—the entirety of what is being asserted. The difficulty of meaningful construal is collective not distributive. Individually regarded, the meaningfulness of those statements is incontestable.

A very special sort of lack of “meaning” is at issue with the “empirical meaningfulness” purported by the logical positivists of the 1930s. For them, a proposition was “meaningless” in the sense of failing to admit of observational disinformation. In adopting this line, they thought they could demolish traditional metaphysics by dismissing it as meaningless nonsense. Unfortunately for this program, it came to light all too soon that a whole host of perfectly meaningful scientific statements would have to be classed as empirically meaningless, so that the baby was being thrown out along with the bath water. For it is clear that discursive verification will be unavailable with statements about the remote past or future, or such generalizations as “X will never happen” or “Caesar would have left the Rubicon uncrossed had he wanted to” and many other sorts of statements whose meaning is intelligible and whose truth is plausible.²

It is tempting to dismiss as meaningless those claims that we simply do not understand—to blame the message, as it were, for our own lack of comprehension. Many or most of us would not hesitate to adopt this line in relation to the explanation that Chinese adepts of acupuncture use in explaining their practice. And while such an argument may well be appropriate, one should nevertheless proceed with caution in these matters. For it is one of the most fundamental facts of epistemology that to those who proceed at a lesser level of understanding, the proceedings of their higher-level interlocutors are bound to seem like magic.

As already noted, meaninglessness is a mode of malfunction. And as such, it is a phenomenon that is both inevitable and instructive. It is inevitable because anything that can be used can also be misused. And it is instructive because in looking to the boundaries between the potential pursue of an instrumentality, we shed light on the nature of its proper use.

3.4 The Corrigibility of Conceptions

It must be stressed that these deliberations regarding cognitive inadequacy are less concerned with the correctness of our particular claims about real things than with our characterizing conceptions of them. And in this connection, it deserves stressing that there is a significant and substantial difference between a true or correct statement or contention, on the one hand, and a true or correct conception, on the other. To make a true contention about a thing, we merely need to get one particular fact about it straight. To have a true conception of the thing, on the other hand, we must get all of the important facts about it straight. And it is clear that this involves a certain normative element—namely what the “important” or “essential” facets of something are.

Anaximander of Miletus presumably made many correct contentions about the sun in the fifth century BC—for example, that its light is brighter than that of the moon. But Anaximander’s conception of the sun (as the flaming spoke of a great wheel of fire encircling the earth) was totally wrong.

To assure the correctness of our conception of a thing, we would have to be sure—as we very seldom are—that nothing further can possibly come along to upset our view of just what its important features are and just what their character is. Thus, the qualifying conditions for true conceptions are far more demanding than those for true claims. With a correct contention about a thing, all is well if we get the single relevant aspect of it right, but with a correct conception of it we must get the essentials right—we must have an overall picture that is basically correct. And this is something we generally cannot ascertain, if only because we cannot say with secure confidence what actually is really important or essential before the end of the proverbial day.

With conceptions—unlike propositions or contentions—incompleteness means incorrectness, or at any rate presumptive incorrectness. Having a correct or adequate conception of something as the object it is requires that we have all the important facts about it right. But since the prospect of discovering further important facts can never be eliminated, the possibility can never be eliminated that matters may so eventuate that we may ultimately (with the wisdom of hindsight) acknowledge the insufficiency or even inappropriateness of our earlier conceptions. A conception based on incomplete data must be assumed to be at least partially incorrect. If we can decipher only half an inscription, our conception of its overall content must be largely conjectural—and thus must be presumed to contain an admixture of error. When our information about something is incomplete, obtaining an overall picture of the thing at

issue becomes a matter of theorizing or guesswork, however sophisticatedly executed. And then we have no alternative but to suppose that this overall picture falls short of being wholly correct in various (unspecifiable) ways. With conceptions, falsity can thus emerge from errors of omission as well as those of commission, resulting from the circumstance that the information at our disposal is merely incomplete, rather than actually false (as would have to be the case with contentions).

To be sure, an inadequate or incomplete *description* of something is not thereby false—the statements we make about it may be perfectly true as far as they go. But an inadequate or incomplete *conception* of a thing is *ipso facto* one that we have no choice but to presume to be incorrect as well,³ seeing that where there is incompleteness, we cannot justifiably take the stance that it relates only to inconsequential matters and touches nothing important. Accordingly, our conceptions of particular things are always to be viewed not just as cognitively *open-ended* but as *corrigible* as well.

We are led back to the thesis of the great idealist philosophers (Spinoza, Hegel, Bradley, Royce) that human knowledge inevitably falls short of “perfected science” (the Idea, the Absolute), and must be presumed deficient both in its completeness and its correctness.⁴

3.5 A “Logic” of Inconceivability

The notion of a “logic of inconceivability” would seem to be a contradiction in terms. For logic looks to what must (or cannot) be true if something related is accepted (or rejected) as such. It deals in relationships among claims in the face of their actual status as true or false—be it actual or assumptive. Any discussion which by virtue of inconceivability lacks a definite truth-status falls outside logic as traditionally conceived.

There are, however, some cognate issues. To facilitate our deliberations, we shall assume that our person variables x, y, z , etc. will range over limited—that is, finite—intelligences at the level of *Homo sapiens*. And we shall adopt the convention that:

Cp abbreviates “ p is conceivable,” that is, it is possible for a human of ordinary intelligence to access the meaning of p ; and Cxp abbreviates “ p is conceivable to x .”

Given that Cxp abbreviates “ x ’s having a meaningful conception of p ,” we will have it that:

$$Cp = p \text{ is conceivable} = \Diamond(\exists x)Cxp$$

And note that this neither states nor entails $(\exists x)\Diamond Cxp$. The conceivability at issue need not be realizable by some actual person. Also, when one can conceive of p , one can conceive of not- p as well, with the result that Cxp iff $Cx \sim p$. (And note, moreover, that by using $\Diamond$ rather than $\Diamond$ in equating Cp with $\Diamond(\exists x)Cxp$ we take purely theoretical rather than effectively practical conceivability into view.)

An assertion may be made with or without affirmative intent. In the former (deliberately affirmative) case, the object is to endorse what the assertion maintains; in the second (merely deliberative) case, the object is only to pose the assertion as an item of consideration.

In the context of the present deliberations, the assertions represented by the variables p, q, r , etc. are also to encompass those made in the merely deliberative rather than substantively affirmative mode. An assertion so made is not being stated as a true affirmation, but merely put forward for consideration. And since the range of our assertion variables, p, q, r , etc. encompasses conceivable propositions in general, so those assertions come to be coordinate with conceivability rather than actual truth. In the context of the present deliberations, we thus do not have the Tarski equivalence:

$$p \text{ iff } |p| = T.$$

Instead, all we have is:

If p then Cp , though not always conversely, since we have:

$$Cp \leftrightarrow \Diamond(\exists x)Cxp$$

There now follows:

$$\sim Cp \leftrightarrow \Box(\forall x)\sim Cxp \text{ and thereby also } \sim Cp \leftrightarrow \Box(\forall x)\sim Cx\sim p$$

Accordingly, that which is inherently inconceivable must be so of necessity for anyone.

It transpires that any claims whose prerequisites or consequences are inconceivable will themselves qualify as such. Thus, $p \vdash q$ and Cp , then $C(q)$; and also if $p \vdash q$ and Cq , then $\sim Cq$. (As usual, $\vdash$ here represents logico-conceptual entailment.) Moreover, impossible or impossibility-entailing claims are not conceivable:

$$\text{If } p \vdash q \text{ \& } \sim \Diamond q, \text{ then } \sim Cxp$$

These principles provide for the rudiments of a quasi-logic of conceivability.

3.6 Inconceivable Possibilities

But are there actually—can there really be—such things as inconceivable objects, facts, or possibilities? Of course, one cannot provide examples. But it is clear on general principles that such items must exist. For we humans have to conduct our conceptualizing business by means of language. And linguistic formulation is a recursive process—exfoliating claims from a finite vocabulary via finite grammatical principles. And this means that we can realize at most a denumerable number of

affirmations. But there is no good reason to think that items, facts, and possibilities are not similarly limited. So—as in Musical Chairs—when the music of language stops, there will yet remain unaccommodated possibilities. The range of what is theoretically conceivable outnumbers the reach of what can possibly be realized.

And so, the idea of identificatory vagrancy considered in the preceding chapter comes into play at this point. We can conceive and indeed be convinced *that* there is an integer no-one ever specifically thinks of, but, of course, we cannot conceive *of* this integer as the specific individual it is.

Moreover, actually conceiving of things is something personal and potentially idiosyncratic. But conceivability as such is something impersonal and objective inherent in the nature of the issues involved and the possibilities of conceptual operation. And even as a chasm may be bridgeable without ever being bridged, so an idea or circumstance may be conceivable without ever being conceived of. Conceivability is a matter of the possibilities of conceptualization: what actually happens within the contingent eventuations of the real world is irrelevant. What individuals can manage to conceive of in practice is a fraction of their range of experience. But what is conceivable in principle is something above and beyond the capabilities of individuals.

But would there actually be a bridgeable chasm if no bridge were ever built—and indeed if the very idea of a bridge were never even conceived of? The answer is, of course, affirmative. The domain of possibility—possibilities of bridging and conceiving included—is independent of and detached from what actually happens in the world. The bridges we build and the concepts we entertain are products of our doings. But the associated possibilities of things are independent of us. Of course, the contemplation and entertainment of these possibilities is a matter of reality and actualization. But not so with the possibilities themselves that are at issue. It is noteworthy and significant that we possess a faculty of imaginative thought that enables us to enter a realm of abstract possibilities whose being we do not produce and whose features we discover rather than create. Like the real word itself, the realm of possibility that lies open to our conception is not of our making but is an independent manifold that we can contemplate but not produce. What we *do* conceive of is up to us, but what we *can* conceive of is not.

3.7 Unrealizable Ideals

Cognitive ideality functions at two levels, that of individual claims and that of collective totality.

The ideals at issue with individual claims are primarily those of

- definite truth
- accurate (detailed) precision
- unchanging stability

In the pursuit of truth the aim of course is to determine what is the case exactly and unchangingly.

The systemic ideals at issue with comprehensive totality are

- completeness
- coherence and consistency
- coordination and holistic unity

The conditions of our access to information about reality constrain us to acknowledge our limitations in this regard. At no particular juncture are we able to claim that the cognitive state of the art of the day has achieved these desiderata. Given the nature of inquiry as a fallible human endeavor it is clear that these ideas are a matter of aspiration rather than realization, of goal rather than achievement. In the realm of cognitive affairs even as in the realm of moral conduct perfection is beyond our grasp—the matter is one of realistic acceptance of the best we can manage under the circumstances.

Notes

1. For relevant material, see also the author's *Metaphysics* (Amherst, NY: Prometheus Books, 2006), pp. 101–104.
2. See C. G. Hempel, "Problems and Changes in the Empiricists Criterion of Meaning," *Révue Internationale de Philosophie*, vol. 4 (1950), pp. 41–63.
3. Compare F. H. Bradley's thesis: "Error is truth, it is partial truth, that is false only because partial and left incomplete," *Appearance and Reality* (Oxford: Clarendon Press, 1893), p. 169.
4. The author's *Empirical Inquiry* (Totowa, NJ: Rowman & Littlefield, 1982) discusses further relevant issues.

Chapter 4

Issues of Identity and Identification



4.1 Identity and Identification

Adequate identification is an indispensable requisite for introducing anything upon the stage of discussion and deliberation. Without it we literally “do not know what we are talking about.”

Bishop Joseph Butler wisely said in his “Fifteen Sermons” that “Everything is what it is, and not another thing.” But just what is it to have an identity—to have a nature of one’s own—to be distinctively different?

Specifically, the identification of concrete particulars (actual or merely possible) will be the prime focus of concern here. And the stage for these deliberations can conveniently be set by addressing a series of preliminary questions:

Question: What is it for something “to have an identity”? *Answer:* To be unique: to differ from anything else, actual or possible.

Question: Does anything and everything have an identity? *Answer:* Certainly anything we can meaningfully consider and deliberate about must do so—it must have a definiteness that enables us to have it be at issue, specifically and individually.

Question: What is it to identify something? *Answer:* To indicate by words and actions that our concern is with something particular and differs from anything else, actual or possible.

Question: How is it that things can be differentiated from one another? *Answer:* Basically there are four ways:

- *descriptively:* to have a distinctive combination of descriptive (and classificatory) feature: to be the only green Ming dynasty vase in existence
- *ostensively:* being *this* (pointing) vase
- *relationally:* being (the only) vase next to two yellow ones
- *positionally:* being the only vase in the room.

There are always different ways of identifying an item. Thus consider some alternative ways of identifying the number 3: The integer that succeeds 2; the (positive) square root of 9; the number of *E*'s in the spelling of "essence," the second smallest prime; the number whose numeral is inscribed in the following square 3, etc.

Identity is coordinate with identification. But this raises the question: Is identity inherent or imputed: is it something an item has in and of itself, or is it merely something that we ascribe to it? The issue is problematic but should become clearer as the discussion unfolds.

Identification cannot take place unless there indeed is an item to be identified. This "there is" does not require physical existence. Abstract objects (Newton's laws, for example), can be identified. And so can fictional or mythological things (the heroine of "Gone with the Wind" or the Sword of Damocles).

4.2 Descriptive Identification

Items admit of several different kinds of descriptive characterization, four in particular.

1. *Sortal Qualifiers*. Answers questions of the format: "Of what sort or type?"

Issue: Is the *A* also a *B*?

Examples:

a Roman legislator
an inebriated sailor
a wireless telephone.

Such sortal qualification looks to the overlap region of two kinds

Roman persons/Legislators
Inebriated persons/Sailors
Wireless Instrument/Telephone.

2. *Taxonomic Qualifiers*

Answers questions of the format: "Of what subcategory or kind?"

Issue: What taxonomic limitation of the *As* yields the *Bs* at issue?

Examples:

A Phillips screwdriver
A mathematical textbook.

Such taxonomic qualification looks to the description of the items at issue.

3. *Modal Qualifiers*. Answers questions of the kind: “Of what differentiating description?”

Issue: What version of *A* are the *Bs* at issue?

Examples:

a gentle touch
a substantial dinner
a familiar story
a wedding feast.

Such model qualification looks to the descriptive factors that differentiate the particular items at issue from the rest.

Touch/Gentle versus rough
Dinners/Substantial versus skimpy
Stories/familiar versus unfamiliar.

4. *Comparative Qualifiers*.

Answers questions: “How compare with the rest?”

Issue: How do the *Bs* at issue stand in relation to how the *As* usually stand—as the *A*’s.

Examples:

a cold month
a competent doctor
an expensive shirt
a lucky outcome
a major birthday
a swift runner
a quick meal
a boring book
an expensive bracelet
a comprehensive survey.

Such communicative qualification looks to the extent to which the items at issue exhibit some failure.

The important thing to note is that throughout the effort of a descriptive proceeding is to begin with a broad grouping (the *A*’s) and then subject them to a relative subgroup by means of some qualifying procedure. But it is (or should be) clear that such a proceeding can actually identify something as an individual when and only when this reductive process issues in a single item, that—so to speak—only one inquire

when in the descriptively last man standing. And of course this critical uniqueness can never be ensured on general principles.

Purely descriptive identification requires ascribing sufficiently many features an item to distinguish anything else within the range of consideration. And identifying description need not be elaborate. George Washington can be identified as “The father of our country.” Again, if Smith is the only club member with a red beard, then attributing these two properties to him (club membership and red beardedness) sufficed to effect an identification. Note that identification here hinges on the contingent facts of the matter. Again, every person has unique fingerprints and can be identified by this means. Identification can only succeed if the requisite information is available—and available *at the time*. It may well turn out that Smith is the only member of the class who will survive to age 100. But we can only identify him as such after the fact. While multiple class members are yet living, that specification fails to identify.

Purely descriptive identification is practicable when (but only when) the Principle of the Identity of Indiscernible applies, i.e., when an item at issue is the unique possessor of the specified manifold of descriptive features. (This proceeding will presumably always be possible at the macro-level of trees and boulders; it may, however, fail at the micro-level of atomic and subatomic particles.)

Of course the identifying properties of a thing must be compatible. If they attribute both being short and long or both young and old, then we fail to realize a coherent identification.

Various key questions arise in connection with descriptive identification:

Question 1: How many properties are needed for successful identification of the items of the given domain? *Answer*. It all depends on the contingent facts of the situation.

Question 2: Is it possible that no finite number of propositions will suffice? *Answer*: Yes, if the domain is sufficiently complex.

Question 3: Is it possible that even an infinite number of properties will not suffice? *Answer*: Theoretically yes. In theory the Principle of the Identity of Indiscernibles could fail to obtain and uniqueness not assured even by infinitely many complex property ascriptions?

However, purely descriptive identification always works for abstractions. Otherwise matters are contingent in this regard. Only in a descriptively cooperative world can items always be identify by finite property specifications.

The descriptive identification of a concrete item always rests on contingent considerations. The separation of such an item from all others of the same descriptive or taxonomic condition is never necessitated by matters of general principle. On the other hand, an abstraction may be identified either by considerations of contingency (“Smith’s favorite color”) or necessity (being an odd integer calls for belonging to the series $1, 3, 5, \dots 2n + 1, \dots$).

4.3 Fallacies of Identification—Variant Identities

Eubulides, a Greek Sophist who flourished around 400 B.C. and taught at Megara near Athens, conducted a prolonged controversy with Aristotle about the value of sophisms and paradoxes. Three of his paradoxes turn on matters of identification, and can all be represented in the form of riddles:

The Masked Man (egkekalummenos): “Do you know this masked man?” “No.” “But he is your father. So—do you not know your own father?”

Electra and Orestes. Electra saw that a man was approaching her. The man was her brother, Orestes. So—Did Electra see that Orestes was approaching her?

The Overlooked Man (dialanthanôn): Alpha ignored the man approaching him and treated him as a stranger. The man was his father. So—did Alpha ignore his own father and treated him as a stranger?

The same equivocation is at work throughout these *Relation Riddles of Eubulides*. In each case, there is, firstly, the individual *as the protagonist of the narrative takes him to be*, and, secondly, the individual *as the narrator takes him to be* (and/or as he actually is). In this regard, the paradoxes at issue are typified by that of the Masked Man. All of them result from maintaining something of an item in one mode of its identification that is derived of it in another mode.

As these considerations indicate, identification by means of any conditional specification as per

The item satisfying condition *C*

requires that this condition be uniquely and meaningfully realizable.

4.4 Ostensive Identification

Ostention—pointing out—is another prime mode of identification. (As, for example, pointing out “this chair,” “that cup,” “this [type of] cheese,” or the like.) Such a proceeding must always be partially descriptive; otherwise it is too indefinite. (Pointing to the table leaves us undetermined whether what is at issue is the table as a whole, the table top, or a particular part of the top, or the color of the top, etc.)

However, ostention has the disadvantage of being limited and localized: one can only point to that which is spatially “within reach” so to speak, and where one’s interlocutor is co-present. This drastically limits its range of application.

4.5 Identification By Placement and Relation

Placement within a framework of reference is akin to ostention in that a contrastive positioning is involved, but it is independent of interlocutor-co-presence. “The vase on the fireplace mantel in the living room.” Once the reference points for a conductive system have been securely fixed, physical pointing out can be replaced via the relationship indicating the appropriate position in the setting of this coordinate system. Such identification requires (1) a pre-arranged positioning system (such as geographic coordinates or the manifold of rooms in a particular house), and (2) descriptive uniqueness in the context of this positioning. (There must not be two vases on the shelf).

Identification via relationships is yet another mode of identification. (E.g. The father of Napoleon). This will only work if two conditions are met: (1) The items being related to must themselves already be pre-identified, and (2) the specified relationships to these items must be unique overall. Thus although “someone’s parent” does certainly describe, but it does not identify; it fails on both scores.

Sometimes an item can be identified solely via its relations. Thus Adam the only human being having the relation of male ancestry to everyone else among *homo sapiens*.

4.6 Respectful Identity

Once all parts of the physical thing have been duly replaced (be it Smith’s 1925 Plymouth, or the Ship of Theseus, or Sir John Cutler’s stockings) is the result identical with what went before? Does the ice cube alter its identity when it becomes a small puddle? Identity then becomes a matter of respect or aspect: same as to *X* (form) but different as to *Y* (matter). Thus DOG and *DOG* are the same word with the same spelling, but differ as to typographic presentation.

This line of thought must be extended to the puzzles of personal identity posed by conjectures regarding brain-transplantation or thought-transfer machines). Is personal identity basically physical and bodily or psychological and mental? The answer is *both*: there just is no nonaspectival personal identity any more than there is a non-comparative personal size. (Same in point of height, different in point of weight; same in matters of thought, different in matters of body). We are both body-persons and mind-persons, the two fused into single individuals by the operative ways of the world rather than by conceptual relationships of general principles.

4.7 Improper Identification

Descriptive identification is improper when the description is predicated on the reality of something that does not exist, or based on presuming that some presume that condition is satisfied which actually is not so. An instance is affirmed by the question: “Are you still married to the woman you used to insult so regularly?” Other instances of such improper identifications based on incorrect presumptions include:

- the present king of France
- the largest integer
- the fastest (theoretically permissible) motion
- the largest set (Cantor).

There simply are not—and cannot be!—items answering to these specifications. France long ago abolished kingship; every integer has a larger successor, and those other items are demonstrably impossible.

4.8 The Prospect of Limited Resources

In developing his theory of probability J. M. Keynes proposed a Principle of Limited Variety stipulating that when a finite mode of properties exists for the description of such things. Now if there at N such properties, each of which can or cannot apply to a given item, then 2^N available complete item-characterizations. And if—as the Principle of the Identity of Indiscernibles has it—at most one single item can possibly answer to a given (complete) description, then there can at most be 2^N items in existence. (Having a richer ontology would require descriptive re-duplications.).

While in theory any item can be identified, this will not always be possible. For all of the various modes of identification require the use of language, and since language is discursively developed (from a finite vocabulary via a finite manifold of grammar), this will never put more than a countable number of formulation at our disposal. But while only countably many items can ever be identified, there are for sure more than a countable number of items in existence. (e.g. real numbers). So while everything has an identity this is not something that can always be specified. It certainly will not do to say that “To be is to be identified”. This is problematic as is the case in Musical Chairs with “To play is to be seatable.” The crux is the difference between: $(\forall x)\Diamond Sx$ and $\Diamond(\forall X)Sx$.

4.9 Fallacies of Identification—Failed Requirements

As the previous considerations indicate, an item can only be identified if certain conditions are met:

1. *Possible realization*: The purportedly identified item must actually or at least *possibly* exist. Thus “The odd square of two” is out, whereas “the only London duplicate Eiffel Tower” is in
2. *Unique instantiation*: The purportedly identified item must be unique. (“The square root of four” is out, since both $+2$ and -2 satisfy this specification)
3. *Actual application*: There must not be a failure to effect reference in communication. Such failure can occur for contingent reasons. (“The prime number of which John is now thinking,” with John actually asleep. Or—for necessary reasons through leading to incoherence or paradox. (“The fastest motion,” “The largest integer.”) Or consider “the wife of the oldest man in the room who has no brothers.” This specification certainly *describes* but does not *identify* since the individual in question may not exist
4. *Viable presuppositions*: The purported identification is based on problematic and likely unavailable presuppositions. (“The customer whom Smith has been cheating.”) Again “the set of all sets” is improper. It presumes without justification that the collectivity of all sets satisfies all conditions needed to qualify it as a proper set. (The same goes for Bertrand Russell’s “set of all sets that don’t include themselves.”) Thus an item cannot be successfully identified if its identification is already presupposed in its identity specification. Thus “the girl who is available to become Herold’s wife” becomes “eventually becoming Herold’s wife” is already required as descriptive condition for identifying the goal in question. That female may well describe someone but does not (as yet) succeed in identifying.

Purported identification that violates such conditions are for this very reason ineffectual and flawed. These, taken together, they constitute what might be called the VIR (valid identification requirement).

4.10 Totalization Problems

Some items can be identified on the basis of universal involvement alone. (Thus as noted above, “the male ancestor of all humans” identifies Adam). But this proceeding will only rarely succeed: in general identifying an item on the basis of universal involvements simply do not work. Such failures are exemplified by:

- being a description that applies to all descriptions not applicable to themselves.
- being a set that contains all sets that do not contain themselves.

Such characterizations are *perfectly meaningful as descriptive characterizations*, but despite this we *cannot* use them in the course of *identifying* (introducing) an item. For if we propose to specify something—say as “the adjective that applies to all adjectives that. . .” or a set as “the set that contains all sets that. . .”—and so make this specification in a way that is *self-inclusive*, then we violate the Illicit Totalization Principle (ITP) in by presupposing the completion of a process on which we are still

engaged. Any totality that is so defined as to include itself—the totality of all *Xs* where the totality if itself an *X*—is problematic because its identification can never be properly effected.

The need for avoiding problematic modes of item-reference is an idea that goes back to Immanuel Kant who realized—and emphasized—that without such a proscription of illicit totalization run into self-defeating perplexity. Accordingly he rejected the “cosmological ideas” of “the world,” or “the cosmos as a whole.”

1. all-inclusive
2. ever-devisable
3. all-producing
4. all-sustaining.

As Kant saw it, *illicit* totalities—such as “the world” as a whole, the totality if existence—produce contradictions (antinomies) with respect to their object via such questions as:

- I. Does the world have limits in space and time—is it confined by something?
- II. Is the world an assemblage of parts?
- III. Are the world’s eventuations categorical or necessary?
- IV. Does the world have a world-external cause?

For Kant argued that an equally good case can be made out either way—affirmatively or negatively—in addressing such questions because in one way they include and in another they exclude the item at issue. Our present aim is not to defend this Kantian position in cosmology, but merely emphasize that the perspective that he adopted via Illicit Totalization Principle (ITP) is able to provide a tool of substantial philosophical utility).

To qualify as an item of some sort requires (1) being a single unified particular individual, and accordingly (2) having a specifiable identity. The former (unity) is up to the item at issue; the latter specifiable identity is a matter of collaboration between the item and our means of conceptualization. It is this latter aspect of identity and identification that makes the issue of itemization a matter not just of metaphysics but of epistemology as well.

Part II
Issues of Truth and Knowledge

Chapter 5

On Explanation and Understanding



5.1 Introductory Preliminaries

Explanation is a matter of accounting for the facts—of elucidating their claims to qualify as a part of the reality of things. It is a prime entryway into the intelligibility of the real, linking the epistemology of knowledge with the metaphysics of reality's access to inquiring man.

To explain something is to give an account of it that answers *why* and *how* questions. In general, this is a many-sided and diversified proceeding. For example, consider:

1. Question. Why does this building have four stories? Answer. They built it Z-wise and Z-wise construction yields 4-story buildings. [Productive result]
2. Question. Why does this building have four stories? Answer it is built in 4-Town and 4-Town requires 4-story buildings. [Rule conformity]
3. Question. Why does this building have four stories? Answer. Buildings with more or less than four stories always collapse on this corner. [Law conformity]
4. Question. Why does this building have four stories? Answer. The people in this area like and favor four-story building. They are superstitious. [Harmonization with Custom, Fashion, Norms].
5. Question. Why does this building have four stories? Answer. More than four stories not financeable, less than four stories not repay/worthwhile. [Contextual Limitations].

Clearly many different approaches are available for answering an explanatory question.

The issues that will particularly occupy us here relate to explaining the occurrences and events of the world. Such factual explanation is a proceeding that admits of a wide range of variable stringency. Thus p helps to explain q whenever given p , it can be shown that q :

- must be the case
- is the case
- is likely the case
- is possibly the case.

These last two are a sort of *faute de mieux* proceeding. Only if no stronger accounting for a fact is available to us would we consider these weaker modes of proceeding.

Our explanatory accountings enable us to deepen our understanding of the relations that bind the world's facts together. However, with explanation as with pretty much anything else we have to settle for the best available. Thus suppose:

I put two acorns on the back porch and overnight the pair is reduced to one. How is this diminution to be explained?

Clearly the following account, if true, will do the job.

(E) During the night an animal came along and made off with one of them.

Note that his "explanation," while adequate, is highly nonspecific. It fails to specify

- (1) the sort of animal at issue
- (2) the time of the removal
- (3) the manner of the removal
- (4) the reason why acorn 1 was removed rather than acorn 2(n) and a vast deal else.

In accepting (E) as the explanation we *presume* (without specific evidence that questions (1)–(n) *have* answers, even when we don't have a clue as to *what* they are. The "explanation" afforded by (E) is highly nonspecific and leaves a vast range of essential matter out of account. Yet in the ordinary course of things such incomplete explanations serve our purposes in asking for "explanation."

Still a schematic explanation is still an explanation. An adequate explanation does not need to answer the whole range of subsidiary questions. We accept very rough and ready explanations as adequate and acceptable in practice. And we presume "on general principles" that the missing detail is somehow appropriately answerable.

In cultivating the explanatory enterprise we confront the decidedly paradoxical-looking situation of Explanatory Regression. Thus consider the argument:

- (1) Explanations require explainers (i.e., explaining premisses).
- (2) Explanations with unexplained explainers (i.e. explainers that themselves lack explanation) are inadequate.
- (3) Therefore: Adequate explanation requires the completion of an undefined regress.
- (4) Unending regresses are incompletionable.
- (5) Therefore: Adequate explanation is unrealizable.

The only promising proceeding here is to attack premiss (2). This can be done in various ways:

- I. One is by arguing that at some point in the explanatory regress a point will be reached where further explanation is not needed either because the premisses at issue *admit* of no further explanation being somehow self-explanatory (and

thereby self-sufficient), or that they are “obvious” and self-evident (and thereby *require* no further explanation).

- II. Another is by convergence, when a point is reached when the contribution made by further steps becomes so small, so trivial, so unproblematic that the law of diminishing returns takes over. These convergent regresses might be deemed as *effectively completable*?
- III. A third is that a point of substantive phase shift sets in where the reasoning at work involves a category transformation—say from the factual to the evaluative or from the informative to the normative. We have reached the end of possibility in mode *A* and have to shift over to mode *B*.

Overall then resolving the paradox calls for particular attention to premiss no. (2).

Explanation stands coordinate with a hierarchical view of the fundamentality and security of the explanatory premisses that are employed. In general, care must be taken to avoid explaining what is problematic, doubtful, or obscure in terms of what is yet more so. When a feet is adequately explained its acceptability should stand on firmer ground at the end of the process.

5.2 Types of Occurrence Explanation: Inevitable Versus Contingent

A coin is tossed. It comes up Heads. How do we explain this? Presumably by means of something like the following account:

There are two possible outcomes for this: Head and Tails. This being a fair coin these outcomes are equi-probable. On this occasion it just happens that the outcome is Heads.

There is really nothing more to be said. That’s all the “explanation” there is—or in the circumstances can be. But there is still another, yet unanswered question. Why Heads rather than Tails: why not Tails.

And this leads to an important realization. For the issue of explanation here has two very different concerns:

- (1) How was it that *X* occurred (on this occasion and in these circumstances)?
- (2) Was *X*’s occurrence necessary (in the circumstances) could something else have occurred in its stead?

The point is that these are different questions and their satisfactory resolution requires different explanatory accounts. And in the present case the reply “By chance” does the job with (1) although fails to provide for the necessity demanded by (4).

Someone could say “But this is only a partial answer; it does not do the whole job! But this complaint is based on the erroneous supposition that only if the answer enjoins circumstantial necessity do we have an adequate explanting account. Explanatory adequacy does not require every relevant issue to be resolved.

The fact is that chance does not prevent explainability as an account of what and how and why. Chance does not block the way to explanation in fact, chance is itself a mode of explanation. In such matters the requirement of negative “why not otherwise?” explanation imposes an inappropriate and unreasonable burden—exactly because it is simply unrealizable. And on this basis we have to accept the reality that randomness does not exclude explanatory understanding. After all, explanation too is realized to the principle that “the most that can possibly be done has to be accounted as good enough,” this will have to be accepted as accomplishing the explanatory job.

5.3 Factual Explanation: Positive and Negative

The standard mode of explanation for a positively formulated fact (p) generally has the format of a *modus ponens* argument:

$$q, (q \rightarrow p) \therefore p$$

Such argumentation is evidentiary and pivots on considerations that imply p ; the later is counter-evidentiary and pivots on considerations that block p . The key factor for a positive explanation (of p) is that sufficient conditions for it to obtain: it, the core feature for a negative explanation (of $\sim p$) that a necessary condition for it to fail to obtain.

But what of negative facts? Given that $\sim p$ obtains, how are we to explain it? After all: Why not p ? There are various possibilities:

1. *incoherence*: p involves a self-contradiction $p \rightarrow (q \ \& \ \sim q)$
2. *exclusion by fact*: There is a fact q such that $q \rightarrow \sim p$
3. *preclusion by chance*. Reality could possibly go either way; in principle there just is no need to go one way or the other— p or not- p ; it’s totally arbitrary, but in this case p lost out.

Explaining negative facts can be a peculiar business. Thus consider explaining that Julius Caesar did not die on the Ides of March of 56 BC. Any of the following can do the job:

- (1) He was too busy doing other things: Every moment of the day is otherwise accounted for.
- (2) He was still living a year later.
- (3) Nothing that happened on that day endangering his life.

Such negative fact-explanations have the logical format of *Modus Tollens* argumentation, though blockage by a continuing fact:

$$(p \rightarrow q), \sim q \therefore \sim p$$

5.4 The Idea of Pan-Explanation

The principle of pan-explicability claims that there is an explanation for everything that is the case. However, the principle encounters problems.

Its difficulty resides in the distinction between an ontological and an epistemological perspective: between facts and presumptive fact claims. For one thing there is a crucial nominal disparity between facts and fact explanations. For the infinitude of particular facts is trans-denumerable, which explanations, being textual, are recursively developed and therefore denumerable. This means that there are simply more facts than explanations.

Fact claims require linguistic embodiment and give the reminiscent nature of language there are only denumerably many possibilities here. Facts, however, are inherently transdenumerable. There is, for example, a uniquely characteristic identifying fact regarding every real number.

So the idea that every fact has an explanation, has to be abandoned. There just are not enough explanations to go around.

And this requires a qualification in the Principle of Sufficient Reason. For at best the most that can reasonably be claimed is that every *stateable* fact has an explanation. There is, however, no cogent reason of general principle why the Principle of Sufficient Reason cannot be claimed in this weakened form.

To be sure, this does not automatically mean that there are unexplainable facts. Consider the analogy of the game of musical chairs. Since there are fewer seats than players, this means that there are bound to be some that go without. And yet that does not mean that they are somehow inherently unseatable. No player is in principle unseatable—destined to go without. It is not that those that will not be seated cannot possibly be so by some inherent factor that constrains unseatability. The fact that not every player can be seated does not mean that some particular player cannot possibly be seated.

But this consideration offers no comfort in the present case. For groups of players are not players themselves. But in our present case, the situation is different. For groups of facts always themselves constitute facts conjunctively. And this means that certain particular facts—and specifically the omnifact that composes all facts whatsoever—that will unavoidably have to go unresolved by factual explanation of the usual sort.

The upshot is clear. Reality as in toto—and thereby characterized by the omnifact—transcends the prospect of explanation in the factual order itself.¹

5.5 Omitting “Why not Otherwise”

It is widely maintained that to explain a fact *F* we require an event that shows why it is the case and not otherwise. This second choice seems highly problematic.

Why did the light go on at 5:30PM? As the day darkened, *X* decided to turn it on? What more is needed for explanation? Surly not: why not otherwise! To require further account of why *X* did not act at 5:49PM and not 5:51PM etc. is surely unnecessary and unreasonable.

We explain why Tom choose the hamburger on the menu we can establish that he has an affinity for beef and has a penchant for hamburgers. Why things don't stand otherwise—e.g. why he does not relish veal cutlets—is something else again, something that is effectively above and beyond the present issue.

Again, we have the coin, which is known to be fair. Fair coins are bound to come up Heads or Tails and in the present occasion this one comes up Heads. Why did it do so? All that we—or anyone—can possibly say by way of explanation is to surmise the preceding account adding the observation that this so how matters worked out in the present occasion. Why it did not do so differently is a meaningless question in these circumstances.

Upon graduating from college, John married Mary. Why did he do so? Well—it was because he fell in love with her. The question “Why no otherwise”—that is, why he did not fall in love with one of the many other dozens of girls he met would be a very long story—perhaps an irresolvable one.

The idea that explanation of a fact requires an additional and separate account of matters do not stand otherwise seems inappropriate and unrealistic. The problem with that “Why not otherwise” requirement is simply that there are too many otherwises. Requiring this would impose unrealizable obligations.

And so, Why the difference in presumption of acts as deliberate and omissions as venial?

Every actions stands irrelative to innumerable omissions. Omissions vastly outnumber actions.

In starting the journey now, I omit starting it 3 s ago—or 3 min. In giving you the apple I omit giving you that one. In marrying Jane I omit marrying Janet.

With actions we have a narrow and definite focus can in general reasonably expect the agent to think about its implications. With omissions however this poses an unrealistic task whose imposition on the agent must be an unreasonable burden.

5.6 Harmonizing Explanation

However, over and above factual explanation (positive or negative) there is yet another, very different mode of explanatory reasoning would look not to necessary or sufficient fact-relationships, to systemic relationships. Thus, for example, consider

1	2
	X
3	4
X	O

Let it be that we find an *O* at position (1). Variant from either mode this could be accounted for because of the five modes of symmetry (numerical, vertical, horizontal, \ diagonal, and / diagonal) only one would be realized by having *X* but three by having *O*. This perspective points to an inductivistic version of explanation through the formation of a harmonious structure of relevant fact (via conformity, simplicity, symmetry, and the like).

Such harmonizing explanation proceeds to explain matters by maximizing the scope of operation of general principles and universalities. It has the format that the fact to be explained obtains because its doing so maximizes the manifold of generalities that obtain in the circumstances.

Since the Pythagoreans of classical antiquity, successive schools of philosophy have subscribed to the metaphysical idea that the real is rational that the world's facts are coordinated in a systemic order that facilitates understanding. The recourse to harmonization as a principle of inductive systematicity inverts this relationship. Instead of a metaphysician that sees reality as a manifold of systemic order, it takes the presence of systemic order as an epistemic evidentiality for being real seeing living as an epistemic invariant for the injunction of reality. The Pythagoreans had it that the real is orderly. Harmonizing epistemology reverses that perspective and takes order to be a test-criterion for being true.

A simple illustration may help to clarify the nature of harmony-gear explanation. Consider the explanatory question:

Why should it be that the length of April (rather than May) fits the rule “30 days has September. ..”? Why are those 5 irregular months positioned as is? In a search for the relevant principles we hit up two rules:

- The year starts with a regular month
- There are never to be two adjacent irregular months.

In working out the possibilities we now draw that the only acceptable series will begin with the Regular/Irregular pattern:

R I R I . . .

In other words, only if April is irregular will a simple and plausible set of general rules become available. Harmonization requires this particular outcome.

Again, consider another sort of example. Suppose the occurrence of the following inscription on an old, weather-eroded tombstone in an old New England graveyard:

T□□s. Ta□lo□
1□□6 - 178□

(Here these squares indicate illegible letters.) It is not difficult to figure out that this must be taken to be

Thos. Taylor
17?6-178?

Here considerations of plausible conjecture can be evolved around harmonizing issues such as

- (1) the usual style and phraseology of inscriptions of the sort at issue;
 - (2) lexical fit of missing or incomplete symbols;
 - (3) making unreasonable sense within the general context;
- and the like, to facilitate the reduction of these possibilities to manageable proportions. The standard parameters of cognitive systematization—uniformity, simplicity, economy, normalcy, etc.—take hold again to supply the considerations needed for the assessment of the data-relative plausibility that guides the process of explanatory gapfilling.

And it is this sort of thing that holds throughout the entire range of harmonizing explanation. The procedure here is one of constructive coordinating—of fitting the pieces together into a harmonious whole—much as with the assembling of a jigsaw puzzle. And this reciprocal accommodation is also a matter of recursive process and thereby eliminable scope.

In sum, while inferential explanation exhibits how the relevant generalities function in favor of the fact to be explained, harmonizing explanation, by contrast, shows how this fact enhances and strengthens the manifold of relevant generalities. In harmonizing explanation we create the material used for explanatory systematization rather than require it to be pre-given.

Standard explanation proceeds via what is demanded *by* lawfulness. By contrast, inductive harmonizing explanation proceeds via what is demanded *for* lawfulness, through the idea that if things stood otherwise, the extent of lawful order would be diminished. Regular explanation proceeds since the purposing that the status quo is as is because this is required *BY* the laws. Harmonizing explanation proceeds on the consideration that this is so required *FOR* the laws. The one asks for harmonization *with* the lawful regularities; the other for harmonization *of* the lawful regularities. Harmonization maximizes the overall fabric of law and order within the relevant domain its operation goes to indicate that a kind of Principle of Rational Economy is at work in the shaping of the real.

5.7 Ultimate Explanation

Ordinary and normally explanation is inferential. But this imposes clear limits. For factual explanations require factual promises. And for this reason it cannot be all inclusive. For it facts are always needed as premisses then no explanation of the all-encompassing omnifact can possibly avoid the circularity of using an explanatory input a part of the output that is to be explained.

So if there is to be an ultimate explanation to assure universal explainability even where pervasive item explanation is unavailable—then we have to look in an entirely different direction.

At this point we have little choice but to embark in what might be called the Axiological Shift which decouples out explaining proceedings from the factual mode altogether and directs it into an altogether different, and now *axiological* dimension. In sum, if there is to be an ultimate explanation of the omnifact corresponding to reality-as-a-whole, then this explanation has to proceed in an evaluative rather than factual mode. Its validation of facts must proceed with reference to value. And so, if ultimate explanation is realizable at all it has to take the line that things are as is because all considered that is for the best. Harmony is the final analysis issue in value.

5.8 The Principle of Optimality

Accustomed as we are to explanations in the mode factual evidentiary, the idea of an ultimate, axiological grounding for explainability on the basis of an evaluative optimalism has a somewhat strange and unfamiliar air about it. Let us consider more closely how it is supposed to work.

The approach rests on adopting what might be called an *axiogenetic optimality principle* to the effect that value represents a decisive advantage in regard to realization.² Accordingly, whenever there is a plurality of alternative possibilities competing for realization in point of truth or of existence the (or an) optimal possibility wins out. (An alternative is *optimal* when no better one exists, although it might have equals.) The result is that things exist, and exist as they do, because this is for the (metaphysically) best.

It may be a complicated matter to appraise from a metaphysical/ontological standpoint that condition *X* is better (inherently more meritorious) than condition *Y*. But, so optimalism maintains, once this evaluative hurdle is overcome the question “Why should it be that *X* rather than *Y* exists?” is automatically settled by this very fact via the ramifications of optimality. In sum, a Principle of Optimality prevails; value (of a suitable sort) enjoys an existential impetus so that it lies in the nature of things that (one of) the best of available alternatives is realized.³

But optimal in what regard? What is the standard of merit at issue with their quest for optimality? The answer is: harmonization itself, so that optimalism’s standard of merit is itself simply rational intelligibility via harmonious systematization.

What we thus have is a consideration of rational economy which has it that merit is of a piece, and that merit for existence and merit of existence stand coordinate with one another.

“But why should it be that such an optimalism obtains? What sort of plausible argument can be given on this position’s behalf? Why should what is for the best exist?” The answer to these questions lies in the very nature of the principle itself. It is self-substantiating, seeing it is automatically for the best that the best alternative should exist rather than an inferior rival.

After all, what explanation of a situation could possibly be better than realizing that this is for the best, with better the merit of reasons coordinated to the merit of

conditions within a unified framework of value made whose aegis the best conditions of being constitutes the best reason for being. But this is just one of its assets;⁴ it also offers significant systemic advantages. For of the various plausible existential principles, it transpires—in the end—that it is optimalism that offers the best available alternative—the rationally superior prospect.

The principle being, as it were, self-explanatory, it transpires that to ask for a different sort of explanation for the principle itself would be inappropriate. We must expect that any ultimate principle should explain itself and cannot, in the very nature of things, admit of an external explanation in terms of something altogether different. And the impetus to realization inherent in authentic value lies in the very nature of value itself. A rational person would not favor the inferior alternative; and there is no reason to think that a rational reality would do so either.

To be sure, could one ask “But why should it be that Reality is rational?” But this is a problematic proceeding. For to ask this question is to ask for a reason. It is *already* to presume or presuppose the rationality of things, taking the stance that what is so is and must be so for a reason. Once one poses the question “But why should it be that nature has the feature *F*?” it is already too late to raise the issue of nature’s rationality. In advancing that question the matter at issue has already been tacitly conceded. Anyone who troubles to ask for a reason why Reality should have a certain feature is thereby proceeding within a framework of thought where nature’s rationality—the amenability of its features to rational explanation—is already presumed.

Yet what is to be the status of an optimality Principle to the effect that “whatever possibility is for the best is ipso facto the possibility that is actualized.” It is certainly not a logico-conceptually *necessary* truth; from the angle of theoretical logic it has to be seen as a contingent fact—albeit one not about nature as such, but rather one about the manifold of *real* possibility that underlies it. Insofar as necessary at all it obtains as a matter of ontological rather than logico-conceptual necessity, while the realm of possibility as a whole is presumably constituted by considerations of logico-metaphysical necessity alone.⁵ But the division of this realm into real vs. merely speculative possibilities can hinge on contingent considerations: there can be logically contingent laws of possibility even as there are logically contingent laws of nature (i.e., of reality). “But if it is contingent then surely it must itself rest on some further explanation.” Granted. It itself presumably has an explanation, seeing that one can and should maintain the Leibnizian Principle of Sufficient Reason to the effect that for every contingent fact there is a reason why it is so rather than otherwise. But there is no decisive reason why that explanation has to be “deeper and different”—that is, no decisive reason why the prospect of *self-explanation* has to be excluded at this fundamental level.⁶ After all, we cannot go on putting the explanatory elephant on the back of the tortoise on the back of the alligator ad infinitum: as Aristotle already saw, the explanatory regress has to stop somewhere at a “final” theory—one that is literally “self-explanatory.” And the ability to meet this challenge is the critical advantage that optimalistic explanation has over its available alternatives.

5.9 Two Modes of Explanation

There are two main sorts of explanation for factual arrangements in the world. The one, by *efficient causation* (as it is generally called) is productive: it looks to the processes by which the state of affairs at issue has come about and explains it as the result of the operative proceedings. It is in this sort of way that we explain matters ranging from the growth of forests or trees to the destruction of buildings by fire or the spread of disease in a population.

The second main sort of explanation—*finalistic explanation*—does not function productively with a view to the process of production, but functionally with a view to outcomes. This sort of account has usually been called *final* because it proceeds with reference to products and results rather than proceedings or processes. Its format is to explain X 's being so on the basis that this is required for a certain result Y to be realized. Taking Y as something fixed, it reasons that X must be so for this outcome to be achievable.

Standard causal explanation brings the operative generalities (the “laws of nature”) to bear on the relevant circumstantial conditions so that the occurrence to be explained follows as a logical consequence. Its generic structure is

- (descriptive + laws) $\rightarrow$ mode of occurrence

This account often called “the Hempelian model of explanation.” Its structure is factual, subsumptive, and deductive. And it typically explains specifics (occurrences or events) through the situational bearing of generalities.

Finalistic explanation is something very different. Its procedure is not formatively conditional but counterfactually hypothetical. Its mode of operation answers to the format “if things did not happen X -wise, then some prominent features of reality [very likely] would/could not obtain.”

A typical explanation of this particular sort would be

- If the freezing part of water were 40°C the polar region would be uninhabitable.
- If women did not constitute roughly half the population the role of families would be...

An example of finalistic explanation runs as follows Question. “Why is there printing on that sheet?” Answer: “Because it comes from a book.” Or again: Question: “Why does that stick have a sharp point?” Answer: “Because it is to be used as a stylus.” However, such explanation is not necessarily purposive. If the subordinate F_1 is required for the superordinate fact F_2 to obtain, one need not think that F_1 obtain *for the purpose* of realizing F_2 . Many raindrops are required to make a rainstorm, but one will not say that they make that rainstorm possible. The connection is one of “and so” rather than “so that!” (It rained and so the street got wet” rather than “it rained so that the street would get wet.”)

Display 5.1

(A)	(B)
To explain what makes <u>something into a</u>	You must elucidate it <u>role in the constitution of a</u>
branch	tree
tine	fork
spoke	wheel
letter	alphabet
page	book
overture	opera
preface	book

To explain what makes something an X requires giving an account of how X 's function in the setting of a larger account Y . (See Display 5.1). Playing a part—physical or finalist. A causal account of how a spoke was produced will not explain its nature as a spoke: this can only be done by interacting its role in a larger context. It is a matter not of its condition but of its function. Its nature and its suitability as such depends not on its productive causality but in its functional finality.

If a complex were not a constituent part of a (B) then it would not qualify for characterization as an (A) .

In effect, then, this version of final causation explains some facet of a complex via the consideration that this has to be so for the characteristic nature of that complex to be realized. Efficient causation is a mode of sufficient conditionalization; final causation is a mode of necessary conditionalization.

Final explanation functions as follows: Let it be that Z is an element or constituent of a larger system S which has a certain characteristic feature

F so that $F(S)$. Then we explain Z 's possession of the characteristic C by the considerations that:

If Z did not have characteristic C , then S could not its characteristic feature F .

And this would function as per

If Z did not have C then its overall context S would possess less symmetry, order, simplicity, regularity, harmony, or the like.

Thus consider

+	—
—	+

If that upper left positon were not $+$ then this configuration would have no symmetry and no $\pm$ balance (equality).

Exactly as with efficiently causal explanation such finalistic explanation accounts for one part [aspect, feature] of reality in terms of others. It does not explain why that reality as a whole is as is, but rather why a part of it must be as is so that the whole (as is) can realize a certain (evidently positive) condition.

It must be stressed that in finalistic explanation it is not claimed that P obtains purposively, in order that Q might be so. This mode of explanation is by nature conditional, not contingent; functional not purposive.

An illustrative example would be

—Seeing that a machine which cannot do addition cannot do mathematics, the explanatory question “How is it that your machine can do arithmetic?” can receive the answer: it is a mathematics machine.

After all, if the machine could not do arithmetic it could not qualify as the sort of thing is actually is.

The idea of *non-purposive* final explanation is that a certain conditional must obtain at one level of complexity if something more encompassing and higher-level is to qualify as harmoniously complete. Thus there cannot be a bladeless knife because without a blade that that physical object just would not count as a *knife* and an organism without the capacity of thought could not count as a human being. (This is not, however to say that thought occurs in order to enable human beings to exist.)

Efficient explanation accounts for the constitution of parts of a system on the basis that this must be as is if the system is to realize some (positive) feature.

In its explanatory modus operandi this prioritizes some general feature of the overall system. Thus consider the “Law of Least Action.” It serves as an explanatory engine in stipulating that given of its characteristics [of in certain specified conditions] the system must exhibited certain others in and for the idea to be minimized.

In general, if a system minimizes or maximizes a certain parameter, on the basis of having some constitutive feature, then that mode of finalistic explanation can come into play.

The “final explanation” of the fact that item x (which is an integral component of the larger system S) the feature F consists in showing that if x did not have F then S could not quality as being the sort of system it is. Such final explanations have the *counterfactual* structure “If not P then Q would not be so.” Accordingly, they pivot on necessary conditions at the local level is a certain status is to obtain more globally.

The Principle of Causality has it that: *Every event in Nature has an explanation for occurring as is in the mode of efficient causality.* Is there a comparably plausible Principle of Finality: Every feature of Nature has an explanation for being as is in the functionalistic mode of explanation.

The answer here if in the negative.

Efficient causality is regressive. When A is the productive cause of B , we can always ask about A s productive cause in turn. However, this is not the situation with final causation. When some situation A completes and unifies a more comprehensive B there need not always be a further C that unifies and integrates A . The sentence

that finishes and completes the story may itself be incomplete in some regard that leave matters hanging open-endedly in the air.

The Principle of Finality would obtain only if it were the case that every item existing at a given taxonomic level in nature there were an associated whole of the next higher taxonomic level in whose being the former plays an essential role. Such a pervasive taxonomic order of functional subordination is conceivable, but scarcely actual.

Notes

1. The author is grateful for the benefit of Patrick Grim's discussion of these issues.
2. The prime spokesman for this line of thought within the Western philosophical tradition was G. W. Leibniz. A present-day exponent is John Leslie. (See the Appendix to this chapter.) See also the present author's *The Riddle of Existence* (Lanham MD: University Press of America, 1984).
3. To make this work out, the value of a disjunction-alternative has to be fixed at the value of its optimal member, lest the disjunctive "bundling" of a good alternative with inferior rivals so operates as to eliminate it from competition.
4. Other principles can also be self-substantiating, seeing that, for example, the Principle of Pessimism (that the worst of possible alternatives is realized) also has this feature. The issue becomes one of plausibility.
5. The operative perspective envisions a threefold order of necessity/possibility: the logico-conceptual, the ontological or proto-physical, and the physical. It accordingly resists the positivistic tendency of the times to dismiss or ignore that second, intermediate order of considerations. And this is only to be expected since people nowadays tend to see this intermediate realm as predicated in value considerations, a theme that is anathema to present-day scientism.
6. After all, there is no reason of logico-theoretical principle why propositions cannot be self-certifying. Nothing vicious need be involved in self-substantiation. Think of "Some statements are true" or "This statement stakes a particular rather than universal claim."

Chapter 6

Alethic Topology



(Issues of Truth, Position, and Paradox)

6.1 Origins of Semantical Topology

Factual claims can be true at one place, false at another; possible in one location and not in another. “It is hot here” is flexible; its being hot in Phoenix is compatible with its being cold in Nome; rain is possible in London yet not in Mecca. The truth-status of claims—and their possibility-status as well—will distribute variously across the positional landscape. This issue of the positionality of truth (and that of modality-status as well) defines the problem-area of semantical topology.

Topology is the mathematical study of place and position. Semantics is the logical study of meaning and truth. Topological semantics blends the term into the conjoint study of positionalized affirmation, be it temporal or spatial “It is sunny” affords one example. “The statement on line (1) is true” is another, where line 1 reads:

$$(1) \quad 2 + 2 = 4$$

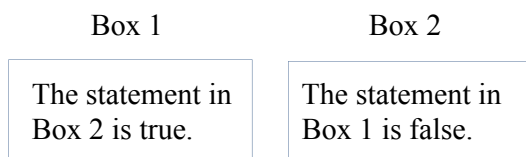
Topological semantics is of special interest for Aporetics because it is a fertile source of paradoxes.¹ This will occur when the selfsame claim is offered in one manner (say via its explicit statement) and yet denied in another (say positionally). What we have here is a matter of *assertoric dissonance* arising when a claim afforded in one manner of inference is denied when afforded in another.

The idea of a positional distribution of propositional truth and falsity originates with the seventeenth-century English logician, Henry Aldrich (1647–1710).² In his logic compendium of 1691, he first articulated a correspondingly devised positional paradox based on the illustration:

*Omne enuntiatum intra
hoc quadratum
scriptum est falsum.*

His consideration of the claim that “Every statement written within this rectangle is false” launched the basic idea of indicating the truth-status of statements by means of the placement of their assertion in some positional framework.

In the Aldrich example, the (paradoxical) claim at issue was self-referential—along the lines of the classic Liar Paradox based on the claim: “This assertion is false.” But soon, analogous paradoxes came to be considered without any overt and explicit self-reference, as per the box-pair paradox:



Along similar lines, consider the statement: “All statements affirmed on page 32 are false.” There is no problem here unless and until that statement itself is affirmed on page 32. Then, if true, it must be deemed false, and if false, it must be deemed true. Here, we have ambivalent truth-status vacillation typical of paradoxes. Just this is the crux of alethic topology.

The problem arises in these cases because of the positioning of the focal statements. And here the issue can become paradoxical, not because of *what* is said—which could be unproblematic in other contexts—but because of *where* this saying occurs. In such instances, the paradoxicality is essentially positional, in that a disconnect can arise between direct and substantive assertion and oblique and position-based assertion.

This way of viewing things involves various interesting variations. Thus, consider:

- (1) p
- (2) (1) is false
- (3) Both (1) and (2) are true

Given the referential indications at issue with the labeling (1), (2), (3), this group is obviously inconsistent and paradoxical. There is no way of arranging for its members—or even the majority of them—to be true.

6.2 Descriptive Basics

All this opens the door to more complex positionally structured truth claims, as with those based on a sequentially positioned sequence:

1	2	3	4	5	etc.
---	---	---	---	---	------

subject (for example) to the rule that position 1’s statement is true, and that otherwise true propositions always both precede and follow false ones. This stipulation will lead to the truth-status series:

$$T\ F\ T\ F\ T\ F\ \text{etc.}$$

Along these lines, positional truth-status specification can generate various complexes of semantical (true/false) distributions across an endless variety of propositional spaces.

As we shall consider it here, a *positional field* or *configuration* will consist of one or more compartments having some sort of structure. And such configuration will secure a semantical aspect whenever its compartments contain a statement that has a definite truth-value, *T* or *F*.

Take any locally measurable quantity *Q* such as daily high temperatures, annual rainfall, height above sea level, the earliest annual time of sunrise, etc. Consider its distribution over a positional field—a region. Identify the locales of this region at issue via an index *i*. Specify a cut-off point *C*. Then characterize those locales propositionally via:

$$S_i = Q_i \geq C$$

This will clearly yield a distribution of truth-values for positions across the entire terrain.

In theory, the assignment of truth-values can be:

- *Normal* in assigning an axiomatized truth-value (*T* or *F*) to each compartment of the field.
- *Abnormal* by way of *vacuity* in assigning no truth-value to some positions, or paradoxical in assigning several different truth-values to some positions.

Our prime concern here will be with normal truth-value assignments, the abnormal being viewed as an extra-ordinary case.

Some semantical truth-rules are omni-determinative, some are only partially determinative, and some not at all. Thus, consider a semantical field that is structured as a 2 x 2 square. The rule:

- *T* may or may not occur somewhere determines nothing. By contrast, the rule:
- Diagonally adjacent positions must be filled alike leaves open just four possibilities:

T	T	T	F	F	T	F	F	F
T	T	F	T	T	F	F	F	F

However the rule:

- Corner positions must be *T*-filled is fully determinative.

The compartmentalized “positions” at issue with w semantical field can be spatial locations, times, or locations in some abstract structure. They will be indicated by some parametric index (here α, β, γ , etc.). The semantical rules at issue will assign to each position $P_\alpha, P_\beta, P_\gamma \dots$ a truth-value, true (*T*) or false (*F*), according as the subscript-indicated positioned statement is true or false. These rules thus have the form:

- Position P_α has *T* (or *F*) according as the situation specified by the statement S_α does (or does not) hold.

An example of such a semantical rule for a linearly ordered position field might be:

- $S_i = i$ is odd

Accordingly S_i is *T* whenever i is odd and *F* whenever i is even. And the resulting truth-situation would thus be:

T F T F T F T . . .

For the sake of further illustration, consider the pair of truth-rules for a 2 x 2 gridwork:

- There can be only one *T* per row
- *T*s and *F*s must be equinumerous in the overall field

These conditions can be satisfied in only two cases:

T	F	or	F	T
F	T		T	F

 Display 4.1

T-pervasive: *T*s everywhere

F-pervasive: *F*s everywhere

Contingent: Some *T*s and some *F*s

Under-determination: The rules leave some positions open

Over-determination (Paradoxical): The rules require some positions to be both *T* and *F*

The semantical rules governing truth-status distributions over the compartmentals of a semantical position-field are subject to the classification of Display 4.1. With a paradox, there is not just falsity but also the contradictory vacillation of truth-status over-determination.

6.3 Preview of Basic Machinery and Illustrations

As the previous considerations indicate, the present deliberations presuppose three key items:

- (1) A (structured) *field or space of positions* P_α , each identified by an index-value α .
- (2) A manifold of *statements*, one of which is assigned to each position of this field, so that S_α can be used to indicate the statement which is assigned to position P_α .
- (3) A family of *semantic rules* that specify the truth-status condition of those variously positioned statements (be it True or False).

For the sake of example, suppose a field of serially ordered positions duly indexed by the successive integers:

1	2	3	4	5	etc.
---	---	---	---	---	------

The statements affiliated to these positions form the series:

$$S_1, S_2, S_3 \dots$$

And we may now suppose that the semantical rule at work requires that a true statement always be followed by true ones:

- If $|S_i| = T$, then also $|S_{i+1}| = T$.
We use here the symbolization $|S|$ to represent the truth-value of the statement S , be it T (true) or F (false). On this basis, if the first occurrence of T were at S_4 , the overall truth-value manifold that arises would be:

F F F T T T T etc.

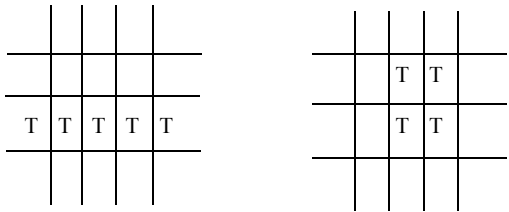
If the rule at issue were

- $S_i = I$ have exactly one true neighbor
then the only series that would fill the bill would be

.... T T F T T F T T F

with every third entry an F .
There will be many different ways in which a positional truth-rule (or family of such truth-rules) can be realized over a given terrain. Thus consider the rule:

- A given position will be T -filled iff exactly two of its adjacent positions are T -filled.
Among the various ways this can be realized in a gridwork we have both of the following:



where the blank spaces are F -filled.
As the preceding example shows, the structure of the field matters crucially for the obtaining of rules. Moreover, the rules are sometimes underdeterminate indeterminate and will leave several different possibilities open. This resembles the situation in metaphysics where the laws of nature underdetermines the phenomena.

6.4 More Illustrations

Consider the following 4×4 gridwork in the light of the rule:

- A position must be T whenever the majority of its adjacent (flanking) neighbors are.

Ⓣ	Ⓣ	F	F
Ⓣ	Ⓣ	Ⓣ	T
F	Ⓣ	T	F
T	Ⓣ	F	T

Here, the circled *T*s are so by virtue of the mandate of that rule. They are situationally rule-necessitated, whereas the remaining four *T*s are contingent. In gridwork situations, the semantical rule:

- Every truth-bearing position is flanked only by others of opposite status
This is readily realized:

T	F
F	T

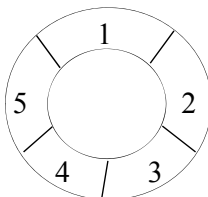
However, realizing the rule:

- Every truth-bearing position is flanked by four others of opposite status is something else again, for this would require an infinite gridwork!

It is clear that one and the same semantical rule can issue in very different distributions of truth-values depending on the generic structure of the positive manifold at issue. Thus, consider the rule:

- Adjacent positions must always have opposite truth-values
This rule can always be realized in a gridwork of any size on the basis of “diagonal alternation.” (All *T*s along a given diagonal with all *F*s along the adjacent diagonals of like orientation.)

However, the rule could not possibly be realized in a circular position-field with an odd number of positions. Thus, suppose that the positional field at issue were:



Then, the basic rule could no longer be satisfied, because compartments 1 and 5 would have to exhibit the same truth-value. And the same story holds for the rule:

- True propositions always precede false ones, and conversely.
This raises no problems with linear position-fields, but claims become paradoxical with odd-numbered circular position fields.

6.5 Paradoxes

A paradox arises in semantic topology when the circumstance of the case requires different truth-value assignments to one selfsame position. Thus consider a 3 x 3 tic-tac-toe square of format

And consider the truth-rules.

- (1) Only corner positions are true
- (2) Diagonals have uniform truth-values

Rule (1) requires the central position to be F. Rule (2), in the context of (1), requires the central position to be T. The situation is clearly paradoxical.

Again, suppose the truth-situation of a propositional field to be as follows:

F	F	F
F		F
F	F	F

And let the statement in the central position be “All of the statements of my assertion field—myself included—are false.” Then, on its own telling that central statement is false. But if this is so, then what that statement affirms is indeed so, and so it would need to be classed as true. Here, we have the vacillating truth-status ambivalence typical of paradoxes: the statement at issue must be seen as truth if classed as false, and as false if classed as true. Neither is a viable option.

6.6 Algebraic Series

Consider two progressive series in algebraic arithmetic:

$$f_i^1 = (i + 1)^2$$

$$f_i^2 = i^2 + 2i + 1$$

Since in general we have:

$$(i + 1)^2 = i^2 + 2i + 1$$

we now have it that the statements of the series:

$$(S_i)f_i^1 - f_i^2 = 0$$

are invariably true, so that for all i :

$$|(S_i)| = T$$

In this manner, the algebra of arithmetically or geometrically progressive series can be recast in terms of the mechanisms of alethic topology. For the claims at issue can all be represented as serially standard assertion-theses whose truth-status is invariably T .

6.7 Incoherence and Paradox in Alethic Topology

Consider a group of statements S_α each of which affirms the self-referential “I am the only true statement of my group”:

$$S_\alpha = (|S_\alpha|) = T \& (\forall \beta) [\beta \neq \alpha \supset |S_\beta| = F]$$

This semantical rule will hold within the trio S_1, S_2, S_3 just in the case of the overall truth-status of its members being one of the trio TFF, FTF , and FFT . Accordingly, we have it that (1) the truth-rule in question is co-satisfiable over a trio of statements, and (2) no conclusion can be drawn from this fact regarding the actual truth-status of those individual statements.

By contrast, consider the truth-rule “Two other statements in my group are true”:

$$S_\alpha = (\exists \beta)(\exists Y)(\beta \neq \alpha \& Y \neq \alpha \& [|S_\beta| = |S_\alpha| = T])$$

This truth-rule will hold throughout the statement trio S_1, S_2, S_3 just in the case of their truth-status being TTT . So, here the operative rules enjoin that all three trio members must be true.

Additionally, consider the truth-rule “None of the statements of my group is true”:

$$S_i = (\forall_j)(|S_j| = F)$$

This is satisfiable throughout the membership of a trio only if the viable truth-status is pervasively F . But when the S_i are thematic marks of the group that renders truth-status is FFF . But since this would rend every S_i true. The situation is clearly paradoxical.

6.8 Self-contradiction

Consider, for example, a color strip of distinct compartments as per:

c_1	c_2	c_3	c_4			
-------	-------	-------	-------	--	--	--

where adjacent compartments are visually indistinguishable in apparent color. So, by hypothesis:

$$S_i = [C(C_i) = C(C_i + 1)]$$

represents claims that always hold good. Nevertheless, with gradual steps the situation can become such that there will be notable color differences among sufficiently remote compartments, so that $C(C_1) \neq C(C_{100})$. And so, while we have $(\forall i)(|S_i| = T)$, we also have $(\exists i)(|S_i| \neq T)$. The alethic topology of the matter shows that the truth-value vacillation characteristic of paradox confronts us. But where is one to place the transition between $C(c_i)$ and $C(c_{100})$? Where does $C(c_i)$ end and where does $C(c_{100})$ begin? Here, we have exactly the same problem as with the classical heap paradox. (At which point does a pile of N sand-grains become a heap?). And exactly the same sort of solution emerges, so that here too a resort to predicative vagrancy is able to do the needed work.

Consider once more the Aldrich paradox that formed the starting-point of deliberations:

This statement—the very statement here as it is written in this box—is false

Internally, by what the statement itself says, it is false. But this being so, then externally, by what is being affirmed about the statement, it is automatically true. In such a situation, statements can have two alethic aspects:

1. The *internal*; its truth-status according to what the statement itself affirms.
2. The *external*; its truth-status according to the positional facts regarding the statement.

Thus, consider the classical “Liar Proposition” of the just-presented box. Internally considered (according to itself), the statement is false, but externally (in the overall context), it appears true because its falsity is just exactly what is being claimed via positionality. (In fact, if it were indeed false it would perversely be on its own telling—and have to be accepted as true since what it claims would indeed be the case.) So here again we have a paradox—with its characteristic vacillation between truth and falsity.

Again, consider the following sequence of statements:

S_i = All my successors are false

As it stands, S_1 claims T F F F ... But if this were in fact so, then S_2 would actually be T. Thus, S_i is in fact self-inconsistent and thereby false. So, here again we have a paradox. If S_i is accepted as true, it must be recast as false seeing that a condition follows. There is just no way to make overall sense of it by getting the internal and external truth-conditions to harmonize.

6.9 A Survey of Generalizations

It is instructive to consider a linear sequence of statements subject to the positional rule:

- My immediate successor is true, but one of my eventual successors is not:

$$S_i = ([|S_{i+1}| = T] \& (\exists j) [|S_{i+j}| = F])$$

This can be true at any and every value of i , but yet not at all of them. We have an illustration of the difference between $(\forall n) \Diamond F(n)$ and $\Diamond (\forall n) F(n)$. The former does *not* imply the latter. This situation illustrates what might be called the Musical Chairs principle. (For in Musical Chairs any given player can be seated but yet not all of them.)

Along these lines, let us consider the following group of serial truth-claims, in each case confronting a series of the format $S_1, S_2, S_3 \dots$ where S_i asserts:

- (A) All of my predecessors are T
- (B) All of my predecessors are F
- (C) All of my successors are T
- (D) All of my successors are F

(E) All of my companions are *T*

(F) All of my companions are *F*

Let us examine these in turn.

(A)

Suppose S_2 is *T*, then by this rule S_1 would also be *T*. And then so would S_3 . And so on. The whole series would be: *T T T T ...*; the series is ever-true.

Suppose S_2 is *F*, then S_1 would also be *F*. And so, all the other S_i would be so also. The whole series would be: *F F F F ...*; the series is ever-false.

Only uniform truth-series are possible given this rule.

(B)

Suppose S_2 is *T*. Then S_1 would be *F*. And so is S_i for all $i > 2$. So, the whole series would be: *F T F F F F ...*

Suppose S_2 is *F*. Then S_1 would be *T*. And all S_i with $i > 2$ are also *F*. So, the whole series would be: *T F F F F F ...*

Given rule (B), the series only admits of two alternatives.

(C)

Suppose S_1 is *T*. Then the entire series would have be *T T T T ...*

Suppose S_2 is *F*. There would have to be an unending recurrence of *F*s. But any further specifics would be contingent.

(D)

Suppose S_1 is *T*. Then the entire series would have be *T F F F*. But then S_2 would be *T*. So, this alternative is ruled out.

Suppose S_1 is *F*. Then some S_j would be *T*. But then all subsequent S_{j+k} would now be *F*. And then any subsequent S_i would be *T*. So, this self-contradictory alternative is ruled out.

There is no possible way of satisfying condition (D). Its specification is *PARADOXICAL*.

(E)

Suppose that *T* occurs for S_j . Then it will have to occur throughout. Yet suppose an *F* occurs for S_j . Then this also must occur throughout. So, again, there are just two possibilities: *T T T T ...* and *F F F F ...*. Ever-true and ever-false are now the only options.

(F)

Suppose that a *T* occurs at some S_j . Then the overall pattern must be of the form *... F F F T F F F ...*. So, this is an unproblematically viable prospect.

By contrast, suppose that a *T* occurs nowhere. Then we would have *F*s everywhere so that every S_i is *T*. Accordingly, this possibility is ruled out as self-inconsistent, and we would be left with the previous viable alternative of one single *T* somewhere.

* * *

And so, in the entire group of truth-rules (A) to (F), it is (D) alone that is unrealizable and paradoxical.

Of course, other more complicated rules of truth distribution can be contemplated. For example, consider:

“My truth-status differs from that of my immediate neighbors.”

$$(\forall i)(|S_{i-1}| \neq |S_i| \neq |S_{i+1}|)$$

This permits only two cases:

$$T F T F \dots$$

$$F T F T \dots$$

With either—but these alone—is what the rule stipulates in fact realized.

6.10 Some Applications

The machinery of topological semantics invites various sorts of application. These include:

Syllogistic Logic

The syllogistic logic of categorical propositions is readily realized within this framework of the present deliberations. Thus, let the A s consist of $a_1, a_2, a_3 \dots$ and consider the propositional sequence:

$$S_i = \alpha_1 \text{ is a } B$$

Then “All A s are B s” because

$$(\forall i)S_i = (\forall i)(|S_i| = T)$$

and “Some A s are B s” because

$$(\exists i)S_i = (\exists i)(|S_i| = T)$$

The truth-status account of the S_i series is a solid row of T s in the first case, and it has a T somewhere or other in the second.

On such a basis, position semantics can accommodate the whole of traditional syllogistic logic.

Grid Searches

Another possible application of this semantical technology is its employment in grid searches. The terrain at issue here is divided into a gridwork and the presence

of some significant feature in each compartment is assessed as true or false, with respect to some deliberative conclusion. Thus, in an archeological application, the concentration in the soil of some chemical or electromagnetic parameter could be taken to indicate underground stonework. And with a grid configuration of:

T
T
T
T

this might indicate a long-buried wall. Or if:

T T T T
T T T T

it might indicate the foundation of a building.

Games

It is possible to devise games that exploit the mechanism of our semantical truth-manifolds. For example, consider a large gridwork. Now, let several players select positions and accordingly be given propositions pre-located there. They have to indicate which they are, *T* or *F*, and their response be marked as correct or incorrect. The first player to get three in a line (row or column) correctly wins. (Throughout the process, the players will try to block each other in the selection of positions.)

All in all, then, the machinery of alethic topology is not only of considerable logico-semantical interest in its own right, but can also be put to instructive work in a variety of applications.

Notes

1. The first general publication in the field would seem to be Nicholas Rescher and James Garson, "Topological Logic," *The Journal of Symbolic Logic*, vol. 33 (1968), pp. 537-598.
2. Henry Aldrich, *Artis logicæ compendium* (Oxonii: E. Theatro Sheldoniano, 1691).

Chapter 7

Relevance and Its Problems



7.1 What Relevance Is and Why It Matters

Philosophers of science face the dual task of elucidating both discourse conducted within science and discourse regarding science. This saddles them with a duty both to interpret what science says and to explain why it does so.

This engages them in a concern within matters not belonging to science itself. *Relevance* is one of these. It may be of intense concern to auditors of scientific journals. But it leave scientists themselves cold. And for that very reason it also leaves most philosophers of science indifferent and unconcerned. They would rather leave it to sociologist of science, rhetoricians of science—anybody else. But in the final analysis they cannot bring this off without some injury to their professional honor.

Relevance is an epistemic merit. It belongs to the same range as other cognitive values such as: charity, verifiability, informativeness.

The core of the conception is that something is relevant to a given item when it provides purported information regarding some aspect of that item. And note that in this context even misinformation—even incorrect information still constitutes information. The erroneous claim that the Eiffel Tower is in London is still relevant to the issue of the Eiffel Tower and its location.

Relevance pivots on the existence of informative connections among facts or ideas: matters are relevant to one another insofar as they are linked in their informative bearing. The key question regarding relevant to a given issue is: “What difference does it make for our understanding of the matter.”

One factor will be relevant to another when it’s being changed evokes changes in this other. Thus the spelling of a word is relevant to its meaning: change the spelling and you get another word (pill/kill). The color of a banana’s peel is relevant to its palatability, the longitude of its placement is not.

And on the same basis one fact can be relevant to another. Change the one (or even its probability) and you will change (the probability of) the other. (“At the time

Jones was absent from the scene of the crime” is relevant to “Jones committed the crime”—and the probability of the later can rise and fall with that of the former.)

Cognitive relevance can also be classified functionally with respect to the purposive context at hand. Thus it can be

- *productive* (as flour is productively relevant to cake)
- *operative* (as batteries are operationally relevant to flashlights)
- *causal* (as causes are explanatorily relevant to their effects)
- *medical* (as medications are medically relevant to curing ailments).

All of these, however, inhere in matters of cognitive relevance, in that they root in these of the format:

A is Xly relevant to B if an A is Xly involved in the realization of a B [as flour is PRODUCTIVELY involved in the realization of a cake, or the antecedent conditions are CAUSALLY involved in the realization of an effect.]

The crux here is that this sort of cognitively functional relevance is always tantamount to and reformulable in terms of propositional relevance.

Unquestionably the most decisive and dramatic mode of relevance is represented by the parametric coordination of mathematico-physical equations. Thus we have it that the circumference (C) of a circle is relation to its radius (r) by the formula $C = 2\pi r$ with its lock-step coordination between stipulates for circumference (C) and radius (R). More generally, what the statisticians call the *sensitivity* of one factor to variation in another is an index of relevance.

Relevance can clearly vary in extent: statements can be relevant to one another in different degrees. The Eiffel Tower is more relevant to the lay-out of Paris than is the mechanism of its elevators.

Relevance is of paramount importance in the management of information because any material not relevant to the issue under consideration can be dismissed from consideration. However interesting or informative it may otherwise be, the rational economy of effort negates the claim to current attention of any material what is irrelevant in its bearing on the present concerns. Relevance accordingly matters for fundamentally economic reasons. For we have only a limited amount of time, energy, and other resources at our disposal in dealing with information. And the extent of relevance to our concerns has to govern our efforts in these regards. “Keep your discussions relevant” is a key principle of communicative rationality.

The items at issue with relevance fall into the range of thought-includes: ideas, conceptions, theses, contentions, and the like. Relevance is a matter of informative communality or overlap. When one has in view information regarding some theme or topic, and yet another item comes to hand the first questions is “Is it relevant?,” and then of course the second becomes: “Does it add anything new?” The former of these questions is a matter of relevance, the latter one of epistemology, broadly construed.

Relevance requires a body of information to serve as a referential context. You can only ask whether A is closer than B relative to some pre-given reference point (be it “here” or elsewhere, e.g. “Paris”). And you can only ask whether A is relevant to

B relative to some pre-given body of information (be it “ours” or someone’s else’s—actual or hypothetical).

Only a body of agreed fact can resolve questions of relevance. Does the phase of the moon affect the suicide rate? Such issues of causal relevancy can only be resolved on the basis of the manifold of accepted knowledge. Background knowledge provides the key to relevance. Thus when casting one particular die the propositions

P: the outcome is 1

Q: the outcome is 2

are totally relevant to one another, albeit negatively, since the one precludes the other. But when casting two dice, *P* and *Q* are totally irrelevant: the one having no evidential differential bearing whatever on the other.

Within any given body of fact there will be only a small subsector that is relevant to a given theme or topic—and especially so if it is a high degree of relevance that we require. And relevance is ambidextrous, as it were, cutting across the theoretical divide between causal and conceptual connection.

What is relevant is a principally factual (empirical, contingent, a positivity) matter and is generally not really a logical (purely theatrical, conceptual, a priori). The connecting list that makes *A* relevant to *B* will usually and for the most part be a matter of contingent fact.

Thought-items are rendered reciprocally relevant to one another by a commonality of comprehension-formulating aspects. Thus, one item is relevant to another insofar as it provides material conducive to its comprehension, explanation, description, production, and the like.

However, relevance is crucial for inductive reasoning. For consider such and inference as

Chicken eggs have yokes

Pigeon eggs have yokes

Robin eggs have yokes

All bird eggs have yokes

All of the premisses in such a formula of substantial induction have the format

X-type bird eggs have yokes

And it is crucial that all the *X*’s here maintain a uniformity of relevance: they must be of the same kind at issue in the conclusion. (Thus only bird eggs are to be involved, with turtle eggs or alligator eggs presently not at issue.) On this basis mutual relevancy is a crucial factor for induction: there is not valid induction without it.

7.2 Putative Versus Actual Relevance

It would be convenient if relevance among theses could be determined on purely mechanical principles: if mere overlap in the use of words or the mention of items could do the job adequately. But unfortunately this will not do.

There is not and cannot be any automatic, algorithmic way of determining whether *X* is relevant to *Y*. For as already noted, relevance is only determinable on the basis of contextual background information and this is something indeterminate and often as yet unavailable. Often as not relevance is a matter of judgment calls, and it is frequently only with the judgement of hindsight that relevancy can be determined. Moreover, people just are not informatively equal. What the expert deems relevant may elude the novice—and conversely. Accordingly, the assessment of relevance will depend critically on the belief system at of the agent at issue

The progress of science and scholarship produces a landscape of shifting sands, and new information provides new perspectives. Relevance between facts can become lost (as when the demise of astrology severed the linkage of astronomy and public affairs).

And in this context it is necessary to stress that distinction between relevance that is real and objective and relevance that is merely apparent and subjective. The former rests on actual connections, the latter on putative ones. (But of course we have no access to the actual stake via what we think to be so!)

The distinction at issue becomes clear by contemplating superstition. In actual fact encountered black cats or underpassed ladders are irrelevancies without effect on people's fate and fortune. But in the realm of personal expectation and belief the matter stands very differently. In commonplace belief and practice where people's conceptions and misconceptions regarding nature's ways are of paramount importance, putative rather than actual relevance stands in the foreground.

In their interesting and instructive book on relevance by Dean Sperba and Deirdre Wilson¹ proceeded in the premiss that "there is an important psychological property . . . of natural processes to which the ordinary notion of relevant roughly approximates" and that this is the "useful theoretical concept" to which their book is developed. However this conception of relevance as a useful psychological property is not at issue in present deliberations. For the relevance presently at issue is a matter not to psychological proceedings and reactions, but of the conceptual relationships among objectively constituted objects of deliberation.

And so for present purposes this otherwise important of the psychology and sociology of relevance can be left aside and its objective dimension taken to be paramount.

7.3 Thematic and Alethic Relevance

Cognitive relevance has two main forms: *alethic* (or *propositional*), and *thematic* (or *topical*). The former—alethic relevance—can be assessed by looking to the percent of *A*-obtaining cases and circumstances where *B* also obtains (is true). The latter, thematic relevance, can be assessed by looking to the percent of *A*-referring) case and circumstances where *B* is also referenced.

Insofar as probabilistic reflection in general look to *A*-obtaining cases where *B* also obtains, both forms of relevancy are probabilistic in effect. For in practice both are assessed by means of consideration that take just this probabilistic format. In this respect, relevance is like causality, seeing that *A* is likely to figure among the causes of *B* when *B* is common where *A* is the case, and *B* is rare when *A* is absent. Correlation serves as a determinative consideration in both cases alike.

By contrast, statement *P* is *alethically* relevant to *Q* whenever *Q*'s truth is inferentially or evidentially connected to *P*'s truth thanks to a commonality of truth-making circumstances. By contrast, the *thematic* relevance of one statement to another turns on the extent to which the former involves claims about objects or types of objects that are also at issue in the latter.

By Contrast, “The Eiffel Tower is in Paris” and “The Louvre is in Paris” are thematically relevant since both pertain to “being a structure located in Paris.” They are, however, alethically irrelevant seeing that the truth of one has no link (be it referential or evidential) to that of the other.

The alethic relevance of *P* to *Q* determined by the ratio of the proportion of cases where *P* obtains when *Q* does so:

$$\text{rel}(P/Q) = \#(Q/P) \div \#(P)$$

The alethic relevance of one statement to another is something contextual: it may not be determined from the substance of this statement, but can require reference to claim-extraneous information.

Moreover, it is important to distinguish between what *relevance* (of some given sort) means and how we *test* for it, between definite and criteriology. Search engines are a great help in the latter regard but we must not forget that the former issue is fundamental.

Lexical commonality does not establish alethic relevance. The same words can occur in totally irrelevant contentions. Thus “Dogs belong to the species Canine” and “Islam prohibits the eating of dogs” are both (in a way) about dogs, but are entirely irrelevant to one another. And two statements can both relate to the same item (say the planning process for constructing the Eiffel Tower and the all-male staffing of its construction workers) and yet turn out to be substantially irrelevant to one another.

Verbal and/or substantive commonalities may make relevance likely, but cannot establish it. For mere commonality of mention is not enough to establish relevance. Thus although both mention *F(X)*, *F(X)* and the following contention are not relevant

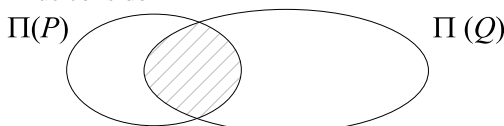
to each other: “ $F(X) \vee (2 \times 2 = 4)$.” For this statement holds true on the basis of $2 \times 2 = 4$, and really provides no information about X .

In utilizing search engines, the topical relevance of a document is assessed by means of key-term frequencies. But, there are problems here. For instance, in employing mention-statistics to assess thematic relevance with encyclopedia references, it is clear that fashion and notoriety can constitute distorting factors.

7.4 Propositional Relevance as Basic

* * *

The proposition P is cognitively relevant to Q to the extent that the range of possibility affiliated to A —which we will symbolize as $\Pi(A)$ —occupies the range affiliated to Q . Thus consider



Here the shadowed overlap region occupies some 20% of $\Pi(Q)$. So P is relevant to Q to that extent. However, the larger relevance of Q to P stands at some 50%.)

In general we have:

$rel(P/Q) = [\Pi(P \& Q) \div \Pi(Q)]$ where $[\Pi(X)]$ measures the size of $\Pi(X)$. The overlap regions are empty, there would be total irrelevance; if it were exhaustive there would be total relevancy.

Relevance also descends from propositions to items. One item A is relevant to another B to the extent that the identifying description which characterizes the one viz. $C(A)$ is propositionally relevant to that of the other $C(B)$.

However, the matter extends beyond group-based relevance. For the connectivity at issue with relevance covers a wide range of different sorts of items: propositions, claims, evaluations, themes, topics subjects, ideas, lines of thought, possibilities, facts, states of affairs, etc.—all issues whose size is not really measurable.

To be sure, logical implication can serve to establish propositional relevance: If $P \rightarrow Q$, then P is relevant to Q . Thus “The Eiffel Tower is in Paris” $\rightarrow$ “The Eiffel Tower is in France.” Location in Paris is highly relevant to location in France. But reverse, of course, is not the case. If it is established that you are in Rouen, this ensures that you are in France. But it sets the likelihood of your being in Paris at nil. However, in general relevance is a matter not of logical entailment but of probabilistic connection with co-variation as the crux of relevance. And so for present purposes we can focus on a positive relevance, propose to focus on this mode.

7.5 Evidential Relevance

Evidential relevance can also be reduced to propositional relevance via the understanding that

- The evidential relevance of a new item of evidence (E) to the contention (C) relative to the previously prevailing information I is simply the difference

$$\text{rel}_1(E/C) = \text{rel}(C/I \& E) \div \text{rel}(C/I)$$

Such evidential or probative relevance can be either positive or negative. With positive relevance P renders Q more likely, with negative less so. Thus satisfaction and content have a positive bearing upon a person's health; dissatisfaction and stress a negative bearing. Attentiveness increases the likelihood of accident-avoidance; distraction decreases it. However, negative bearing is simply positive bearing upon the negation.

In the presently prevailing sociological conditions a person's gender is certainly relevant to their profession. To be sure, there is no lock-step determinism here. There certainly are female merchant-ship captains and female fighter pilots; but only relatively few of them. Being an X is relevant to being a Y to the extent that the percent of X 's are Y s: if most X s are Y s, being an X is positively relevant to being a Y ; if less than half, negatively relevant, if 90% highly probably relevant; if 10% highly negatively so.

Evidential Relevance can thus be a largely statistical matter. Is your cat ownership relevant to your owing a dog? It all depends on your belonging to a groups most (or fewest) of whose cat-owing members are also dog owners. This item of background information is crucial for relevance here (both as to direction and degree).

The relevance of claims and propositions is generally grounded in nomic or lawful relationships of some kind (as when the amount of rainfall is relevant to the size of crops.) Thus it required considerable contextual knowledge to establish that a person's finger prints are relevant to his identity. However, the mere possession of 10 fingers is not particularly relevant for identification (unless it happens that the individual we are concerned with is known to have lost some of his fingers). One's income is relevant to the size of one's dwelling, but one's body temperature is not.

The meaning of "relevance" in U.S. law is stated in Rule 401 of the Federal Rules of Evidence via the specification that the relevance of information given is evidence consists in its

having any tendency that is of consequence in rendering the determination of the action more probable or less probable than is would be without the evidence

This specification amounts to what our previous exposition has termed *evidential relevance*, so that evidence given in support of a claim is relevant where it renders this claim more (or less) likely than it would otherwise be. This specification effectively replicates the position of J. M. Keynes who in his classic *Treatise on Probability*

(London: Macmillan, 1921), proposed assessing the (positive or negative) extent to which a new item of evidence e is relevant to a proposition p in the information context the initial information complex I by the difference between the posterior probability and the prior probability.

Keynes accordingly proposed that:

New evidence (e^+) is relevant to a proposition (p)—whose probability relative to the earlier, preexisting body of evidence (e^-) was $pr(p|e^-)$ —to the extent that the new probability changes this status quo ante:

$$Rel(e^+/p) = pr(p|e^+) - pr(p/e^-)$$

Thus the relevance is great when this posterior probability is substantial and the prior probability small. However, it is zero when e makes no contribution whatever to c 's determinable probability; and it is negative when it detracts from c 's prior probability.

7.6 The Systemic Integrity of Fact

Classical logic addresses relevance only with respect to two extreme possibilities: full inclusion (implication) or total exclusion (contradiction). The weaker modes of relatedness that are at issue with relevancy remain outside its purview. And from another point of view the matter is even worse.

Ironically from the standpoint of classical logic, any fact F_1 is alethically connected with any other fact F_2 by deductive interlinkages. Thus consider the following line of reason:

- (1) F_1 is a true fact (by hypothesis)
- (2) F_2 is an established fact (by hypothesis)
- (3) $\sim(\sim F)$ follows from (1)
- (4) $\sim F_1 \vee F_2$ follows from (2)
- (5) F_2 follows from (3) (4).

As far as classical logic goes, this effectively renders inferential truth-relatedness ("alethic relevance") trivial and thereby failing to establish any meaningful mode of connection.

Just this situation led to the development of so-called "relevance logic" in whose development former Pittsburgh colleagues Alan Anderson and Nuel Belnap played a central role. They saw it as critical for relevance logic that a condition of *variable sharing* obtains, so that $A \rightarrow B$ holds in such a system only when these two propositions both contain a free variable in common. This variable sharing principle (VSP) doubtless holds for certain systems of formalized logic. It does not, however, hold throughout wider contexts of deliberation. Thus the relevant implication ("Most As are Cs & Most Bs are Cs) $\rightarrow$ (Something exists)" obtains since its consequent is

entailed by $(\exists x)(Ax \& Bx)$. But in this valid entailment the conclusion does not share a variable with the antecedent.

But even beyond this formal issue of logico-propositional relevant-interconnection, then lies the consideration that given the structure of fundamental physical law as we have it, the universe is an incredibly fine-tuned, system of such a kind that if its fundamental forces (gravitation, nuclear binding force, etc.) were even the least bit different, a world of stable objects could never have developed.² Reality, it is fair to say, is a *chaotic* system in the physicists' sense that every hypothetical change in the physical make-up of the real—however small—sets in motion a vast cascade of further such changes either in regard to the world's states or in the laws of nature. The causal relevancy of physical process is all-pervasive within the bounds of *a/the* universe.

Thus suppose that we make only a very small alteration in the descriptive composition of the real, say by adding one pebble to the river bank. But which pebble? Where are we to get it and what are we to put in its place? And where are we to put the air or the water that this new pebble displaces? And when we put that material in a new spot, just how are we to make room for it. And how are we to make room to the so-displaced material. Moreover, the region within six inches of the new pebble used to hold N pebbles. It now holds $N + 1$. Of which region are we to say that it holds $N - 1$. If it is that region yonder, then how did the pebble get here from there? By a miraculous instantaneous transport? By a little boy picking it up and throwing it. But then, which little boy? And how did he get there? And if he threw it, then what happened to the air that his throw displaced which would otherwise have gone undisturbed?

Here problems arise without end. For the more deeply one examines how things work in the world, the more one comes to realize the limitless interconnectedness of things and the pervasiveness of the coordination of everything that goes on with everything else. The consequences of any change in the arrangement of the real is unfathomable in nature and extent. If that cup were at present located there rather than here it would change not only the configuration of my body, but the status of gravitational, thermal, and electromagnetic interaction that would unify throughout the universe. And accounting for how and why it comes to be so different would lead to stories of unending length and complexity.

The implications of those changed arrangements would ramify throughout space and time diffusing their consequences throughout the totality of things. And this endows nature with an interwoven fabric where the severing of any thread unravels the whole with results and consequences that are virtually impossible to discern in advance. In the universe as we see it, everything is carefully bound up to everything else. A web of reciprocal relevance encompasses all. The universe is a plenum of causal interconnectedness.

* * *

Overall, then, the present deliberations carry four main lessons:

- A comprehensive philosophy of science that addresses both its claims and their validating rationale must also have a concern for issues of relevance.
- This methodological commitment unseparably links the philosophy of science to issues of general epistemology.
- Such metascientific matters cannot be resolved by intra-scientific (i.e., science-internal) means. They are part of what makes the philosophy of science into an inevitably philosophical enterprise.
- On virtually any given issue, the range of relevance expands with the growth of knowledge, and in the end renders the management of relevant information a challenge of Sisyphean scope.

7.7 Relevancy Limits and Diminishing Returns

The final part deserves special emphasis. With the progress of science and the expansion of learning, the tentacles of relevance reach ever further across the range of information.

In modern times the growth of publications and information in every branch of science and scholarship has been exponential. And as the body of information as a whole grows exponentially, the volume of relevant information in a given topic is also bound to grow on average to an exponential (even if lesser) rate. The task of information management becomes increasingly daunting as more ramifications and interconnections come to view.³ And yet this expansion of knowledge is not an unmixed blessing. With the growth of the scientific and scholarly literature, the tower of learning has become increasingly unsteady in the wake of accommodating growth. In a way, cognitive progress is its own worst enemy; building up higher and higher obstacles to further progress until we apparently approach a situation where the development of knowledge eventually collapses under its own weight. The result has been a vast increase in specialization and division of labor, with a consequent fragmentation of knowledge. For the individual, the achievement of expertise becomes increasingly difficult. And this holds also for the community with respect to the advancement of significant knowledge. Extension exceeds a case in comprehensibility.

Notes

1. *Relevance: Commemoration and Cognition* (Oxford: Blackwell, 1986; 2nd ed., 1995).
2. See Martin Rees, *Just Six Numbers* (New York: Basic Books, 2000).
3. For details see N. Rescher, *Scientific Progress* (Oxford: Blackwell, 1976).

Chapter 8

The Logic of Knowledge Distribution



8.1 Knowledge Quantity

Different people, of course, have different knowledge, varying from one another in regard to what is known to them.

To clarify matters, let us use x, y, z , etc. for finite knowers, and f, f_1, f_2 , etc. for facts (true claims of something) employing formulas of the format Kxf to mean “the individual x knows the fact f .” (Of course, one can only actually know what is true; otherwise, one merely *thinks* that one knows. So, $(\exists x)Kxf \rightarrow f$).

The range of our knower-variables x is to extend over human—and thus finite—intelligences. If (contrary to this stipulation) God were within the range, we would have the otherwise false thesis:

$$(\forall f)(f \rightarrow (\exists x)Kxf)$$

Not only will certain facts not be known to particular individuals—as per $\sim Kxf$ —but certain facts are actually unknowable to an individual as per $\sim \Diamond Kxf$. (For example, no-one can know that f is a fact they do not know, seeing that $Kx(f \ \& \ \sim Kxf)$ is self-contradictory.)

The realm of *actual* facts is, of course, transdenumerable, seeing that there are unique facts about each and every real number. However, only *specifiable* facts can possibly be known specifically and individually. And their range is going to be denumerable in view of the recursive nature of human languages. This too means that there are some facts that will never be known to anyone:

$$(\exists f)(\forall x) \sim Kxf$$

It does not, however, mean that there are some facts that cannot be known, facts too complex and recondite for access to human knowers:

$$(\exists f)(\forall x) \sim \Diamond Kxf \text{ or equivalently } (\exists f)(\forall x)\Box \sim Kxf$$

Still, this thesis is likely true. Think here of the aforementioned omnifact. However, consider such variant theses as:

- (a) $(\exists f)\Diamond(\forall x) \sim Kxf$ some facts are possibly unknown (to everyone).
- (b) $(\exists f)(\forall x)\Diamond \sim Kxf$ some facts are as such that anyone may possibly

or equivalently not know them. Or there are some facts such $(\exists f) \sim (\exists x)\Box Kxf$ that no-one necessarily knows them.

Both (a) and (b) are true, seeing that they follow from the true $(\exists f)(\forall x) \sim Kxf$: “There is a universally unknown fact.”

It is also instructive to introduce the majoritative quantifier M with $(Mx)Fx$ to be construed as “Most items of the x -range at issue have the property F .”¹ Use of this mostly-quantifier M allows for the articulations of the great many true contentions such as $(Mx)(\exists f)Kxf$: “Most people know something.” This, of course, will be true and indeed follows from the truth of $(\forall x)(\exists f)Kxf$: “All people know something.” But it differs from $(\exists f)(Mx)Kxf$ which claims what is true generally (albeit not necessarily universally). (There clearly are such facts as, for example, “The earth is not flat.”).

Difficulty arises, however, with theses involving (Mf) because the individuation and counting of facts becomes problematic. It would, in fact, seem that a thesis of the format $(Mf)Zf$ will never be true unless $(\forall f)Zf$ obtains.

Interesting, however, is the thesis:

$$[(Mx)Kxf_1 \& (Mx)Kxf_2] \rightarrow (\exists x)Kx(f_1 \& f_2)$$

which holds, but would fail to do so if the existential quantifier $(\exists x)$ in the consequent were changed to (Mx) .

8.2 Any Versus Every

In matters of knowledge, as elsewhere, there is an important difference between any and every.

The somewhat inebriated customer who proposed “To fight *any* man in the house” is certainly not offering to fight *every* man in the house taken together (and likely not even taken sequentially). Note that “every” in and of itself is indefinite as to order and thus invites further specification. To promise “A chicken in every pot” taken simultaneously is quite generous, but “at some time or other” offers far less. The cruise line that assures that “Any passenger can be accommodated in our lifeboats” does not sound quite as safe as the one assuring that *every* passenger can be so accommodated.

Display 8.1

ANY VS. EVERY GENERALIZATIONS

Distributive Generality

any candidate can win

any day next year can rain

eating any item on the menu

any student can outperform the rest

any member of the group fits in the car

any player at Musical Chairs can be seated

Collective Generality

every candidate can win

every day next year can rain

eating every item on the menu

every student can outperform the rest

every member of the group fits in the car

every player at Musical Chairs can be seated

Accordingly, as the contrasts at issue in Display 8.1 illustrate, there is a significant difference between distributive and collective versions of possibilistic generality, between what any members of a group might be and what every one might. Careful attention to that logical form of assertion clarifies the difference at issue. Thus, “Any *A* can possibly be a *B*” takes the distributive format:

$$(\forall x)(x \in A \supset \Diamond x \in B)$$

While “It is possible that every *A* can be a *B*” takes the collective format:

$$\Diamond(\forall x)(x \in A \supset x \in B)$$

The placement of that possibility operator ($\Diamond$) makes a substantial difference in what is being said. The second implies the first ($\Diamond$ every $\rightarrow$ $\Diamond$ any) but as Display 3 shows, the reverse is certainly not the case. (The difference tracks that of the medieval distinction between *necessitas consequentiae* and *necessitas consequentis*.)

This difference at issue here becomes especially significant in cognitive contexts. This emerges when we compare:

Someone might know any fact (about something) $(\exists x)\Diamond(\forall f)Kxf$

Someone might know every fact (about something) $\Diamond(\exists x)(\forall f)Kxf$

Someone can know any fact (about something) $(\forall f)\Diamond(\exists x)Kxf$

Someone can know every fact (about something) $\Diamond(\forall f)(\exists x)Kxf$

Let us consider the relationship between:

$$(1) (\exists x)\Diamond Kxp$$

and:

$$(2) \Diamond(\exists x)Kxp$$

The inference from (2) to (1) does not hold. Outside theology at least, one can never infer actual existence from the merely possible. (*De posse ad esse non valet cosequentia.*) On the other hand, the inference from (1) to (2) is clearly valid. Note, however, that the inference from (2) to p is a valid one. Only what is true can possibly be known.

These considerations mean that the inference from (1) to p also obtains so that:

$$(1) \rightarrow (2) \rightarrow p$$

We thus have it that:

$$(\exists x)\Diamond Kxp \rightarrow p \text{ or equivalently } \sim p \rightarrow (\forall x)\Box \sim Kxp \text{ or equivalently}$$

$$p \rightarrow (\forall x)\Box K * xp \text{ (when } K * xp = \sim Kx \sim p)$$

If p is true, then this must be so for all anyone knows to the contrary (which is what $\sim Kx \sim$ asserts).

8.3 Cognitive Incompleteness

But what about universal ignorance? With finite knowers we will surely have $(\forall x)(\exists f) \sim Kxf$ or equivalently $(\forall x) \sim (\forall f)Kxf$. This too must be accepted, since, as noted above, no one knows *of a particular* fact that he does not know it.

But now consider the variant situation where our individual variable x ranges not over persons but rather over propositional systems, with knowledge now taken to be a matter of systemic provability, so that $\vdash_x f$ (that is, “ f is demonstrable in the system x ”) stands in place of Kxf .

There is no reason to think that the acceptability situation (as regards True/False) that emerges in the preceding deliberations is now any different. And if this is indeed so, then the following thesis holds true:

$$(\forall x)(\exists f) \sim Kxf \text{ or equivalently } \sim (\exists x)(\forall f)Kxf$$

With axiomatized systems of formal demonstration, just as with personal knowledge systems, a condition of cognitive incompleteness must be expected to obtain. Now, one can be a “know-it-all.”

To see that this is so, consider that as long as the claims affirmed in a cognitive system are formulated linguistically in recursive articulation, there will be at most a denumerable number of them. And this will, of course, also hold for the subset of *demonstrable* claims. So, now consider the (infinite) inventory of all these demonstrable claims: F_1, F_2, F_3 , etc. But note that the omnifact—the conjointly compiled totality of all of the true facts—can itself have no place in this series.

Accordingly, it will represent improvable/indemonstrable truth. It follows that no deductive systemization of facts can possibly be complete in encompassing all of them.

8.4 The Distribution of Knowledge

In regard to the distribution of knowledge, very different sorts of situations can, of course, obtain. In this context, it is interesting to consider the following array of contentions:

- | | | |
|-----|--|---------|
| (1) | $(\exists x)(\exists f)Kxf$: Someone knows something | (True) |
| (2) | $(\exists x)(\forall f)Kxf$: Someone knows everything | (False) |
| (3) | $(\forall x)(\exists f)Kxf$: Everyone knows something | (True) |
| (4) | $(\forall x)(\forall f)Kxf$: Everyone knows everything | (False) |
| (5) | $(\exists f)(\exists x)Kxf = (1) \text{ above}$ | (True) |
| (6) | $(\exists f)(\forall x)Kxf$: Something is known by everyone | (True) |
| (7) | $(\forall f)(\exists x)Kxf$: Everything is known to someone | (False) |
| (8) | $(\forall f)(\forall x)Kxf = (4) \text{ above}$ | (False) |

Here, (7) is false, since for any given individual there is some fact that they do not know (no-one is omniscient) and then the *conjunction* of all these facts is a fact no-one knows.

The negation of (6) is:

$\sim(6) \quad (\forall f)(\exists x) \sim Kxf$: Every fact is unknown to someone

Since there are some absolutely obvious and “evident” facts—“ $1 + 1 = 2$ ” for example, or “Everything is self-identical” or “Nothing can be larger than itself”—it would seem that some facts are universally acknowledged.

As regards (1) to (8), observe that only and exactly those theses containing $(\forall f)$ are false. In this range, no true universal generalizations regarding facts are available. Facts are cognitively diversified. Only if we resort to dual possibilization as per the following can we achieve a plausible truth:

$\Diamond(\exists x)(\forall f)\Diamond Kxf$: It is possible that there is someone to whom any fact can be known.

And note that this is very different from:

$\Diamond(\exists x)\Diamond(\forall f)Kxf$: It is possible that there is someone to whom every fact is known

which is untenable in the realm of finite knowers.

Note that nothing changes with regard to tenability if we adjoin the possibility operator ($\Diamond$) to the front of those previous eight theses. Their truth-status combines axioms if we adjoin the possibility operator ($\Diamond$) after the first initial quantifier. We then obtain:

- (9) $(\exists x)\Diamond(\exists f)Kxf$: Someone might know something (True)
- (10) $(\exists x)\Diamond(\forall f)Kxf$: Someone might know everything (False)
- (11) $(\forall x)\Diamond(\exists f)Kxf$: Everyone might know something (True)
- (12) $(\forall x)\Diamond(\forall f)Kxf$: Everyone might know everything (False)
- (13) $(\exists f)\Diamond(\exists x)Kxf$: Something might be known by someone (True)
- (14) $(\exists f)\Diamond(\forall x)Kxf$: Something might be known by everyone (True)
- (15) $(\forall f)\Diamond(\exists x)Kxf$: Anything might be known to someone (False)
- (16) $(\forall f)\Diamond(\forall x)Kxf$: Anything might be known to everyone (False)

Here, (10) is false because no-one (save God) can possibly be omniscient; (14) is false *a fortiori*; and (16) is false for the same reason as (7) above.

Here, too, our previous finding continues in place: All (but only) those theses not involving ($\forall f$) are true.

Let us next consider the situation when the possibility operator ($\Diamond$) is inserted after all the initial quantifiers, just before the knowledge claim itself:

- (17) $(\exists x)(\exists f)\Diamond Kxf$: Someone might know something (True)
- (18) $(\exists x)(\forall f)\Diamond Kxf$: Someone might know everything (False)
- (19) $(\forall x)(\exists f)\Diamond Kxf$: Anyone might know something (True)
- (20) $(\forall x)(\forall f)\Diamond Kxf$: Anyone might know everything (False)
- (21) $(\exists f)(\exists x)\Diamond Kxf$: = (17) (True)
- (22) $(\exists f)(\forall x)\Diamond Kxf$: Everyone can know a certain fact (True)
- (23) $(\forall f)(\exists x)\Diamond Kxf$: Any fact can be known to someone (False)
- (24) $(\forall f)(\forall x)\Diamond Kxf$: = (20) (False)

Both (18) and (20) are different versions of the equivocal (but always false) contention that “Someone might know everything.”

But, of course, unknowing is something else again. Thus, consider “Any and every fact is possibly unknown to someone,” symbolically:

$$(\forall f)\Diamond(\exists x) \sim Kxf \text{ or equivalently } \sim (\exists f)\Box(\forall x)Kxf$$

This negative variant of (15) is perfectly in order.

Observe that (23) should be deemed false for much the same reason as the problematic (6) above. The negative of (23) is:

$$\sim(23) \quad (\exists f)(\forall x)\Box \sim Kxf$$

This will, of course, be true since in a world of finite intelligences no-one can possibly know the omnifact consisting of the collective amalgamation.

8.5 A Conjecture

Observe that throughout the preceding analysis all the theses considered were systematically false when having the format $\sim(\forall f) \sim Kxf$, where the blanks contained some mixture of $(\exists x)$, $(\forall x)$, and $\Diamond$. On this basis, our previous deliberations suggest that no universal generalizations regarding the factual knowledge of facts can possibly be true. Within the range under consideration, universalizing with regard to the distribution of factual knowledge invariably goes wrong.

To render this plausible, consider what would happen if:

$$\sim(\forall f) \sim Kxf \text{ were true}$$

The weakest version of such a thesis to satisfy this requirement is:

$$\Diamond(\forall f)(\exists x)Kxf$$

But in view of the aforementioned consideration that there are more facts than can possibly be known in a universe of finite knowers, this thesis is untenable.

As long as the range of specifiable knowers is at most countably infinite, there is a corresponding limit for the possible knowers at issue with $\Diamond\exists x$. But the range of fact transcends countably. And so, with more facts than possible knowers, some facts must remain unknown (much as with the game of Musical Chairs). Thus, even the weakest version of our thesis format proves to be untenable and, in consequence, the whole range is bound to be so.

It would appear that the realm of fact is indeed so variegated and recondite that no universal generalization regarding knowledge about it is ever warranted.²

Notes

1. This should be construed as holding that the cardinality of the set of F s is greater than that of the set of not- F s.

2. This chapter builds on considerations in the modal logic of knowledge set out in the author's *Epistemic Logic* (Pittsburgh: University of Pittsburgh Press, 2005). The lines of thought at issue go back to Frederic B. Fitch's classic paper on "A Logical Analysis of Some Value Concepts," in the *Journal of Symbolic Logic*, vol. 28 (1963), pp. 135–42.

Part III
Issues of Paradox and Cognitive
Incompleteness

Chapter 9

Cognitive Reflexivity and Objective Knowledge



9.1 Factual Knowledge and Its Modes

What do we know in the way of general principles regarding our knowledge itself? Cognitive reflexivity—knowledge about cognition itself—is a critical issue for the philosophy of knowledge.

In reasoning about cognitive matters one must begin with acknowledging that knowledge has many modes. However, the knowledge that concerns us here is specifically informative (or “factual”) knowledge. “Practical” or “performative” knowledge is not on the agenda). In particular one must differentiate among various modes of factual knowledge:

- *Immediate knowledge.* The information that is “at the tip of one’s tongue,” so to speak, relating to questions which one can answer “in a flash.”
- *Available or “potential” knowledge.* The facts one can come up with some effort, relating to questions one should be able answer (on one’s own!) upon giving the matter thought though possibly a good deal of it.

For of course one’s knowledge need not be immediately accessible to conscious awareness. One can know something without being able to produce the information on the spot, “I know that man’s name [or “I know the French for caterpillar”] but just can’t bring it to mind at the moment, it will come to me shortly”—still exemplify modes of factual knowledge.

When one knows something, does one also know its consequences? Do we have it that if someone knows something they thereby also know its logical consequences:

$$(Kxp \ \& \ [p \vdash q]) \rightarrow Kxq$$

It should be clear that this idea of “inferential obviousness” only holds with respect to *potential* (available) knowledge. It would be unrealistic to ask for immediate access to the totality of the inferential commands of what we know.

Can one know something that one has never even considered, something entirely outside the range of one’s awareness? With respect to potential knowledge the answer has to be yes. It may never have occurred to you that if you had four more sisters you would have five in all—you who have just one sister never looked at the matter in this light. And yet it is true as you would instantly acknowledge it if asked. The shift from immediate to potential knowledge opens the floodgates.

9.2 Error

Only the truth can be known. Actual knowledge must be error free.

$$Kxp \rightarrow p$$

But it is not because we are so smart that our knowledge cannot err. Rather it is because one could not meaningfully say that someone knows what is false, but only that they (mistakenly) *think* they know. For of course an agent be wrong/mistaken/in error regarding his cognitive condition. He can mistakenly think that he knows

“I know that p is so but might be wrong about it.” This just doesn’t make sense. Only facts can be known ($Kxp \rightarrow p$): in claiming to know that something is so, one claims this as a fact. Knowledge—actual knowledge—cannot go wrong. We can, of course, mistakenly *think* something to be known that is in fact false: but if indeed known then it has to be so. Our knowledge cannot be mistaken because if it were, it just could not be so characterized. We could not appropriately term something as known that turned out false; we would have to say that they thought they knew it but were mistaken.

In deliberating about knowledge one must also consider its cousins such as belief and other cognitive inclinations. And as Display A indicates, cognitive stances and dispositions can be both positive and negative.

Display A

COGNITIVE STANCES AND DISPOSITIONS

<u>POSITIVE</u>	<u>NEGATIVE</u>
Know (K)	Be unknowing or ignorant regarding] (a)
Believe (B)	Disbelieve (X)
Accept (A)	Reject (R)
Incline to accept (deem plausible) (I)	Doubt (D)
Suspend judgment (S)	$\sim A$ & $\sim R$

NOTE: An analogous duality obtains with respect to action and evaluation: performance (do/refrain) and preference (favor/reject)

Of course acceptance does not constrain truth: when we believe ourselves to know something we may well be wrong. We certainly do not have it that $BxKxp$ or $AxKxp$ entail that actually Kxp . As in all other objective matters one can err regarding one's knowledge: here as elsewhere error is possible.

Mere belief lacks the objective import of knowing: We have $Kxp \rightarrow p$ but not $Bxp \rightarrow p$ or $Axp \rightarrow p$. Nor do we have $BxKxp \rightarrow Kxp$. But we do have $BxBxp \rightarrow Bxp$.

This raises a questions. Can the agent be deluded about his subjective cognitive condition? Or are agents themselves always the decisive arbiters regarding their own cognitive stances? Is it possible that someone

- mistakenly think he believes
- mistakenly think he doubts
- mistakenly think he accepts.

Throughout the answer is negative. For in all such purely subjective matters the agent is indeed the final authority. While the objective " X is the case" is always fallible, such merely subjective avowals as "I am under the impression that X is the case" (or "I believe that X is the case") are, when candid, totally secure. There are innumerable problematic issues about which we can be mistaken, but our own subjective cognitive stances are not among them.¹

9.3 Knowledge Reflexivity

When one takes oneself to know a fact, this may certainly fail to be so. But *knowing* that I know a fact is something else again. This leads to the so-called *KK* thesis, namely

$$Kxp \leftrightarrow KxKxp$$

However, this holds only with respect to *available* knowledge. Whenever the facts at issue are in fact available realizing that this is so is itself always available and in prospect. Availability is itself always available: Whenever I can come to realize something, I can also come to realize this circumstance itself.

However when an agent actually knows something, that has to be the case. But what renders knowledge error-free is not that knowers are so smart but simply that we would not and will not speak of knowledge with regard to erroneous matters.

Note that the reflexivity of the *K* theses to the effect that $K \rightarrow KK$ will itself yield $KK \rightarrow KKK$ and thereby $K \rightarrow KK \dots K$. Is this not a problem? No—not as long as we deal in potential *K* rather than giving *K* an active and immediate construal.

And we can go well beyond this. For whenever *C* is itself a cognition-involving condition then one can know of its realization. For every sort of “cognitive stance”—be it positive or negative—is reflexively knowledge-demanding so that

$$Cxp \rightarrow KxCxp$$

And this will hold not only for knowledge for belief as well ($C = B$) and indeed also for negative stances, such as rejection (*R*) or doubt (*D*) or uncertainty (*U*)? But of course all this holds only when those cognitive stances are construed potentialistically.

9.4 Subjectivity/Objectivity

Cognitive reflexivity calls for being aware of our own cognitive operations. Is this realistic? Two sorts of “awareness” are at issue here:

Subjective awareness: *personally think* something to be so

Observative awareness: *correctly think* something to be so

It deserves stress that there just is no valid transit from the subjective order of what is thought to be so to the objective order of what actually is so UNLESS what is at issue itself lies in the subjective-order of what merely is thought to be so. We ourselves are the decisive arbiter with regard to our own thought. Granted, when I believe that the cat is on the mat, I am indeed open to correction as to the cat and the mat—but not as to the belief itself. I can be incorrigibly certain as to what it is

that believe of accept. (But not of course *know* since this reaches outside the range of subjectivity.).

This circumstance does not obtain because I am so knowledgeable a person. Rather, this autodecisiveness obtains not for cognitive but only for strictly linguistic reasons. For it just would not be correct to claim that I believe something if it were not in fact the case.

- Levels of Thought

We humans are able to think about Reality at two levels. First there is

- I. *The level of immediacy*. This relates to *what actually is the case* regarding Reality. Here we address what take to be matters of objective fact.

But there is also

- II. *The level of reflexivity*. This relates to matters *regarding* what is thought about what is the case regarding Reality. Here we address (what we take to be) our thought about things. [Subjectivity makes that parenthesis dispensable!]

Two key considerations come to the foreground here:

- We have no cognitive access to I save via II, that is, we have no access to the facts save via what we take them to be (i.e., what we *think* they are).
- Our situation is such that we cannot avoid or eliminate the prospect of error in our thoughts about the objective facts, both as regards omission and commission.

When we take account of the difference in levels with respect to the status of our knowledge we realize that we cannot but distinguish between

- the *objective* issue on nonevidence with respect to the facts as they are
and
- the *subjective* issue of reflexivity regarding the facts as we think them to be

The difference is substantial, in theory, but set in practice we have no way of getting at the former save via the mediation of the latter.

At the subjective level of deliberation I stand committed to my correctness regarding particular specifics—that is to accepting as fact that what I think to be is so:

$$Tip \rightarrow p$$

But neither at the level of generality $(\forall p)(Tip \rightarrow p)$ now that of objective factuality is this acceptable. How is this paradox to be resolved?

The answer lies in separating those just-mentioned spheres of theory and practice. With duality we need in dual perspectives. In practice (in matters of practical reasoning and procedure), I accept *my* truth as *the* truth. In theory (in matters of abstract principle and theoretical generality), I realize that there is a crucial discrepancy here.

It must, accordingly, be acknowledged that we cannot evert the distinction between R , reality as it is, and R^* , reality as we think it to be

R = the totality of *actual* fact

R^* = the totality of *putative* fact

While the equation

$$R = R^*$$

is certainly not a theoretical truth, it is nevertheless but a practical principle—an instrumentality of practical procedure.

When we buy a car we do so in the realization that “Some cars are lemons.” But we proceed via the idea of accepting

- *This* car is not a lemon.

This contention is not a secure fact, but a plausible presumption, something we see ourselves as authorized to accept in the circumstances. We endorse it not on theoretical but on practical grounds.

What we have here is emphatically not a line of thought validated not by considerations of general principles, but rather as a practical step justified by its communicative utility in managing information and coordinating action. It is not an inference but a presumption—not a *given* but a *taken*.

What this means for the transit from subjectivity to objectivity is simply that I resort to the practical policy of a suspension of disbelief with regard to the theoretical (general purported) difficulties between the two.

Two considerations justify this practical policy:

- (1) There is no better alternative in sight
- (2) And by and large we “get away with it” in practice.

To be sure, when we look reflexively at the manifold of our knowledge two crucial considerations thus come to the fore:

- We are *not infallible*. Our thinking something to be so does not mean that it actually is. It is not the case that: $Txp \rightarrow p$. (Indeed this would only hold if x were God.)
- We are *not omniscient*. Something’s being so does not mean that I realize it to be so. It is not the case that: $p \rightarrow Tip$. (Indeed this too holds only for God.).

And even *general* acceptance does not necessarily make for truth. People can go wrong collectively as well as individually: we realize full well that in factual matters it is not the case that: *Securus iudicat omnia terrarum*.

9.5 From Subjectivity to Objectivity

The only conclusions that follow securely from premises relating to the order of thought are facts regarding this order of thought itself. What follows from premisses of the format

X thinks that p is the case

is never p itself, unless this claim p relates only to the order of thought, as per “Somebody thinks something,” or—as with Descartes—“There are thinking beings.” The irony of it is that the objective experiential certitude is available only with respect to subjective matters.

The only objective facts that can securely be based upon matters of subjectivity are facts about subjectivity itself. I can know with objective cogency that I *subjectively* take something to be so. But objectivity cannot be extracted from subjectivity. There is no factual-generation *ex nihilo*. The inferential step from subjectivity to objectivity is possible only when the item at issue is something subjective.

Notes

1. What about evaluative (unlike cognitive) stances: liking (L), disliking (X), etc., rather than believing (B), disbelieving (D), etc.? Is subjectivity decisive here: do we have the takes of

$$Lxp \rightarrow KxLxp$$

or even

$$Lxp \rightarrow BxLxp$$

With regard to the potentialistic readings of K and B the answer here too will again be in the affirmative. And this also holds for the converse:

$$BxLxp \rightarrow Lxp$$

Here too the agent's convictions are (potentialistically) determinative in matters of subjectivity.

Chapter 10

Leibniz and “The Liar”



10.1 The Liar

Concern for logical problems of self-reference originated with the Liar Riddle (*pseudomenos*) of the Greek dialectician Eubulides of Megara (ca. 440–ca. 380 BC), who included it in his register of seven paradoxes. On his telling, it posed the puzzle: “Does the person who says ‘I am lying’ actually lie?” (Also: “Does the witness who declares ‘I am perjuring myself’ thereby perjure himself?”)¹ The problem that arises here can be posed via the following dilemma:

The declaration that I lie will be either true or false. But if this declaration is true, then I lie, and my declaration will be false. But if that declaration is false, then what it says—namely that I lie—is not the case, so I must be speaking the truth. Thus, either way the truth-status of the contention is inappropriate, unstable, and self-defeating.

Accordingly, what we have here is an untenable contention that involves an inherent conflict of truth-claims.

The situation is paradoxical because individually plausible theses are collectively inconsistent here. And this situation can be set in train by a single proposition, as per the Liar’s:

L: This statement is false.

For it now emerges that we have both

(1) *L* is true

and

(2) *L* is false.

Here, (2) is simply what *L* itself affirms. And (1) follows because (2) is exactly what (1) itself maintains. *L* is accordingly paradoxical in that it creates this conflict of affirmation among the various claims it plausibly authorizes.

As logicians address it, the Liar Paradox pivots on the puzzling truth-status of statements that affirm their own falsity (be it directly or obliquely), and thereby will

apparently be true when false, and false when true. This situation arises not only with the Liar but also with such contentions as:

(A) Statement (B) is false.

(B) Statement (A) is true.

Here, if (A) is true, then (B) is false, so (A) is not true. And if (A) is false, then (B) is not false, so (A) is true.

Such contentions have puzzled theorists since classical antiquity when Greek philosophers approached the problem via the ancient story of Epimendes the Cretan, who is supposed to have said that “All Cretans are liars”—with “liar” being understood in the sense of a “*congenital* liar,” someone incapable of telling the truth.² (To be sure, if by “liar” one meant someone who lies frequently but not always, there would be nothing paradoxical about the matter.) The Liar Paradox has been the subject of ongoing preoccupation throughout the history of philosophy, alike in the West and in the Islamic world.³

10.2 Leibniz on “The Liar”

Leibniz was prominent among those theorists who have deliberated about the “Liar.” He addressed the issue already in his early (aet. 18) master’s thesis, *Specimen questionum philosophicarum ex jure collectarum*, submitted at the University of Leipzig in 1664.⁴ Here, Leibniz treated the Liar in the context of his discussion of issues of legal perplexity where an equally good case can be made out either way, pro or con. He dealt with it by proceeding in the manner favored by the medieval scholastics, namely distinctions. His analysis sets out from the commentary of the “Doctors of Coimbra” on Aristotle’s *De interpretatione* (Chap. IV, Question 3), which dismissed reflexive statements as invariably meaningless (*non significativa*). Leibniz rejected this position as overly general in overlooking what he saw as a crucial distinction between two modes of reflexivity, namely:

- *Declarer reflexivity* in relation to the individual who makes that reflexive statement (*reflexiva ratione subjecti*).
- *Declaration reflexivity* in relation to the statement itself that is being made (*reflexiva ratione [enuntiationis] ipsius*).

As Leibniz saw it, declarer reflexivity is something that can be harmless. Thus, consider such reflexive sentences as:

- All sentences have some subject matter.
- This sentence is about sentences.

These are innocuous truisms. By contrast, however, such statements as:

- What *X* is now saying is false
- Everything that *X* ever says is false

are differently situated. As long as *X* is not the declarer himself, such statements are unproblematic. But when *X* is the declarer himself, then they become problematic. But this does not, however, make them true because the “false” is not the only alternative to “not true.” For Leibniz clearly saw that the domain of what language can express “*contiendrait non seulement les vérités, mais aussi les faussetés que les homme peuvent inventer, et même des expressions qui ne signifient rien.*”⁵

Statements in general, and statements about statements in particular, have two importantly distinguishable aspects. There is the matter of *explicit assertion* that pivots on just what it is that the statement affirms, its assertoric *content*. And there is also the *status-condition* of the statement, its classificatory *status* in point of various descriptive features: its truth-status above all.

Thus, consider, for example, the self-referential statement: “This statement is brief.” Of course, if the statement had actually been long, its status will change to false, and would disagree with what it itself explicitly says. On the other hand, “This statement is true” would presumably be true since its explicit claim and its status will then agree.

In point of explicit assertion, the initial statement affirms that it itself is brief, and as regards its status and condition, it transpires that the statement at issue is indeed brief. So, here again agreement means that in point of truth-condition the statement is actually true.

However, when a statement’s status and its explicit declaration disagree (as with “This statement is made in French”), the statement is automatically falsified. That is bad enough. But even worse is the situation where no fixed alethic status (be it as *true* or *false*) can be found to square with the statement’s explicit declaration. This problem arises with “This statement is false” when no stable status can be found for the claim. (If false it is true, and if true it is false.) Such self-refuting claims are paradoxical, and Leibniz has it that such claims should be seen as meaningless.

Thus, consider the Liar contention in its communicative context. Suppose someone says:

- *X* never says anything.

This is, of course, a perfectly meaningful contention—unless and until the declarer is *X* himself, in which case the declaration becomes reflexive and baffling. If the declarer is anyone other than *X*, then the claim will be as true or false as the case may be. But if it is *X* himself, it becomes problematic and effectively meaningless—unable to transmit any intelligible information.

And exactly the same situation obtains with respect to:

- What *X* is (now) saying is meaningless
and:
- Anything that *X* ever says is false.

With anyone other than *X* himself such a claim is perfectly meaningful and bound to be either true or false. But with *X* himself as the assertor, the claim becomes baffling and effectively meaningless.

In such cases, the form of words at issue, while grammatically meaningful, becomes substantially unintelligible—and so effectively meaningless and informatively empty.

Leibniz proceeded to show just how and why it is that such problematic statements can become meaningless. Thus, consider the contention:

- This statement is false.
But just what is it that this statement affirms? Given that what is at issue is the statement itself, the only available response is that it comes to:
- It is false that this statement is false.
But now just what is it that *this* statement affirms? Again, we have no alternative but to repeat the preceding step, so as to come to:
- It is false that it is false that this statement is false.
And now the writing is on the wall. The endeavor to specify just what it is that our original falsification then asserts embarks us on an infinite and thereby unrealizable task. In the face of its regressive dissolution, there is no alternative but to reject such a self-referential statement as meaningless.⁶

And indeed, Leibniz himself rejected such endlessly conflicting statements as meaningless (*absurdum*) because they have an infinite—and thereby unachievable—regress of referential presuppositions. There simply is no specifiable referent to accommodate such contentions within the realm of meaning.

10.3 Leibniz in Context

Twentieth-century logic and semantics have given a great deal of attention to the Liar Paradox, and a vast mass of publications has yielded a challengingly large literature of the topic.⁷ These modern treatments of the Liar Paradox have generally proceeded along one of two lines. They pivot either on the Liar statement’s semantical truth-status or on its communicative meaning-status. With respect to the former (truth-status), they pursue one of three prospects:

- *potential plurality*: the alethic status of a statement need not just be True or False but can encompass one of several variant Intermediate conditions
- *superposed duality*: it is possible for statements to have both the status of Truth and that of a Falsehood
- *potential vacuity*: statements may have no truth-status whatsoever; they can be alethically stateless, as it were.

And with respect to meaning-status, recent theorizing envisions primarily two possibilities:

- *equivocation*: duality of meaning in being true in one sense and false in another; being alethically variegated relative to different angles of consideration.

- *meaninglessness* (not in the *linguistic* sense of the term but in the *informative* sense): lacking any informative substance.
In implementing this second prospect, Leibniz addressed the Liar by means of a semantical distinction, effectively that between:

Sentences: linguistic complexes that are either meaningful or meaningless and provide symbolic formulation for claims

Statements: contentions that stake a claim and accordingly have a truth-status of True or False or (perhaps) Other.

And now in the face of this distinction, consider the Liar contention:

- This sentence is false.
This contention will now be something meaningless. It commits a category mistake, making an invalid misattribution, much like “This sentence is green.” For sentences are meaningful or not; only statements can be true or false.

By contrast, consider:

- This statement is false.
As Leibniz in effect argued, there simply is no definite referent for “this statement” to be about. And in consequence, there is actually no identifiable statement at issue. The sentence at issue is actually without meaning.

Leibniz’s thus proposed resolving of the Liar Paradox problem on the basis of a vitiating meaninglessness. And while this general line of approach had also appeared earlier, Leibniz’s explanation for taking this position was original and innovative. For he held that the self-referential claim of the liar thesis is without meaning through lacking any objective referent, seeing that any attempt at its specification leads to an informative regress of meaning-presuppositions. And this crucial consideration constitutes an original contribution to the problem that possesses ongoing utility and interest.⁸

10.4 Closing Considerations

As Leibniz realized, any putative informative statement will have two distinct aspects: (1) its “internal” asserted *substance or content* in relation to what it maintains, and (2) its “external” descriptively manifest *status or condition* as indicated by a description such as “true” or “interesting” or “formulated in French.” Approached from this standpoint, consider the Tarski equivalence:

$T(p)$ if and only if p

It becomes at once clear that this thesis becomes problematic when the propositions at issue are self-referentially reflexive. For then these two aspects can conflict when a statement’s substance clashes with its descriptive status. (This holds not only with “This statement is false” but also with “This statement is without truth-status.”)

To see this clearly, let us use $|p|$ to symbolize the *truth-status* of p (over the range including T for “true” and F for “false”). And now, consider that Tarski equivalence to the effect that in general p amounts to: p is true. Symbolically formulated, we thus have the general principle:

$$p \equiv (|p| = T)$$

But we now cannot coherently proceed on the assumption of a self-negating statement q that denies its own truth: so that $q = (|q| \neq T)$. For this would now lead straightaway to the absurd consequence:

$$|q| = T \text{ and } |q| \neq T$$

And so, there now arises a forced choice between two alternatives:

- (1) Regarding at least some logically incompatible propositions as conjointly acceptable as true.⁹
- (2) Rejecting the Tarski equivalence: p iff p is true. Or, rather, restricting its applicability to “ordinary” propositions and excluding some or all reflexive theses from this range as simply meaningless.

In theory, both of these are open possibilities. But given that Leibniz joins traditional logic in rejecting (1), he becomes bound to (2).

And so, for Leibniz the way to manage the Liar Paradox is not by recourse to a Tarskian infinitely constructive *progress* of meta-languages, but rather by resort to an infinitely destructive *regress* of meaning-presuppositions. And it deserves to be acknowledged that this Leibnizian approach on the basis of rejecting an endless regress of referential presuppositions represents an original and viable way to address the problem.

Notes

1. See Aristotle, *Sophistical Refutations*, 180a35; *Nicomachean Ethics*, 1146a71. Or again: *Si dicis te mentiri verumque dicis, mentiris* (Cicero, *Academica*, II, 30, 96; and compare, *De divinatione*, II, 11). On the history of the paradox, see N. Rescher, *Paradoxes* (Chicago: Open Court, 2001), and R. Sorensen, *A Brief History of the Paradox* (Oxford: Oxford University Press, 2005). Not only the ancients considered this paradox but the medieval schoolmen examined it as well. (Thus, John Buridan (ca. 1280–1350) analyzed the Liar in the eleventh sophism treated in Chap. 8 of his *Sophismata*.) An English translation and commentary are given in G. E. Hughes, *John Buridan in Self-Reference* (Cambridge: Cambridge University Press, 1982).
2. Such issues were discussed not only by Aristotle and Cicero but also even the Bible contains an oblique reference to them. In the New Testament, the Epistle of Paul to Titus at Crete echoes this in warning him about “those many unruly and vain [Cretan] talkers and deceivers whose mouths must be stopped, who subvert whole houses, teaching things which they ought not, for filthy lucre’s sake. One of themselves, even a prophet of their own [viz. Epimenides], said the Cretans are always liars ...” (1, 12–13). In medieval times, the Liar became a staple in the extensive discussions of insolubilia. John Buridan and Thomas Aquinas were among those who concerned themselves with it.

3. For an illuminating and widely informed discussion of the Liar in the Arabic tradition, see Ahmed Alwishah and David Samson, "The Early Arabic Liar: The Liar Paradox in the Islamic World from the Mid-Nineteenth to the Mid-Twentieth Centuries CE," *Vivarium*, vol. 47 (2009), pp. 97–127.
4. The original Latin text is presented in Alberto Artosi et al., *Leibniz: Logico-Philosophical Puzzles in the Law Philosophical Questions and Perplexing Cases in the Law* (Dordrecht: Springer, 2013), pp. 49–70, which is here cited as L. An English translation is also given there on pp. 1–39, and is here cited as E. The Liar is addressed in Question XII (L, p. 64; E, pp. 29–30).
5. Louis Couturat, *Opuscles et fragments inédits de Leibniz* (Paris: Vrin, 1903), p. 532.
6. *Dabitur processus in infinitum, quod cum absurdum sit, priones omnes corruunt, quippe super nulla fundatur.* (L, p. 65).
7. The literature on the Liar Paradox is vast. Recent books that deal extensively with it include:

Barwise, J. and J. Etchemendy, *The Liar: An Essay on Truth and Circularity* (Oxford: Oxford University Press, 1987).

Beall, J. C., *Revenge of the Liar* (Oxford: Oxford University Press, 2008).

Beall, J. C. (ed.), *Liars and Heaps: New Essays in Paradox* (Oxford: Clarendon Press, 2003).

Beall, J. C. and Bas Van Fraassen, *Possibilities and Paradox: An Introduction to Modal and Many-Valued Logic* (Oxford: Oxford University Press, 2003).

Bottin, F., *Le antinomie semantiche nella logica medievale* (Padova: Antenore, 1976).

Hughes, G. E., *John Buridan on Self-Reference* (Cambridge: Cambridge University Press, 1982).

Kirkham, R., *Theories of Truth: A Critical Introduction* (Cambridge, Mass: MIT Press, 1992).

Martin, R., *Recent Essays on Truth and the Liar Paradox* (Oxford: Clarendon Press, 1984).

Martin, R., *The Paradox of the Liar* (New Haven: Yale University Press, 1970).

McGee, V., *Truth, Vagueness, and Paradox: An Essay on the Logic of Truth* (Indianapolis: Hackett, 1991).

Simmons, K., *Universality and the Liar* (Cambridge: Cambridge University Press, 1993).

Sorensen, R., *A Brief History of the Paradox* (Oxford: Oxford University Press, 2003).

8. Leibniz's position in this regard appears analogous to David Lewis's later idea of rejecting claims whose truth status cannot be determined until some further claim of exactly the same nature has already been determined, and so ad infinitum. See his "Outline of a Theory of Truth" in *The Journal of Philosophy*, vol. 72 (1975), pp. 690–716 (see pp. 693–94).
9. Regarding this prospect, see N. Rescher and R. Brandom, *The Logic of Inconsistency* (Oxford: Blackwell, 1980).

Chapter 11

Did Leibniz Anticipate Gödel?



11.1 Gödel's Belief in a Leibnizian Conspiracy

Kurt Gödel's 1930s demonstration of the provability incompleteness of axiomatic arithmetic was a monumental achievement in mathematical logic and marked him as "one of the most significant logicians in history."¹ In the mid-1940s, Kurt Gödel embarked on a systemic study of Leibniz's logic which continued for at least another decade. During this time, I myself was writing my Princeton doctoral dissertation on Leibniz's Cosmology and we had something of a tug-of-war over the Leibniz material of Firestone Library—each recalling for his own use material out on loan to the other. (Unfortunately for me, we never made any direct contact.)

Gödel described himself as "following Leibniz rather than Spinoza (like Einstein)."² As Gödel studied Leibniz via Louis Couturat's classic *La Logique de Leibniz*, he became convinced that resistance to the logico-mathematical Platonic realism of his own position was prefigured in a conspiracy of suppression and silence that had kept Leibniz's similar insights from being properly understood and appreciated. And the more Gödel studied Leibniz, the more keenly he suspected that Leibniz might have anticipated kindred aspects of his own work—and especially his demonstration of the provability incompleteness of mathematics.³ Gödel came to this view because he saw Leibniz as a precursor and a kindred spirit whose problematic reception was a foreshadowing of his own difficulties.

So while there is little doubt that Gödel saw Leibniz as a precursor engaged on an analogous inquiry, there remained in his mind questions about the extent of anticipation in point of findings. Moreover, in this regard there remain for us questions about his attitudes toward the prospect of anticipation—whatever their extent. However, the matter of motivation remains somewhat obscure. Was he worried at the idea of having been anticipated? (After all, in mathematics all the credit goes to him who gets there first.) Or was he hopeful of finding that he had succeeded in a common project where the great Leibniz had tried and failed? Perhaps we will never

know. But either way Leibniz's work on matters of provability and demonstrative systematization in mathematics were of deep concern to Gödel.

Curiously, Gödel not only saw Leibniz as a precursor in logic and the foundations and epistemology of mathematics but in metaphysics (i.e., the general theory of reality). Gödel described his philosophical position as "a monadology with a central monad . . . like the monadology of Leibniz in its general structure."⁴ The idea of complexly integrated reality comprising of units coordinated into a systemic unity functioning under the aegis of natural laws unquestionably had an appeal for Gödel, although this particular aspect of his Leibnizian sensibility was something he never addressed in detail at length.⁵ Accordingly, one acute Gödel scholar tells us that "his most profound sense of identification was with the über-rationalist Leibniz,"⁶ and that at the very core of his thought was his "interesting axiom" to the effect that the world is rational⁷—i.e., always stands in such a way that its doings are explicable on rational principles—which, of course, is nothing but Leibniz's Principle of Sufficient Reason.⁸ Gödel felt that this conception alienated him from the dominant majority of his mathematical contemporaries. And he was concerned that Leibniz had suffered a similar fate.

While most contemporary mathematical theorists proposed to interpret Gödelian incompleteness along relativistic and intuitionistic lines, he himself was an unabashed Platonist. Leibniz, like most philosophers, held the view that to be is to have a *locus standi* of some sort—a place in Reality's scheme of things. Thus, for him even mere possibilities will, if authentic, exist (or "subsist") in the mind of God. And in this regard, "One idea of Leibniz that appealed to Gödel was that the objects of mathematics exist in God's mind"⁹.

Thus, in declining an invitation from Paul Cohen to a conference in 1967, Gödel wrote "For many years my own thinking was moved along lines entirely different from those of the conference."¹⁰ He felt estranged and isolated from the contemporary philosophy-of-mathematics community. And in this regard, he persisted in viewing Leibniz as a comparably misunderstood fellow spirit. Thus, Gödel told a skeptical Oskar Morgenstern in 1945 that Leibniz was "systematically sabotaged by his editors."¹¹ And Gödel went on to claim for Leibniz (1) a recognition of the scientific importance of developing "a theory of games," (2) a discovery of antinomies of set theory "cloaked in the language of concepts, but exactly the same," (3) anticipations of Helmholtz's resonance theory of hearing, and (4) the law of the conservation of energy. While Morgenstern inclined to see such claims as "fantasies," the fact remains that they are all perfectly true—and far from exhaustive of the range of scientific, mathematical, and logical innovation that can be ascribed to Leibniz.

Much of what we know about Gödel's belief in a Leibniz conspiracy comes from Karl Menger's recollections.¹² After noting that "Gödel had always been most intensely interested in Leibniz,"¹³ Menger informs us that during the late 1930s and 1940s:

Gödel was more and more preoccupied with Leibniz. He was now completely convinced that important writings of this philosopher had not only failed to be published, but were destroyed in manuscript. Once I said to him teasingly, "You have a vicarious persecution complex on Leibniz' behalf. . . Who had an *interest* in destroying Leibniz' writings?" "Naturally those

people who do not want man to become more intelligent," he replied. Since it was unclear to me whom he suspected, I asked after groping for a response. "Don't you think that they would sooner have destroyed Voltaire's writings?" Gödel's astonishing answer was: "Who ever became more intelligent by reading the writings of Voltaire?" Unfortunately, at that moment someone stepped into the room and the conversation was never concluded.¹⁴

11.2 Leibnizian Anticipations

It is unquestionable that the work of Leibniz has various significant points of anticipation of Gödel. Hilbert and the logicist tradition sought to embed mathematics in logic; Leibniz—and Gödel with him—contrarwise sought to mathematicize logic. Let us look at the matter more closely.

11.2.1 Propositional Id Numbering

The arithmetization of discourse provided the basis for Gödel's demonstration of the incompleteness of axiomatic arithmetic. And in this matter of discourse arithmetization, Gödel was right in surmising Leibnizian anticipations.¹⁵

Gödel devised his characteristic method for identifying arithmetical propositions by a two-step process. First, he formulated these propositions in the symbolism of *Principia Mathematica*. Then he numerized the resultant symbolic recasting by means of the special algorithm that came to be called Gödel numbering.

Leibniz managed to achieve much the same ends by other means. This called for a rather different approach; first, formulating the mathematical thesis at issue in language, and then encoding the resultant linguistic statement by any of the readily available numerical encryption procedures familiar from secret diplomatic communication.¹⁶ Leibniz himself devoted considerable efforts to cryptological methods and was acutely aware of various procedures for representing any sort of text by numerical means.

Both approaches, Leibniz's and Gödel's, achieve their results in essentially analogous ways: assigning a unique and characteristic Identification Number to every well-formed arithmetical (i.e., mathematical) proposition.

All in all, then, there can be little question that in this matter of the numerical presentation of logical and mathematical propositions, Leibniz—and with him modern cryptography in general—anticipated Gödel in providing different machinery for achieving the same results.¹⁷

11.2.2 *Mathematical Platonism*

Gödel deemed it plausible that: “Classes and (other mathematical) concepts may. . . be conceived as real objects. . . existing independently of our definitions and constructions.”¹⁸ But although Gödel was drawn to such a Platonic realism in mathematics, he never claimed to have built a conclusive case for this position.¹⁹ In the early 1950s he conceded:

Of course I do not claim that the foregoing considerations amount to a real proof of this view (i.e., Platonism) about the nature of mathematics. The most I could assert would be to have disproved the nominalistic view, which considers mathematics to consist solely in syntactical conventions and their consequences. Moreover, I have adduced strong arguments against the more general view that mathematics is our own creation. There are, however, other alternatives to Platonism.²⁰

All the same, Gödel was convinced that none of these alternatives had anything like as much to be said on their behalf as did Platonism.²¹ In 1938 he wrote:

. . . even if one should succeed in proving [the independence of the continuum hypothesis], this would. . . by no means settle the question definitively. Only someone. . . who denies that the concepts and axioms of classical set theory have any meaning (or any well-defined meaning) could be satisfied with such a solution, not someone who believes them to describe some well-determined reality. For in this reality Cantor’s conjecture must be either true or false, and its undecidability from the axioms as known today can only mean that these axioms do not contain a complete description of this reality. . .²²

It is repeatedly asserted that Gödel’s discovery of the deductive incompleteness of arithmetic—establishing that some arithmetical truths simply cannot be demonstrated—somehow undermines foundations of this field and casts a shadow of uncertainty over arithmetic as a cognitive discipline. Arithmetical truth must now supposedly be deemed dependent on human inquirers and ceases to be a matter of objective fact about a mind-independent reality. But, as Gödel himself firmly maintained, the very opposite is actually the case. As he saw it, the fact that no matter how we might twist and turn in our attempts at axiomatization we will be unable to derive the entire realm of arithmetical truth, establishing the field’s independence of our cogitation. As he viewed it, this means that this realm of arithmetical fact that will inevitably outrun the reach of axiomatization bespeaks realism rather than man-controlled relativism.

If finite provability were the sole standard for rational assertability, then as John von Neumann put it “there is no rigorous justification for (all of) classical mathematics.”²³ But such a conclusion was anathema to Gödel.

Gödel believed in a region of truth that we cannot reach by our ventures in axiomatization—a Platonic region of reality above and beyond our contrivance. And it is against this background that one must understand his 1975 avowal to Bernays that:

I’m pleased that. . . you advocate a cautiously [*vorsichtig*] Platonistic point of view. To me a Platonism of this kind (also with respect to mathematical concepts) seems to be obvious and its rejection to border on feeble-mindedness [*an Schwachsinn zu grenzen*].²⁴

And as Gödel saw it, his mathematical Platonism had been foreshadowed—alike in general terms and in substantial detail—in the thought of Leibniz.²⁵

Leibniz held a theory of necessary truths independent not only of the artifice of human thought but even of dealing specifically with the *actual* world. We humans are wholly reliant upon experience for our acquaintance with such truths, for we can obtain knowledge of them only by perception. But while the resultant truths of fact hinge upon the will of God—they would be falsehoods rather than truths had He chosen it so—truths of reason, by contrast, do not deal with matters of contingent existence but with absolute truths that must hold good in every possible world.²⁶ Knowledge of such necessary truths is available, even to man, by conceptual analysis of a finitistic character. The status of these truths is absolute and unconditional; they are and must be as is because the concepts involved in them are what they are. And the truths of logic and mathematics are paradigm examples here.

All this is something that Gödel regarded as essentially correct. Like Leibniz, he grounded the reality of abstract ideas and the necessary truth of their relations not in a self-subsistent Platonic Realm of ideas but specifically in one embodied in the thoughts of God. And this is why for him a “God of the philosophers” became a crucial requisite for the philosophy of mathematics.²⁷

11.2.3 *Mind Not a Machine*

On the basis of a shared conviction that mathematical understanding stands decisively in the way, Gödel and Leibniz both held a deep ideological conviction that the workings of the human organism transcended the automaticity of mechanism. As Leibniz wrote in his 1716 *Monadology*:

Each organic body of a living being is a kind of divine machine or natural automaton which infinitely surpasses all artificial automata. For a machine made by human artifice is not a machine in each of its parts. For example, the tooth of a brass wheel has parts or pieces which to us are no longer artificial things, and no longer have something recognizably machine-like about them, reflecting the use for which the wheel is intended. But the machines of nature, namely living organisms, are still machines even in their smallest parts, ad infinitum. It is this that constitutes the difference between nature and artifice, that is, between divine artifice and ours. (*Monadology*, Sect. 64)

In consequence, Leibniz envisioned a considerable gap between thought (conscious awareness or “perception”) and purely mechanical operations:

One must admit that perception and what depends upon it is inexplicable on mechanical principles, that is, by figures and motions. In imagining that there is a machine whose construction would enable it to think, to sense, and to have perception, one could conceive it enlarged while retaining the same proportions, so that one could enter into it, just like into a windmill. Supposing this, one should, when visiting within it, find only parts pushing one another, and never anything by which to explain a perception. (*Monadology*, Sect. 17)

In a decidedly similar vein, Gödel drew comparable lessons from the incapacity of automatized calculation procedures to realize the entirety of mathematical truth. He construed his proof of incompleteness to show that “the kind of reasoning necessary in mathematics cannot be completely mechanized.”²⁸

And in his 1951 Gibbs lecture, Gödel suggested that his incompleteness theorems furnished strong evidence for an idealist philosophical stance, maintaining that they confront us with a disjunction:

Either mathematics is incompletable in [the] sense. . . [that] the human mind (even within the realm of pure mathematics) infinitely surpasses the power of any finite machine, or else there exist absolutely undecidable diophantine problems. . .²⁹

Accordingly, in a letter to Gottard Günther dated June 30, 1954, Gödel wrote:

When I say that one can (or should) develop a theory of classes as objectively existing entities, I do indeed mean by that existence in the sense of ontological metaphysics, by which, however, I do not want to say that abstract objects are present in nature. They seem rather to form a second plane or level [*Ebene*] of reality, which confronts us just as objectively and independently of our thinking as nature.³⁰

Gödel was clearly prepared to endorse a dualistic, Plato-reminiscent realism that envisioned the ontological realms of both a concrete (physical) and an abstract (mathematical) sector of reality.

On one occasion Gödel remarked “that Leibniz’s project of the *characteristica universalis* was not utopian”—i.e. could in fact be realized.³¹ But in the end, he changed his mind, declaring:

The universal characteristic claimed by Leibniz (1677)—if interpreted as a formal system—does not exist. Any systemic procedure for solving problems of all kinds [and thus for resolving issues of provability] would have to be non-mechanical.³²

And Gödel surmised that Leibniz himself came to realize that in ultimately abandoning his *characteristica universalis* project in the 1690s.

As Gödel saw it, his demonstration of the provability incompleteness of arithmetic puts paid to Leibniz’s prospect of a *calculus ratiocinator*. But Gödel surmised that this was a conclusion ultimately reached by Leibniz himself, and he strongly suspected that the problems with his reasoning calculus led Leibniz to realize its incompleteness. After all, he did pretty much abandon the project after the 1690s?³³

Commitment to the belief in the incapacity to manage mathematics by purely mechanical means represents a deep ideological affinity between Gödel and Leibniz. Both shared the conviction that our limited capacity for calculation and mathematical knowledge provided the basis for a materialism-negating idealism, a realm of abstract principles that underlie and yet somehow transcend physical reality as the level of purely mechanical principles.

Alike for Gödel and Leibniz, the crux of philosophical adequacy lies in the proper understanding of the personal self. For only when this is achieved will we be able to say how it is that human are able to acquire knowledge of reality since this can

only be achieved (by us) through the mediation of our own capacities, theorists thought objectivity can only be reached through the doorway of subjectivity. But not Leibniz—or Gödel. For if—as they thought—a sufficiently powerful intuition is among our resources, then a Platonic realism becomes viable.

11.2.4 Intuition

One key role of intuition is to provide for self-knowledge, and this played a critical role alike in the thoughts of Leibniz and of Gödel.

In a 1702 letter to his friend Queen Sophie Charlotte of Prussia, Leibniz wrote: “It is through this (reflexive self-awareness) that I conceive what is *substance* in general. And it is also the consideration of myself that provides me with the other concepts of *metaphysics* such as cause, effect, action, similarity, etc., and even those of logic and ethics.”³⁴ And Gödel also surmised that the basic principles of mathematics were rooted in the human mind awaiting their uncovering by intuition.

It was, of course, no news to the mathematical community that mathematical truth cannot be encompassed by demonstration. Since the dawn of formalized mathematics in classical antiquity, it has been noted that the basic fundamentals of the subject—the axioms, postulates, and definitions—were indemonstrable. But these basic fundamentals are modest exceptions to the rule, supposedly unproblematic because of their self-evident obviousness and informative vacuity.

For Leibniz, the fundamental and basic concepts and facts at issue in transcendental (i.e., nonempirical) knowledge are provided to the human mind on an experience-independent, a priori basis by inherently implanted “innate ideas.” No doubt it takes experience to realize them, but their constitution and nature is experience-independent, and our access to them is not sensorially mediated but intellectually immediate. Leibniz calls this immediate apprehension of objective fact by the name *intuition* and correspondingly speaks of “*les vérités primitives qu’on sait par intuition*,” contrasting it with the discursive knowledge we obtain by demonstration.³⁵ Analogously, Gödel viewed the overall lessons of his own work as “decidedly opposed to materialistic philosophy”:

Namely, if the first alternative holds, this seems to imply that the working of the human mind cannot be reduced to the working of the brain. . . . On the other hand, the second alternative. . . seems to disprove the view that mathematics is only our own creation. . . . So [it] seems to imply that mathematical objects. . . exist objectively. . . that is to say [it seems to imply] some form or other of Platonism or “realism” as to the mathematical objects.³⁶

Accordingly, Gödel insisted that “Any systemic procedure for solving (mathematical) problems of all kinds would have to be non-mechanical.”³⁷ As he wrote to Paul Tillich in 1963: “in mathematical reasoning the non-computational (i.e., intuitive) element (is unavoidable and) consists in intuitions of higher and higher infinities.”³⁸

This, of course, is the core of Leibniz’s doctrine of innate ideas. The line of thought at issue envisions an internally oriented intuition—a mode of apprehensive capacity

able to provide cognitive access to basics. Gödel is fully on board with regard to this Leibnizian doctrine: “If you know everything about yourself, you know everything of philosophical interest.”³⁹ Reflexive intuition furnishes the basics for inference, so that mathematics has no access ways to fact, viz. immediate (nondiscursive) apprehension or intuition and inferentially demonstrative proof.

Gödel calls upon (mathematical) intuition to accomplish two missions: (1) to authenticate the axioms that provide the basis of the deductive system at issue, and (2) to authenticate certain other mathematical truths outside the reach of deductive derivability from those axioms. (Otherwise there would, of course, be no justification for speaking of deductive incompleteness with its commitment to nondemonstrable truths.)

For Gödel, too, the reality of indemonstrable arithmetic truth reveals something basic about the nature of arithmetical epistemology, viz. the existence of a characteristic mode of insight into mind-independently objective arithmetical fact. This position is the basis of what is widely characterized as Gödel’s mathematical Platonism.

As Gödel observed to Alonzo Church in 1966:

You know that I disagree about the philosophical consequences of Cohen’s result [regarding the provability independence of the continuum hypothesis]. In particular I don’t think realists need expect any permanent ramifications. . . as long as they are guarded, in the choice of the axioms, by mathematical intuition and by other [similarly cogent] criteria of rationality [such as consistency].⁴⁰

The subsequent mathematical tradition down to Husserl and beyond assigns to “intuition” (however conceived) this exact task of providing us with valid cognition of the axiomatic basics of demonstration. And Gödel was fully on board here.⁴¹ Regarding the Continuum Hypotheses, Gödel wrote in 1947: “(A demonstration of nonprovability will) by no means settle the question (of truth) definitively. . . For (an unprovable theses) must be either true or false and its underivability from the axioms as known today can only mean that those axioms do not contain a complete description of this reality.”⁴² As Gödel saw it, discursive rationality can only yield information about the conditional make-up of a system of (humanly endorsed) hypotheses, while intuition can yield insight into the unconditional truth of things.⁴³ And this view of the matter is substantially consonant with Leibniz’s.

To be sure, Husserl sees the role of intuition as constructive. We have a priori only what we construct, be it physically or conceptually; either way, we secure “only what we ourselves put into it.” But neither Leibniz nor Gödel held this view. For what we ourselves make, we could always unmake and/or make differently. Classic intuitionism is reportorial, yielding accurate insight into the Platonic realm. (Believing that this could only be provided for us by a higher potency Gödel’s viewed God as guarantor of mathematical knowledge—to the horror of most of his mathematical contemporaries.)

Gödel never abandoned his Leibnizian commitment to intuition when he subsequently turned to Husserl to facilitate and potentiate the Leibnizian program by broadening the reach of Leibnizian innateness beyond the limited range of totally

obvious fact. And Gödel was convinced—that this means for extracting mathematical basics from reflexive self-understanding was going to be pivotal for Husserl phenomenology. As Hao Wang reports:

Gödel told me that the most important of Husserl's published works are *Ideas* and *Cartesian Meditations* (the *Paris Lectures*). He said: "The latter is closest to real phenomenology—investigating how we arrive at the idea of self."⁴⁴

And, of course, this conception of the self and its self-evident cognition that lay at the basis of Leibnizian apperception. As he put it in the *Monadology*:

It is also via the knowledge of necessary truths and their abstract [from merely sensuous matters] that we are raised to *Reflexive Acts* that enables to think of what is called I and to consider that this is what lies within ourselves. . . . And these reflective acts furnish the principal objects of our [present metaphysical] reasoning.⁴⁵

11.2.5 Truth by Calculation

Leibniz's conception of a *calculus ratiocinator* constituted another significant factor in Gödel's thought. For Leibniz not only wanted to represent propositions numerically, but also sought to determine their truth by arithmetical means. This idea lay at the very basis of the system of systemic logic he devised to track by numerical means. This proceeding was based on a coding that coordinated concepts with numbers using primes to represent primitive concepts and products thereof to represent complexity. In consequence, the analytical decomposition of concepts could be achieved by the arithmetical relations of integers; a way that (for example) when "All As are Bs," then the integer assigned to the As is a divisor of that aspect to the Bs: $\#A \div \#B$. Thus, the fact that when the As are Bs and the Bs are Cs, then that the As must be Cs is an immediate consequence of the transitivity of division. And in analogous ways, the entire system was to be so developed that the acceptability of propositions can be determined by means of calculation: truth-determination via calculation was the object of the enterprise.

His incompleteness theorem led Gödel to maintain that "the universal characteristic deemed (possible) by Leibniz (1677) if interpreted as a formal (i.e., axiomatized) system does not exist."⁴⁶ After all, Gödel's findings indicate that formalized assessment methods cannot resolve issues of arithmetical truth.

But after the late 1690s, Leibniz pretty much abandoned this project and Gödel seems to have worried that he did so because he realized that calculation simply cannot always settle matters of mathematical truth. And if Leibniz realized this, did he not effectively anticipate Gödel's own findings?

Leibniz would not have found Gödel's provability incompleteness of axiomatic arithmetic all that shocking. Granted, he was drawn to the idea that calculation can always settle issues of mathematical proof—i.e. that provability can (somehow) be arithmetized. (That, after all, was the mission of his project of *calculus ratiocinator*.) But he also realized that the realm of mathematical *truth* outran that of *provability*,

seeing that—to formulate it anachronistically—since probability is recursive, the totality of provable propositions is enumerable, while that of mathematical truths (specifically those regarding reals) is not.

11.2.6 *Quantitative Disparity and Provability Incompleteness*

Leibniz too was able to envision the provability incompleteness of arithmetic, albeit on grounds radically different from those at issue with Gödel. For with Gödel, incompleteness is a matter of qualitative specificity, while for Leibniz it was one of quantitative generality.

The somewhat complicated story at issue here begins with Leibniz's thoughts about infinite pluralities. Here, Leibniz's argument against infinite numbers rests on the premiss that numbers must measure the size of units, which must be definite (and thus finite) boundaries. This, of course, separates Leibniz from post-Cantorian mathematicians such as Gödel. Thus, for Leibniz, while there are infinite *manifolds* having countless individual units as members, there are no infinite *magnitudes* or infinite *numbers* to measure their size. Such infinite manifolds are literally innumerable: their vast size does not admit of numerical quantification. Accordingly, Leibniz maintained: "I grant (the prospect of) an infinite multitude, but this multitude does not form a number or total unity, but means no more than there are more terms than a number can designate."⁴⁷

However, Leibniz realized that infinite multitudes can differ immensely—literally immeasurably—in quantity. In specific, these infinite manifolds can be of two sorts:

(1) the *discrete* involving steps of endlessly ongoing repetition, which can be of two sorts, viz.

- by *addition* as per 1, 2, 3. . . subject to the operation $+$ 1
- by *division* as per 1, $\frac{1}{2}$, $\frac{1}{4}$. . . subject to the operation $\div 2$

and:

(2) the *continuous* as per a geometric line segment or a temporal interval.

After all, "It has been known since Aristotle that treating a (continuous) line as a set of discrete points cannot do justice to its continuous nature . . . (because) points are (always) isolated from one another"⁴⁸

The relationship between these two modes of infinitude preoccupied Leibniz a great deal, for these matters lie at the core of his extensive discussion of what he called the *Labyrinthus continui*, the Labyrinth of the Continuum.⁴⁹ And as he saw it, although the point on a line segment or the real numbers of an interval altogether constitute a continuity or plenum, nevertheless "*Le continue, quoyqu'il ait partout de tels indivisibles [sc. points] n'en est point composé.*"⁵⁰ And specifically, Leibniz realized that successively aggregating points by repeatedly adding more and more—even infinitely often—one will never arrive at an interval. (And by successively

dividing intervals—even infinitely often—making them smaller and smaller, you will never arrive at a point.) There are too many points in an interval to be reached by successive additions. A continuum such as that of the real numbers “*n’est point formé par l’addition des parties.*”⁵¹ No matter how many points you add successively, there will always be too few. There is a quantitative (albeit not numerical) disparity between resolution point aggregations and the points constituting a continuous interval.⁵²

The matter at issue is helpfully clarified by means of the story of Hilbert’s Hotel, which has it that this establishment has an unending corridor of rooms as per:

1	2	3	4	5	6	
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The hotel is never full but can always accommodate more guests. For when n new guests arrive, the hotel simply puts them into Rooms 1 to n , and moves the former Room 1 guest to Room $n + 1$, the Room 2 guest to Room $n + 2$, and so on. The example typifies the problems and conceiving of such a thing as an infinite number. For, as the example shows, addition will not here alter size, $N + n = N$.

Or again consider multiplication. Let it be that the size of the rooms is rather large. The management decides to increase the establishment by a remodeling that always creates two rooms where there had only been one before, changing Room 1 into 1A and 1B, Room 2 into 2A and 2B, etc.

But now consider the following theses:

- (1) Duplicating yields a result that is larger. So, if there is an infinite number N of rooms in the hotel’s “before” condition, there must be $2N$ afterwards with $2N > N$.
- (2) A 1-to-1 matching determines equality. And this can be effected before and after via the matching arrangement:

1	2	3	4	5	6	7	8	etc.
1A	1B	2A	2B	3A	3B	4A	4B	etc.

- (3) Universally measurable (numerological) determination of the hotel’s rooms is indeed possible. The idea of “the number of rooms” also holds good in the infinite case.

It is clear that (1)–(3) are incompatible. At least one of them has to go.

So, let R^- = the number of rooms before

R^+ = the number of rooms after.

But given these specifications, we must now abandon one of the following:

- (1) $R^+ > R^-$
- (2) $R^+ = R^-$
- (3) R^+, R^- are meaningful numbers: well-defined quantities.

There are three alternatives here:

- (1) abandonment (G. Cantor)
- (2) abandonment (No-one: Everyone accepts one by one equalization)
- (3) abandonment (G. W. Leibniz).

Cantor accepted infinite numbers as measures of set size, but abandoned the rules that define their traditional interconnection. Leibniz, by contrast, accepted the standard rules (as defining what numbers are) and consequently rejected the numerical measurement of infinite quantities. While Leibniz held a pre-Cantorian conception of numbers, fact remains that he fully realized that there are “just as many” even integers as integers altogether) was to his mind a decisive block to numerosity, because numbers had to obey the authentically laws.

Leibniz thus wanted to distinguish between quantities as such and specifically *measurable* (i.e., numerically assessable) quantities. For him, those infinite multitudes are innumerable because, for him, size comparison of quantitative extent is one thing and numerical measurement quite another. Thus, he writes:

*Et non obstant mon Calcul Infinitésimal, je n'admetes point de véritable nombre infini, quoique je confesse que la multitude des choses passe tout nombre fini, ou plutôt, tout nombre.*⁵³

For Leibniz, there are not and cannot be any infinite numbers, be they infinitely large or infinitely small.

This does not, however, mean that those infinite multitudes are of the same size. For Leibniz as for us, a discrete infinitude is one thing and a continuous infinitude another. And a discretely infinite multitude that can be counted off as per 1, 2, 3 etc. is vastly smaller than an infinite multitude that constitutes a continuum—a difference that was crucial for Leibniz's theory of knowledge because the manifold of humanly knowable (identifiable, linguistically expressible) *truth* constitutes an at most discrete infinitude, whereas the realm of *fact* involves continuities.⁵⁴

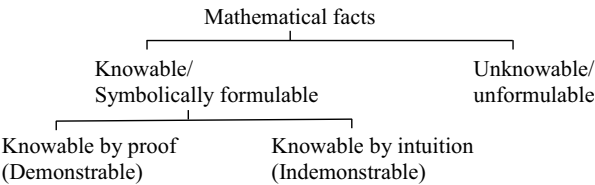
This view of the matter has profound epistemic implications when one considers that language is fundamentally discrete—its assertions articulated in the stepwise manner that one would nowadays characterize as recursive.⁵⁵ But the realm of fact—of what is the case in the objective, thought-independent mode of reality—is characterized by the larger, continuous mode of quantity. (For example, there will be a uniquely coordinate descriptive face for every real number.) There is thus a critical size disparity between assertable truths and objective facts in mathematics, as well as demonstrative truth and actual facts. (The infinite manifold of denumerable truth is countably infinite, seeing that truth must be articulated in a recursively developed language. But infinite manifold of fact is not subject to this limitation.)

There is an epistemic disconnect not just between truth and demonstrability but even between truth and facticity—and accordingly, knowability. Knowable truth cannot encompass actual fact. There are bound to be unknowable truths.

The crucial fact for Leibnizian epistemology is that we humans are finite beings who must use a discretely finitistic intelligence for knowledge regarding the endlessly

Display 11.1

LEIBNIZIAN VS GÖDELIAN INCOMPLETENESS



Leibnizian Incompleteness: Not all actual mathematical facts are knowable.

Gödelian Incompleteness: Not all knowable mathematical facts are demonstrable.

complex world contrived by an infinite intelligence. The limited resources of our logic and language confine us to the realm of the discrete, finitistic, recursive thought. Reality is not subject to this limitation. For Leibniz saw it, the supreme intelligence that has designed Reality operates at a level of complexity and sophistication that runs beyond our purview, its detail lost in the statistical blending. We humans must do digital reasoning in an analogue world, to put the matter anachronistically. For Leibniz, approximation and analogy are the best that we can manage in our efforts to secure a cognitive hold on the infinitely complex nature of reality.

Both Leibniz and Gödel were incompleteness aficionados, albeit of somewhat different kinds. For Leibniz, human intelligence cannot master the world’s complexities; for Gödel, inferential (i.e., axiom-based) processes cannot fully grasp the arithmetical complexities. And both agree that mathematics cannot be fully axiomatized—with Leibniz, in relation to the continuum of reals; with Gödel, even in relation to the arithmetic of integers.

11.3 Variant Perspectives Regarding Provability Incompleteness

In their concern for mathematical completeness, Leibniz and Gödel were concerned with distinct matters (See Display 11.1). The justificatory rationale for Leibnizian incompleteness is based on three considerations:

1. In view of the essentially recursive and thus enumerative nature of linguistic formulation, it follows that the manifold of linguistically/symbolically formulable mathematical propositions are discretely (i.e., countably) infinite. And so, *a posteriori* the manifold of demonstrable (possible) mathematical propositions is also only discretely infinite.⁵⁶
2. That there must be at least as many distinct arithmetical truths as there are real numbers because there has to be at least one uniquely characteristic truth about each real number—viz. its identifying characterization.
3. That the totality of real numbers constitutes a continuum, so that there are quantitatively more points on the real number axis than can ever be coordinated with rational numbers (let alone integers).

The upshot of these three considerations is a quantitative disparity: there must be more mathematical facts than can ever be specifically and individually demonstrated.

However, Gödel's rationale for provability-incompleteness is something quite different.⁵⁷ To get at the core of his thinking let us suppose that the characteristic hope of mathematical logicism were realized and that membership in the class of provable arithmetic propositions could be determined by calculation along the lines envisioned in Leibniz's *calculus ratiocinator*. Arithmetical provability could then be arithmetically internalized. Claims to provability in axiomatized arithmetic would thus be reformatable as an axiomatized arithmetical thesis. One would accordingly be in the position to claim that for a suitable arithmetical function F we would have it that:

$$\vdash p \text{ iff } F(p) = 1$$

(Here, $\vdash$ symbolizes provability in the envisioned deductive systematization of arithmetic.)

But as Gödel noted, there is now a crucial fly in the ointment, based on analogy with the Liar Paradox of classical antiquity. The problem is posed by thesis (1), which affirms its own unprovability:

$$(1) \sim \vdash (1) \text{ or now } F(1) \neq 1$$

Consider the question: Is (1) arithmetically provable? Obviously not. For if it were, then we would have $\vdash (1)$ so that $F(1) = 1$. And this is exactly what (1) itself denies. Thus, assuming arithmetic to be consistent, we have to acknowledge that (1) is not provable, so that $\sim \vdash (1)$. But just exactly this is what (1) itself asserts. So (1) must be true.

Accordingly, we have it that (1) is an unprovable arithmetical truth. Our axiomatized proof system (as represented by $\vdash$) must be deductively incomplete in that not every arithmetical truth is demonstrable. In the end, therefore, the arithmetical internalization of arithmetical fact defeats the prospect of provability completeness in the face of the sort of self-negation instantiated by our Liar Paradox variation.

And the same conclusion can be reached by other means. Since all propositions of the arithmetical system can be encoded, they can all be listed in the numerical order of their coded ID numbers. And since all of the propositions can be enumerated, so can the subset of provable propositions, i.e., theorems. So, let it be that $T_1, T_2, T_3,$

etc. is an inventory of all the theorems, so that $\vdash T_1$, and $\vdash T_2$, and $\vdash T_3$, etc. But now consider the thesis $(\forall i)(T_i \& T_{i+1})$. By hypothesis this will certainly be true. And yet it cannot figure on the inventory of provable theorems of the system since it differs from the i -th entry on the register for every value of i .

This variant “diagonalization” perspective thus yields yet another route to the provability incompleteness of arithmetic. It is, however, fundamentally different from everything considered by Leibniz.

11.4 Establishing Incompleteness: Similarities and Differences

Notwithstanding various analogies and similarities between Gödel’s and Leibniz’s views on logico-mathematical issues, Leibniz was not really in a position to anticipate the detailed nature of Gödelian incompleteness. This is so for two reasons:

- (1) The idea of using self-referential inconsistency as per the Liar Paradox was ruled out by Leibniz’s rejection of its meaningfulness.
- (2) Leibniz did not yet have access to Georg Cantor’s idea of diagonal argumentation.

The second of these (diagonal argumentation) is critical for Gödelian incompleteness because Gödel (unlike Leibniz) needs an arithmetical pathway to propositional self-reference. And the first of these (self-referential *inconsistency*) is critical for Gödel’s argument because the indemonstrable truth being constructed takes just this form. However, this entire line of thought was unavailable to Leibniz because he adopted a very different and indeed incompatible stance that rejected self-referential propositions of the sort at issue in the Liar Paradox as effectively meaningless.⁵⁸

Accordingly, the turn to Liar-like reflexivity which enabled Gödel’s argumentation to realize its aim is not something that Leibniz could possibly have anticipated because he viewed the sort of self-reference at issue in the Liar Paradox as meaningless and conceptually flawed.

And so, there is no place for any possible concern that Leibniz had anticipated a demonstration of provability-incompleteness along Gödelian lines. Leibniz came nowhere near to the technical details of Gödelian argumentation. And while Leibniz was alive to the demonstrability incompleteness of (real-number) arithmetic, he had no means of realizing Gödel’s more detailed and case-specific demonstrability incompleteness of integer arithmetic.

Ironically, the very circumstance of imperfect access to Leibniz’s work of which Gödel complained prevented him from realizing that concerns about anticipation were groundless.

To be sure, Gödel’s reasoning implements a much more general insight to which Leibniz might well have subscribed. Already in the early 1930s Gödel hit on a very simple way of getting to the crux of the matter, viz. that “the ‘truth’ of the propositions of a language (or system) *cannot be expressed* in the same language,

while provability (being an arithmetical proceeding) *can* (be so expressed). Hence true $\neq$ provable.”⁵⁹ The claim “proposition *P* is true” as formulated within the system can only address its system-internal status, while actual truth is something else that is inherently system-external. And so, truth as such will be something substantially distinct from intra-systemic provability and truth determination cannot possibly be achieved by demonstrability in a coherent axiomatic systemization.

11.5 Was There a Leibniz Conspiracy?

Apparently, Gödel thought he had telling evidence for this Leibniz conspiracy, as per the following episode reported by Karl Menger:

I once discussed Gödel’s ideas on Leibniz with a common friend, the economist Oskar Morgenstern. He described to me how Gödel one day took him into the Princeton University Library and piled up two stacks of publications: on one side, books and articles that appeared during or shortly after Leibniz’ lifetime and contained exact references to writings of the philosopher published in collections or series (with places and years of publication, volume and page numbers, etc.); on the other side, those very collections or series. But in some cases, neither on the cited page nor elsewhere was there any writing by Leibniz; in other cases, the series broke off just before the cited volume or the volume ended before the cited page; in still other cases, the volumes containing the cited writings never appeared. “The material was really highly astonishing,” Morgenstern said.⁶⁰

Gödel believed that Leibniz was an even greater thinker than posterity has realized. For Gödel viewed Leibniz’s conception of *characteristica universalis*—or an alphabet of thought—as preliminary to his (Gödel’s) own plan for the symbolic representation of propositions in a manner that renders their logical relations transparent. And Gödel believed that Leibniz had actually carried this project out much further than the written record suggests. And he apparently thought that Leibniz’s pioneering ideas evoked the opposition of a hidebound establishment. Thus he told Oskar Morgenstern that he believed that “Leibniz had been systematically sabotaged by his editors,”⁶¹ and confided in Karl Menger his suspicions that some of Leibniz’s “important writings. . . had not only failed to be published, but (had been) destroyed in manuscript.”⁶² Gödel was particularly struck by the absence of any discussion of Leibniz’s universal characteristic in any publications dating from his lifetime and well beyond.⁶³

However, Gödel’s charges are decidedly problematic. By all visible indications, Leibniz’s writings were preserved conscientiously, and were published more and more fully and carefully as the years went on.⁶⁴ Rather than tolerating the destruction of Leibniz manuscripts, the Hanoverian archivists went to great lengths to preserve them. (Their Leibniz material runs to several hundred-thousand items!) And the shortfall of published material looks to be more a product of ignorance than of malign and conspiratorial intent.⁶⁵

The severest censor of Leibniz’s work in logic was the man himself, who simply did not publish it. The vast amount of ground-breaking work that he did in this field

was simply kept out of people's sight. But for this Leibniz was himself responsible, and not some hostile conspiracy. When he could not persuade an otherwise sympathetic Christiaan Huygens of the value of his logical projects, Leibniz gave up and turned attentions elsewhere.

Generations of editors have labored over the centuries to bring Leibniz's *Nachlass* into the light of print.⁶⁶ And if Leibniz's logical work was underrepresented it was only because logic itself had not developed to a point where it could be understood and appreciated. The reality of it is that the earlier editors did not appreciate Leibniz's works on symbolic logic because it was "all Greek" to them. Only when the discipline itself had been rediscovered did there come into existence the wisdom-of-hindsight contextualization requisite for appreciating Leibniz's contributions. And the lack of a proper appreciation of Leibniz's logical work is due simply to the fact that before a great deal of Leibniz's own logical innovations had been rediscovered, it was somewhere between difficult and impossible for people to see what he had actually accomplished.

Gradually intensifying paranoia led Gödel to believe that malign forces were astir in the world not only to make men slaves (as per Hitlerian fascism and Stalinesque communism) but to render them unthinking as well—forces so powerful as to operate even in democratic societies. As one Gödel scholar puts it, "He came to believe that there was a vast conspiracy, apparently in place for centuries, to suppress the truth 'and make men stupid'."⁶⁷ In this regard, he felt that those thinkers who, like Leibniz and himself, were persuaded of nature's fundamental rationality were destined to have those ideas suppressed and distorted and go "rejected and despised" (as Händel's *Messiah* puts it).

All the same, the idea of a Leibniz-suppressing conspiracy is untenable for many reasons. Adequate appreciation of Leibniz's logic would have required knowledge of Leibniz's work to be of a quantity and quality that was effectively unavailable until Louis Couturat in the early 1900s. Only with Couturat and Russell did working logicians begin to manifest a concern with Leibniz, and even Russell—whose Leibniz book antedated Couturat's pioneering work—did not really understand Leibniz's logic properly. Moreover, the forces "dumbing the world down" were not the work of conspiratorial intellectuals but the vast socio-cultural movement energized by the disasters of the 1914–18 war and the consequent economic and societal disruptions throughout the world. Overall, it seems that the natural shortcomings of man—both individually and socially—suffice to account for the phenomena that troubled Gödel; no malign conspiracy theory was required here.

And so, the fact remains that Gödel viewed himself as a fellow victim to an ideological antagonism of the sort which (as he thought) blighted the acceptance of Leibniz's pioneering ideas in logic. As he saw it, both he and Leibniz were kindred spirits embarked in the thankless mission of promoting reason in an irrational world,⁶⁸ fellow victims of the doctrinal hostility of an uncomprehending and unsympathetic ideological environment opposed.⁶⁹

And indeed, with regard to the reception of their idea on the foundations of logic, both Leibniz and Gödel were caught up in a similar difficulty. However, this was not a matter of conspiratorial opposition by doctrinally hostile opponents, but rather by

being out of tune with the ideologically alien spirit of the times—victimization, if you will, not by opposition but by indifference. The mathematical treatment of logical discourse and inference that fascinated Leibniz had little appeal for his contemporaries and none for his immediate successors. And something of the same sort was the case with Gödel, whose rationalism and idealism was simply out of synch with the rampant relativism of the day.

Notes

1. *Wikipedia*, art. “Kurt Gödel.”
2. Dawson 1997, p. 6 (my interpolations).
3. For details, see van Atten 2015, Chap. I.
4. van Atten 2015, p. 33.
5. Relevant data are given in the chapter “On the Philosophical Development of Kurt Gödel” by Mark van Atten and Juliette Kennedy in S. Feferman et al., *Kurt Gödel: Essays for his Centennial* (Cambridge: Cambridge University Press 2010), pp. 275–325.
6. Goldstein 2005, p. 48.
7. “*Die Welt ist vernünftig*.” See Goldstein 2005, pp. 20–21, 30–31, 40, 55, and 231.
8. On these matters, see Dawson 1997, p. 263.
9. van Atten 2015, p. 273, n. 99.
10. van Atten, 2015, p. 139.
11. van Atten, 2015, p. 23.
12. Karl Menger, *Reminiscence of the Vienna Circle and the Mathematical Colloquium*, by L. Golland et al. (ed.) (Dordrecht: Kluwer, 1994).
13. *Ibid*.
14. Menger 1994, pp. 122–123.
15. For details of Leibniz’s position see Louis Couturat’s *La logique de Leibniz* (Paris: Vrin, 1901), pp. 327 ff.
16. Leibniz’s elaborate forays into the cryptographic arithmetization of text are described in detail in N. Rescher, *Leibniz and Cryptography* (Pittsburgh: University Library System, University of Pittsburgh, 2012).
17. See Fichant 1991.
18. Kurt Gödel, “Russell’s Mathematical Logic,” in his *Collected Works* (ed.), Feferman et al., 1986, p. 128.
19. The various facets of Gödel’s Platonism are exhaustively detailed in van Atten 2015.
20. Kurt Gödel, *Collected Works III: Unpublished Lectures and Essays*, ed. Feferman et al. (Oxford: Oxford University Press, 1995), p. 321.
21. For a clear and comprehensive account of Gödel’s Platonism, see Parsons 2010.
22. Davis 2010, p. 235.
23. *Stanford Encyclopedia of Philosophy*, art. “Kurt Gödel,” p. 11.
24. Davis 2010, p. 239.
25. On the deep analogies between set theory and Leibniz’s monadism, see van Atten 2015, Chap. 3.

26. “Truths of fact, unlike truths of reason, thus have an essential existential component to the effect that the substances of this possible World are the ones that actually exist. The necessary, eternal truths, insofar as they deal with items of contingent existence at all, deal with them in a strictly hypothetical way” (Leibniz, P V, 428–429). “Caesar crossed the Rubicon” typifies the one case; “If Caesar crossed the Rubicon, then someone crossed a river” typifies the other.
27. See van Atten 2015, pp. 37–39.
28. Dawson 1997, p. 263.
29. Davis 2010, p. 239.
30. Davis 2010, p. 239.
31. Fefferman et al., 2010, p. 280.
32. van Atten 2015, p. 102.
33. See the chapter on “*Le Cacul Logique*” in Couturat 1901. Between 1675 and 1695 Leibniz developed three distinct approaches, none of which satisfied him completely. (He sought in vain for collaborators on their development.)
34. van Atten 2015, p. 128.
35. P III 343. “Il est vray, que nous connaissons notre existence par un intuition immediate, et celle de Dieu par demonstration” (Leibniz, P V, 343).
36. Davis Davis 2010, p. 239.
37. van Atten 2015, p. 102.
38. van Atten 2015, p. 160.
39. van Atten 2015, p. 129.
40. van Atten 2015, p. 139 (my interpolations). By “ramifications,” Gödel presumably means *obstacles*.
41. See Parsons in Feferman et al. 2010, p. 352. Some theorists have it that the axioms can be demonstrated by “indirect proof” via the untenability of their negation. But the materials for such a demonstration have to come from somewhere, and intuition of some sort is the best (and perhaps only) option here.
42. Kurt Gödel, *Collected Works* (ed. S. Feferman et al.), Vol II (Oxford: OUP, 1990), p. 186. Compare Martin Davis in Feferman et al. 2010, pp. 324–236.
43. See Chap. VI, “*La Science Générale*” in Louis Couturat, *La logique de Leibniz* (Paris: Vrin, 1901).
44. Wang 1996, p. 164; quoted in van Atten 2010, p. 152
45. *Monadology*, Sect. 30; my interpolations.
46. van Atten 2015, p. 102.
47. As Leibniz told John Bernoulli: “*Concedo multitudinem infinitam, sed haec multitudo non facit numerum seu unum totum; nec aliud significat, quam plures esse terminos, quam numero designari possint, prorsus quemadmodum datur multitudino seu complexus omnium numerorum; sed haec multitudo non est numerus, nec unum totum.*” (Leibniz, M III/2, p. 575)
48. van Atten 2015, p. 87.
49. Leibniz, P I 416.
50. See, for example, Leibniz, M VII 326.
51. Leibniz, P III 583.

52. See Leibniz, P I 338.
53. Leibniz, P VI 629.
54. L. E. J. Brouwer came to much the same conclusion, seeing that he argued that no language with countably many expressions can exhaust the continuum of real numbers, hence one needs further appeal to intuition. Of course, the theorems that Gödel went on to demonstrate are of a different and more specific nature (van Atten 2015, p. 195). This, of course, explains why Brouwer—unlike many other theoreticians—did not think Gödelian incompleteness changes anything fundamental.
55. See Fichant 1991.
56. See N. Rescher, “Leibniz and Issues of Eternal Recurrence” in his *Philosophical Episodes* (Frankfurt: Onto, 2011a, b), pp. 33–55. Also, Michel Fichant, *G. W. Leibniz: De l’horizon de la doctrine humaine* (Paris: Vrin, 1991).
57. Gödel’s original paper is available in English as *On Formally Undecidable Propositions of Principia Mathematica and Related Systems*, tr. B. Meltzer, (New York: Basic Books 1962; Dover reprint, 1992.) And also in Martin Davis (ed.), *The Undecidable* (Mineola, NY: Dover, 1993). For an accessible informal exposition of the reasoning involved, see Rebecca Goldstein, *Incompleteness: The Proof and Paradox of Kurt Gödel* (New York: W.W. Norton, 2005). See also E. Nagel and J. R. Newman, *Gödel’s Proof* (New York & London: New York University Press, 1945; revised ed. 2001).
58. See Rescher 2020, Leibniz’s account if the matter is Artosi et al. (eds.), 2013.
59. Kurt Gödel, *Collected Works* (ed. S. Feferman et al.), vol. VI (Oxford: Oxford University Press, 1995), pp. 9–11 (my interpolations). Compare Mark Davis in Feferman et al. 2010, pp. 230–231.
60. Menger 1994, pp. 223–224. Compare another report: (The Princeton economist Oskar Morgenstern) had been alerted by Gödel as to the deliberate suppression of Leibniz’s contributions and had tried to argue the logician out of his conviction. Finally, to convince Morgenstern, Gödel had taken the economist to the university’s Firestone Library and gathered together “an abundance of really astonishing material,” in Morgenstern’s words. The material consisted of books and articles with exact reference to published writings of Leibniz, on the one hand, and the very works cited, on the other. The primary sources were all missing the material that has been cited in the secondary sources. “This material was highly astonishing,” a flabbergasted (if unconvinced) Morgenstern admitted. See Goldstein 2005, p. 248 and also Dawson 1997, pp. 166–167.
61. Dawson 1997, p. 166.
62. Goldstein 2005, p. 247. Further aspects of the matter are discussed in N. Rescher, “Gödel’s Leibniz Conspiracy,” Chap. 13 of his *Philosophical Episodes* (Frankfurt: ONTOS Verlag, 2011a, b).
63. See Dawson 1997. Gödel seems not to have appreciated the fact that Leibniz himself was the severest censor of his own work and that he deliberately abstained from publishing many of his ideas and projects. Accordingly, he wrote: *Qui ni non nisi editis novit, non novit* (“Who knows me only by published work does not know me at all”).

64. This is amply attested by Emile Ravier's magisterial Leibniz Bibliography and by the archival records published in two substantial volumes by Eduard Bodemann.
65. Scholars often underappreciated various of Leibniz's discussions because in their day the ideas at issue themselves yet awaited rediscovery.
66. Emile Ravier's monumental *Bibliographie des oeuvres de Leibniz* (Paris: Alcan, 1937) runs to some 700 pages.
67. Goldstein 2005, p. 48.
68. One very positive effect of Gödel's Leibniz conspiracy was its contribution to bringing copies of Hannover's Leibniz manuscripts across the Atlantic. Gödel energized Morgenstern to efforts in this direction, which ultimately came into confluence with the cognate efforts of Paul Schrecker of the University of Pennsylvania and ultimately led—with Rockefeller Foundation funding—to securing copies of the Hanoverian Leibniz material for that university (See Dawson 1997, pp. 189–190).
69. "(Gödel's mathematical colleagues) weren't amused by (what he called) his 'strange axiom,' his version of Leibniz's principle of sufficient reason, which disposed him to believe that everything that happens has a thoroughly logical explanation." (Goldstein 2005, p. 236).

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Chapter 12

Reification Fallacies and Inappropriate Totalities



12.1 Improperly Totalized Wholes and Illicit Reification

Philosophers have generally felt ambivalent about the idea of totalities. There is no problem with limited totalities as per generalization made about all elephants or all electrons. But virtually unrestricted totalities—say, of sets, of truths, of facts, let alone of *everything*—are another matter. Ironically, it appears that totalization has its limits: that nothing can sensibly be said about everything. The task of the present deliberations is to explore some of the instructive ramifications of these difficult ideas.

It is constructive that some of the problems and difficulties that logicians generally seek to overcome by elaborate *formal* devices such as the theory of types can be averted by straightforward *informal* devices, namely the rejection of inappropriate anaphora, the mistake committed in communication when a back-reference treats an item yet to be specified as already identified.

The circumstance that questions can rest on inappropriate suppositions has far-reaching implications. In particular, it means that there are—or can be—items of discussion that are no more than pseudo-realities which fail to give rise to meaningful questions and do not qualify as a viable subject of further inquiry. And it is noteworthy that various sorts of putative totalities constitute a case in point.

To introduce something into deliberation or discussion as a meaningful item of consideration, we have to proceed by way of expository definitions or explanatory specifications that employ only discursive materials that themselves have *already* been introduced—at any rate, apart from whatever may qualify as a *primitive* on the basis of being so self-evident as to require no further introduction. And, above all, the *identification* of something can be achieved satisfactorily only through reference to items that have already been identified themselves. It does us little good to be told that something we seek is sited next to something else whose location we do not know.

Thus, consider the following identification-purporting specifications:

- The individual in question is the first child of the mother who is *this person's* female parent.
- The position in question is the one to the left of that position which is to *its* immediate right.
- The woman in question is the wife of the man who is *her* husband.

Although there is some description going on in all such cases, there is certainly no question of any effective identification. For in each case the formula's anaphoric back-reference to the item in question itself supposes an *already* effected identification. All of these formulations fail as identifications because they require a prior item-identification which (by hypothesis) is unavailable in view of the identification processes that are being embarked upon. In each case, the purported identification manifestly presupposes a yet-unavailable answer to the identificatory question that is being addressed.

Here, as elsewhere, what that answer is depends on just exactly what the question at issue happens to be. If the question is one of introductory individuation—of identificatory specification—then referential *self*-involvement is inappropriate because the “self” at issue is not yet in hand. On the other hand, if it is only a matter of the description or characterization of some pre-identified item, then self-involvement is harmless because the “self” at issue has already been specified. As long as the self-reference at issue is merely a matter of dealing with something that is already specified, there is no problem. But whenever we are dealing with the *introduction* of items by way of an *initial* specification or definition, then it is clear that the availability of these items cannot be appropriately assumed *at this stage*.¹ Self-referential item-specifications and identifications are by their very nature ineffectual because until the item supposedly at issue has been properly identified, there is nothing to discuss.

Now, what holds here of *identification* also holds of any meaningful *totalization*. Totalization as such is harmless enough. The totality of *Xs* is the putative item that includes all of the *Xs*—that is, contains or embraces them in whatever way is appropriate to the sort of thing at issue. Thus, the totality of letters of the alphabet is the alphabet as such, the totality of colors is the entire color spectrum, or the totality of plants is the entire kingdom of flora. However, *self-referential* totalization is by nature an illicit process, because identification or specification of a totality is meaningfully practicable only when the item being totalized has already been identified *antecedently* to the totalization process itself.

It is one thing—a harmless one—to be *given* a total and thereupon to learn that it is duly self-inclusive—a “fractal” rectangular pattern, say, within each of whose four quadrants this very pattern is repeated all over again. But it is something very different—and altogether vitiating—to attempt to *introduce* (define, identify, specify) a totality in a way that demands or presupposes that this putative item is *already* available through previous introduction.

Perplexity is thus bound to arise when there is *homogenous* totalization, that is, when the items being totalized are of the same type (thing-kind) as the totality that is supposedly at issue. There is thus a decisive difference between the identificatory

specification of “The set of all sets that include less than five members,” which is innocuous since this set certainly cannot include itself, and the specification of “The set of all sets that include more than five members,” which—were it to exist—would have to include itself. The homogeneous totalization at issue accordingly represents a putative item that the formula at issue fails to specify in a meaningful way. Self-referential identifications are by nature inappropriate, and we must accordingly place a conceptual embargo on self-inclusively characterized totalities.

This occurs in such cases as when: the totality of sets is supposed to be a set that includes *all* sets; the totality of regions is supposed to be a region that includes *all* regions; or the totality of timespans is supposed to be a timespan that includes *all* timespans, and the like for such things as facts, items, events, etc. All such attempts at identification are uniformly problematic because no meaningful item-introduction is effected when the totality that is being introduced is itself one of the items being totalized: when it is, so to speak, one of its own members (elements, constituents, components, or the like). For then it is implicitly supposed (via the presence of that “all”) that the total being introduced is itself *already* available. And the difficulty that arises with homogenous totalization is clearly the problem of self-inclusion. The membership of a meaningfully identified totality must have a pre-established identity: those envisioned members must be identified prior to and thereby independently of the totalization that is in progress.

Just as the object of a meaningful item-specification must have a pre-established identity independent of that specification itself, so must the objects that are being gathered together in an item-totalization. But when the totality supposedly at issue is homogenous with the items being totalized, then of course we would have to have it that the totalization at issue is vitiated. It is clearly inappropriate to offer *a specification based on the supposition that the very item at issue in this specification has already been specified*. For meaningful (proper) identification, we cannot presuppose the availability as *input* into the identification process the very item itself that is supposed to be the *output* of that process. The illegitimacy of self-involving specifications roots in the fact that self-reference presupposes that the item in question is *already* available as a meaningful unit of discussion.²

Let us adopt the notation that when C is an item-characterization, $\langle C \rangle$ will abbreviate “the totality that embraces all of the C -type items.” Thus, for example, if $C(x) = “x$ is a letter of the (Roman) alphabet,” then $\langle C \rangle$ is the totality of letters of the (Roman) alphabet, which is to say it is that alphabet itself. If $C(x) = “x$ is a cat,” then $\langle C \rangle$ is the totality of the genus Felix that includes all the felines there are. Or again, if $C(x) = “x$ is a color,” then $\langle C \rangle$ is the totality of colors, which is to say it is the entire color spectrum.

On the basis of the preceding deliberations, it is clear that whenever the totality $\langle C \rangle$ that is presumably being defined is “itself” seen as something that meets condition C —which is emphatically *not* the case with any of the preceding examples—then the totalization process in question goes awry since it is now unable to realize a well-defined result. A purported item that is not subject to a discussion-introducing identification in terms of reference independent of itself has simply not been introduced meaningfully into the discussion at all.

Examples of putatively totalized wholes that violate the proscription of self-presupposition include the following pseudo-totalities:

A *supertruth* understood as the truth that conjoins all truths.³

A *cosmic fact* about the world understood as a fact that encompasses all facts about the world.

A *megaset* understood as the set that includes all sets.

An *eternity* understood as the timespan that contains all timespans.

A *pan-explanation* understood as the explanation that encompasses all explanations (that explains everything).

A *protocause* understood as the cause of all causes.

A *mega-story* understood as the story that encompasses all stories.

A *superadjective* understood as the adjective that applies to all adjectives.

A *world-all* (“*Weltall*”) understood as the physical region that includes all physical regions.

An *omni-cause* understood as the cause that causes literally everything that is real.

In each case we have a putative item that is identified by means of a common format: “The *C*-type item that ‘embraces’ all *C*-type items.” And this clearly presupposes not only that there indeed is something that “embraces” all *C*-type items, but that this something is itself of type *C*. Throughout, the proscription of self-involving totalization is thus violated.

And so, there yet remains the prospect of objecting: “But if it is *defined* as an *X*, if its *X*-hood is something that is explicitly specified in its definition, then surely it’s got to *be* an *X*!” This plausible product does not, however, hold water. As the longstanding critique of the Ontological Argument shows, substantive questions cannot be settled by definitional fiat. (This is a cardinal principle of rational inquiry.) *Calling* something an *X* does not mean that this is an actual item that is actually an *X*. Thus, “the integer that is larger than any other” is a formula that purports to specify an integer but actually fails to do so, seeing that there is no integer larger than all others. And similarly, to *call* a supertruth a truth or a megaset a set does not mean that this is actually so—that there really is a set or a truth that answers to the specification at issue. For that item may simply not exist as such. The assumption that something exists under a certain description—for example, as the set of all sets that do not include themselves—may well be false. Anything that answers to this description—whatever it might be—cannot be a set. That supposed specification simply fails to identify.

To be sure, *hypostatization* (item-introduction) is one thing and mere *description* (of a pre-identified item) something else again.

It warrants remark that there is a crucial disanalogy between identifying, on the one hand, and seeing or describing, on the other. We can only consider “the thing identified” once some “identification of the thing” has been given. By contrast, nothing whatever about the describability of something rests upon the description of this

thing. The hypothetical removal of its identification creates problems for discourse about something in view in a way that the hypothetical removal of its description would not.

To see more clearly what is at issue here, we may begin by noting that a formula along the lines of:

(A) a total that satisfies condition C in relation to itself

is always viable as a *description*. Thus, for example, the indefinite description:

- *X* is a set that satisfies condition C in relation to itself

is always meaningful in that its ascription to any well-defined set *S* results in a significant claim that is going to be true or false. However, the definitely descriptive identification:

(B) the total *X* that satisfies condition C in relation to itself

is inappropriate as a way of identifying or specifying a set on grounds of self-involvement. The only exception to this rule arises when the (B)-corresponding indefinite (A)-style description is such that it can be shown that there is a definite particular item (independently specified) which is such that (1) it answers to the (A)-description, and (2) that it is the *only* item that does so. For meaningfulness, however, both of these conditions—exemplification and uniqueness—must be met.

For example, consider:

- the set that includes all sets having one element in common with itself.

This is an inappropriate item-specification on grounds of self-involvement. But the situation is quite different with:

- the set that has no member in common with itself.

For here we can show that the null set, and it alone, answers to the description of being:

- a set that has no member in common with itself.

The preceding exception condition is thus satisfied. To be sure, we could not *define* (or introduce) the null set in this way. But once it is available, the indicated formula can unproblematically be applied to it by way of definite description.

All of those indicated specifications purporting to identify a totalized item of some sort are in fact inappropriate, and when *introduced* in the indicated manner, all of these supposed items are no more than illusions. Their specificatory self-presupposition

precludes appropriateness throughout. As noted from the outset, meaningful totalizations cannot involve an anaphoric back-reference that is self-referential; such reference must proceed with respect to items that have an antecedently established identity.⁴ And we therefore cannot appropriately effect a totalization whenever its purported product is itself a candidate for inclusion among the items being totalized. Only when its constituents are *antecedently* identified can we have a meaningfully defined totality that is to be comprised of “them.” And we are certainly not entitled to presuppose the *prior* specification of a (totalized) item whose specification is only now in progress.

A significant lesson emerges. Not only are there *pseudo-questions* like “Have you stopped beating your wife?” that should not be asked (because they rest on inappropriate suppositions), but there are also *pseudo-things* that should not be considered (investigated, taken seriously) because their very conception is flawed in that it rests on the erroneous presupposition of a particular answer to an inappropriate question.

However, it is important to stress the limited bearing of the point at issue. Nothing is necessarily amiss with a self-referential characterization when this is seen *as a descriptive remark* about an independently pre-identified item. There is no problem about (say) characterizing a pre-identified being (i.e., God) as that which is “the ultimate reason of all being” or again as that which “self-caused—*causa sui*—the cause of itself.” *Describing* an already individuated God by such formulas could in theory qualify as perfectly meaningful. But we nevertheless cannot *identify* God by a decision-introducing characterization as “the reason-for-being of all reasons-for-being” or as “that being which is the cause of all being, itself included.” For when seen *as an identifying specification*, self-reference must be rejected as counterproductively vitiating. We can certainly say—truly and meaningfully—of some pre-identified item that “it is the result of the cause that produced it.” But we cannot use this sort of formula when the initial identification of the item is at issue (“the item that is to be at issue is the product of its cause”). For here, nothing is as yet available to serve as referential for that ultimate anaphoric back-reference.

What is at issue here is the elemental principle that if an identificatory specification itself involves a reference to some item, then it can only succeed in its identificatory mission if the said item has *already* been identified. For successful identification requires and presupposes an affirmative answer to the question: “Have all the items being referred to already been identified effectively?”

Let us be somewhat more explicit about the ramifications of this state of affairs.

12.2 Antinomies

The avoidance of improper hypostatizations is an idea that goes back to Immanuel Kant who realized—and emphasized—that without such a proscription of illicit totalization, we are going run into self-destructive perplexity. For instance, let us suppose the following definition:

T = the conjunction-of-truths that conjoins all (but only) those conjunctions-of-truths that do not include themselves as conjuncts.

Clearly, then, paradox ensues. For:

If T includes T as a conjunct, then it does not.

If T does not include T as a conjunct, then it does.

The way out here is, of course, to say that—its overt specification as a conjunction notwithstanding— T is not a proper conjunction at all: an item of the specified sort simply does not exist. Neither T nor anything else fits the bill. Which is to say that T is no more than an illicit or improper pseudo-specification of an item purportedly at issue. Put differently, that purported “conjunction-of-truths” does not in fact qualify as something that can be characterized in this way at all: there can be no such proposition.

And analogous paradoxes arise in relation to the other sorts of illicit totalities—with strictly analogous results. Without a proscription of illicit totalization, our reasoning about such matters is bound to fall into paradox. In particular, when we engage in unrestricted totalization, we run into the dead-end of the Russell paradox by having to contend with:

- the totality of all (and only) totalities that do not include themselves.

For this totality, once admitted as such, must either include itself (in which case it does not), or not include itself (in which case it does). Unrestricted totalization is a logically unacceptable process.

However, what is wrong with the Russell paradox set—the “set of all sets that do not include themselves”—is not so much that it does not exist but rather that it is not even well-defined and thereby fails to represent a meaningful specification of something. For that would-be specification incorrectly presupposes an affirmative answer to the question: “Can the totality of sets that do not include themselves be characterized as a set?” And the flaw here does not arise from mere self-reference as such but from the fact that *the particular way* in which self-reference occurs involves an inappropriate totalization and thereby runs into paradox.

To speak of the aggregate of all sets meeting a certain condition, or the conjunction of all propositions meeting a certain condition, or the like, is to beg a question. It is, in each case, to presuppose that sets can meet the condition in question or that conjunctions of propositions can do so. And this presupposition can prove to be false and this bears with special force on the deliberations at issue here. For to see totalization as homogenous—to represent the totality of all X s as itself an X —is through this very step to commit an identification error.

And here we slide down a slippery slope towards further problems. The illicit totalizations we have been considering are a prolific source of inappropriate questions. For clearly we can only ever be asked (appropriately) to explain the origin, existence, and interrelationships of such items as have actually been identified. With

respect to improperly specified pseudo-entities, to raise questions that by their nature presuppose existence is to open the doorway to fallacy.

12.3 The Root of the Problem

Recall that the identification operator $(\iota x)Px$ so functions that: (1) it is undefined unless there is just exactly one specifiable item that has the property P , and (2) when there is just exactly one such item, then $(\iota x)Px$ is (identical with) that item. And, here, saying that $(\iota x)Px$ is properly defined—symbolically $E!(\iota x)Px$ —is to say that it represents a unique item within the range to our universal quantifier $\forall$. Thus understood, $(\iota x)Px$ will be defined only when P plainly and unequivocally applies to a single object. And it remains *undefined* whenever there fails to be just exactly one single unique x such that Px obtains and in particular when P is an “unsuitable” predicate—say, because it is equivocal or ill-defined. Specifically, this means that that “the set of all x such that Px ”—namely $[\int x]Px$ —will be defined only if all of its members “logically pre-exist” this putative total itself. That is, only once all of its putative members are identified independently of and without reference to anything that requires or entails the existence of $[\int x]Px$ itself will this putative totality be a meaningful item.⁵ The infeasibility of totalization in unsuitable conditions deserves to be seen as a natural consequence of the definitional specifications at work: unsuitable predicates demonstrably do not totalize.

Given this approach, no special machinery along the lines of a theory of set-types need be adopted to resolve the paradoxes posed by “illicit totalities”: those paradoxes simply do not arise because these problematic totalities are simply not well-defined.

12.4 Russell’s Vicious Circle Principle

What is at issue here is closely allied to—but yet not identical with—what Bertrand Russell characterized as the:

Vicious Circle Principle (VCP): No collection (whole or totality) can contain members that are defined in terms of itself: specifically, no collection can ever be a constitutive part of itself.⁶

As it stands, this is clearly a limitation upon the *constitution* of totalities—and a very strong limitation at that. To quote Russell: “Whatever involves *all* of a collection must not itself be one of the collection.”⁷ Saying that such a “collection” “has no total” is to say that it does not exist as a collection. What we have here is a restriction on the sorts of collections that can exist—that is, upon how authentic collections can validly be constituted.

Russell’s idea here also goes back to Kant. For as Kant saw it, the transcendence purported by classical metaphysics is simply unavailable. We humans have to operate

within the world and therefore from a particular (specifically human) perspective. We have a cognition-external Archimedean fulcrum outside the realm of our thought from which to move the entire domain of thought as a whole.

And this Kantian perspective relates to the relationship between inner and outer. We humans are emplaced *within* nature and by this very fact are limited as to the sort of information we can obtain *about* nature. All of our observation-based judgments are partial and thereby incomplete. Every observer has a body of information that is well short of anything like an all-inclusive view of things. As far as actual knowledge goes, totalities are off limits—be it the totality of actual experience (the self) or of its objects (the world) or of its rationale (ultimate purpose or God).⁸ Such ideas represent useful mind-contrivances for the development of actual knowledge but do not—cannot—themselves constitute actual objects of knowledge: as Kant saw it, they are not *entities* at all, but only practical resources, instrumentalities of cognitive procedure useful only for contrasting what we actually have with what we would ideally like. Those improperly totalized items simply do not exist as such.

By contrast, the principle at issue with the present approach is a limitation on the introduction (or specification or definition) of totalities. It reads:

Illicit Totalization Principle (ITP): To be meaningfully and viably introduced into the discussion (that is, adequately specified, identified, defined), a totality (collection or whole) must not be purported to include itself.

This is not a principle that governs the membership of totalities—let alone their existence as such—but one that merely addresses the proprieties of how totalities can meaningfully be introduced upon the stage of discussion and consideration. It deals with issues of communicative procedure, not issues of existence as such, and is accordingly a principle of logical grammar rather than one of ontology.⁹

An instructive lesson thus emerges. Russell sought a rationale for rejecting certain totalities. And he thought to find this in a conception of inappropriate existence claims based on a complex theory of types. But in fact, a much simpler rationale is available. It lies in construing totalization (τ) in terms of definite description (ι). For then the fact that definite descriptions fail to identify in certain conditions automatically provides a rationale for seeing totalization in the same light. What is now at issue with improper totals is a problem of meaning or definition rather than one of being or existence—an epistemic rather than ontological principle.

And Russell's approach has a serious drawback. As he saw it, we must not stake claims about "all propositions" or about "all properties" because such locutions involve violations of his Vicious Circle Principle.¹⁰ However, he was so intent on barring *illicit* self-involvement that he insisted on barring self-involvement in general. In seeking to eliminate paradox, Russell also similarly dismisses a great deal of innocuous stuff as well—including such tautologically harmless universalizations as "All meaningful propositions make an assertion of some sort," or "All properties can be attributes to some sort of object," or "All objects can be members of collections." Someone prepared to subject logic to the requirements of common sense might well see this consequence alone as vitiating his version of the Vicious Circle Principle on grounds of throwing out the baby with the bath water.

To recapitulate: while the Russellian Vicious Circle Principle is *ontological* in its nature, the presently operative Illicit Totalization Principle is merely communicative or *semantical*—it only deals with the expository proprieties of how items of discussion can meaningfully be placed upon the agenda of consideration. One principle deals with matters of *actual existence*; the other merely with matters of *appropriate specification*. Against this background it should be observed that the bearing of our present analysis of the implications of “vicious circularity” is purely *terminological*. It pivots upon showing that various putatively identificatory specifications will not succeed in placing a certain putative item upon the stage of consideration. And the rationale at issue is simply that a particular individuating presupposition is violated by the use of such an expression, thereby failing to enable the expression in question successfully to establish its intended reference.

Russell’s approach and that of the present discussion would accordingly come to the same thing only if the thesis “To be is to be specified” obtained—that is, only if existence stood coordinate with actual specifications. But this thesis is not plausible—unlike its revision “To be is to be *specifiable*.” And this opens the doorway to another line of thought.

12.5 Impredicativity

The present approach to paradox avoidance is thus substantially different from Russell’s own. The problem, as Russell saw it, is that the predicate at issue here, namely “being a set that does not contain itself,” is *impredicative* in that it offends against his Vicious Circle Principle by countenancing something general that is included within its own range. However, this line of approach led Russell to proscribe not only the clearly problematic “set of all sets that do not contain themselves” but also the (seemingly innocuous) “set of all sets that have more than five members.” Any such self-inclusive set is simply too large for legitimacy on Russellian principles.

It is, accordingly, fortunate that the present approach to untenability is far less drastic. For we do so only by an embargo on properties so formulated that what is supposedly being specified has in fact *already* been specified. Thus, what is wrong with the “set of all sets that do not contain themselves” is—on the present account—not its excessive inclusiveness as a presumptive set but rather the anaphorically self-invoking back-reference—via the expression “themselves”—that is at work in its formulation. In short, what the present approach proscribes is not self-referential impredicativity in general, but only its presence in *identificatory* contexts. (There is nothing wrong with a descriptive report along the lines that some pre-identified set is “a set that does (or does not) contain itself” as such.)

The proscription of impredicative self-involvement in item-specification is thus to all appearances a sensible policy. But now, in taking this line in the present context we place a restriction *not on existing totalities as such, but on the sorts of predications that can meaningfully be employed in the course of identifying totalities*. The restriction to certain *suitable* properties means that we cannot simply identify sets with properties

without let or hindrance. For now, the question always arises: Is the property in question one that is capable of providing for the introducing definition of a set? And this is something that is not the case with self-referential properties.

Now, in order to maintain the distinction and keep these two approaches apart we must reject the idea of an automatic equivalence between “the totality of *Xs* exists” and “the predicate of ‘*being an X*’ represents a meaningful condition.” That is, we must abandon the Principle of Set Constitution in its altogether unrestricted construction. After all, this principle is supposed to be a condition upon *introducing and specifying* certain sets. But if this specification is framed in terms that already presume the specificatory availability of the set in question, then it is for this very reason problematic and of questionable legitimacy.

And, of course, not only are *set specifications* by means of impredicative properties in theoretical hot water, but so—clearly—are *totalizations* that are presented by means of impredicative specifications. For when the very term being defined is resorted to in the course of its own definition, it is clear that the entire process is thereby vitiated. It is not that such self-reference is meaningless, but rather that their objectification—their attribution to an object that is in the course of introduction—is systematically inappropriate.

Accordingly, consider such descriptive characterizations as:

- Being a description that applies to all descriptions not applicable to themselves.
- Being a set that contains all sets that do not contain themselves.

Such characterizations are *perfectly meaningful as descriptive characterizations* (notwithstanding the fact that they are systemically inapplicable: attribution to any item uniformly results in demonstrable falsity). However, despite their potential meaningfulness we *cannot* use them in the course of *identifying* (introducing) an item. For if we propose to specify (define) an adjective as “the adjective that applies to all adjectives that...” or a set as “the set that contains all sets that...” in a way that is going to be self-inclusive, then we violate the Illicit Totalization Principle (ITP) in a way that opens the door to inconsistency.

To avoid paradoxes, then, we need not reject the existence of certain *kinds of thing*; it suffices to reject the appropriateness of certain particular *ways of talking about things*. The inappropriateness at issue turns on answering questions based on untenable presuppositions. For the principle we are violating in matters of that totalization is not that of faithfulness to the facts of existence, but that of keeping to conformity with the conditions of discursive meaningfulness. It is—to reemphasize—the linguistic proprieties that are being violated. And this perspective makes it possible to take a more narrowly targeted approach that is not committed to wholesale object dismissal in the manner of Russell’s approach. We need not proscribe *all* talk about “all sets” or “all propositions” but rather must merely be careful about what we endeavor to say with these locutions.¹¹ We need to embargo impredicative characterizations only in identificatory, definitional, and similarly item-specificatory contexts where ontological applications are at issue. Russell’s wholesale rejection

of self-inclusion—dismissing, for example, the contention “All propositions have a meaningfulness status”—simply slays the innocent along with the guilty.

In the final analysis, then, it is not the *meaningfulness* of impredicative characterizations but rather their *employment in context of hypostatization* that needs to be called into question, since a fallacy or illicitly presupposition is then unavoidable. For when the answer to the presuppositional question “Are the conditions for a meaningful item-identification met?” is “No,” then we cannot blindly barrel ahead as though it were “Yes.”

The principal lesson of this discussion, however, is that some of the difficulties that logicians traditionally try to overcome by an elaborate *formal* device such as the theory of types can be sidelined by the employment of a very straightforward and far less elaborate *informal* device, namely the eminently plausible proscription of identificatory self-reference.

12.6 Eliminating Grelling’s Paradox

One further consequence of proscribing self-referential item-introduction arises in connection with the specification of ranges of reference. Kurt Grelling’s well-known heterology paradox is illuminating in this regard. Some predicates are self-applicable (“expressible,” for example) and some are not (“inexpressible,” for example). Predicates of the latter sort that do not apply to themselves may be characterized as *heterological*. And now the question can be posed: What is the status of this predicate itself: is “heterological” itself heterological or not? At this point, we are seemingly plunged into paradox: If heterological is heterological, then it is not self-applicable, contrary to supposition. On the other hand, if it is not heterological, then it is self-inapplicable and thereby in fact heterological. Either way, we seem to be plunged into self-contradiction.

Let us examine the situation in a precise formulation.

We begin with the definition of the heterologicality of predicates:

$$(H) \text{ Het } (F) \text{ iff } \sim F(F)$$

And now, by instantiating the generic F to Het in this definition we have it that:

$$\text{Het}(\text{Het}) \text{ iff } \sim \text{Het}(\text{Het})$$

And so, if we have Het(Het) then we arrive at $\sim \text{Het}(\text{Het})$. And if we have $\sim \text{Het}(\text{Het})$, then we arrive at Het(Het). Self-contradiction ensues either way.

The difficulty that we encounter here traces straight back to that initial definitional specification (H). For note that this is predicated on the assumption that F is a viable variable relative to a well-defined range of predicates—that is, that there is a well-defined totality of predicates for the variable F to range over. For if Het is to be a predicate that is defined by (H) relative to that range, then this range must be

specified independently of (and, so to speak, “antecedently to”) the introduction of (H) upon the stage of consideration. But this condition of requisite pre-specification is violated by introducing Het relative to a presumably pre-specified range to which it is the added *ex post facto*, as it were. In adopting (H) as a specificatory definition, we augment the *F*-range improperly. Once again, we commit a fallacy of improper totalization.

And so, heed of the basic principles that have figured in our discussion suffices to expel Grelling's “heterological paradox” into the realm of fallacies.

12.7 Historical Postscript: Kant as a Critic of Inappropriate Totalization

Immanuel Kant deserves to be acknowledged as the philosopher who, in his *Critique of Pure Reason*, first objected to illicit totalization. And he took this step because he saw it as a means for freeing himself from what he regarded as problematic and theoretically unwelcome entities. For as Kant saw it, such ideas as that of “the ultimate constituents of spatial regions” or “the ultimate causes of events” represent the products of illicit totalization. He rejects totalities that are not closed as it were and thereby impossible to survey *in toto*, since he held that experience is the key to our knowledge of existence, and, of course, we never experientially survey certain totalities as such. As he saw it, a fundamental fallacy is involved in such totalitarian conceptions:

The concept of totality is in this case [of the world as a whole] simply the representation of the completed synthesis of its parts; for we cannot obtain the concept from the apprehension of the whole—that being in this case impossible. ... (CPuR, B456)

For Kant, such closure-defying, unsurveyable conceptions as that of the world as a whole one, whose content goes beyond the range of that which could ever be given in experience, is something ill-defined and thereby inappropriate. Only experiential interaction can assure actual existence—description alone can never do the job:

[It is inappropriate to suppose] an absolute totality of a series that has no beginning or end [such as would be at issue with “the terminus of all successive divisions of a region” or “the initiation of all the causes of an event”]. In its empirical meaning, the term “whole” is always only comparative. The absolute whole of quantity (the universe), the whole division [of a line segment], or of [causal] origination or of the condition of existence in general. ... along with all questions as to whether this whole is brought about through finite synthesis or through a synthesis requiring infinite extension. ... [are something altogether inappropriate]. (CPuR, A483-84 = B511-12)

With experiential unification unachievable, we can never appropriately reify such a totalitarian conception into that of an *object* that has a well-defined identity of its own.

For Kant, such improperly totalized items as “the physical universe fails to represent a well-defined object; it is an inherently fallacious conception that leads to inconsistency:

[As a sum-total of existence] the world does not exist in itself, independently of the regressive series of my representations, it exists *in itself* neither as an *infinite* whole nor as a *finite* whole. It exists only in the empirical regress of the series of appearances, and is not to be met with as something in itself. If, then, this series is always conditioned, and therefore can never be given as complete, the world is not an unconditioned whole. And does not exist as such a whole, either of infinite or of finite magnitude. (CPuR A503-05 = B531-33)

Kant has it that the endorsement of such unsurveyable, unbounded totalizations will result in untenable concepts that engender *antinomies*—actual logical self-contradictions. For if we violate this proscription of ungrounded totalization, trouble is bound to ensue by way of inconsistency:

If the world is a whole existing in itself, it is either finite or infinite. But both alternatives are false (as was shown in the [previous] proofs of the antithesis and thesis respectively). It is therefore also false that the world (the sum of all appearances) is a whole existing in itself. (CPuR, A506 = B534)

Kant insisted that illicitly totalization fails to yield objects that have a well-defined identity and that to accept such pseudo-objects as actual—to reify or hypothesize them—is to commit a fallacy that engenders the self-contradiction reflected in his antinomies. And there is, accordingly, a substantial parallelism between Kant’s position and the line of thought set out above.

However, another aspect of the matter must be noted. Kant thought that by representing such items as “the world” or “the deity” as illicit totalizations, he could eliminate the salient issues of traditional (“pre-critical”) metaphysics. In particular, his critique of inappropriate totalization was an integral part of his dismissal of the cosmological argument for the existence of an all-creative God to serve as producer of the world. As he saw it, if there is no such thing as “reality at large” or “the world as a whole,” then the issue of its causal origination is rendered moot, subject to the plausible principle that: *Only an item already known to exist can (let alone needs) to have its existence explained.*

Although all this may seem sensible enough, it is nevertheless questionable whether this line of argumentation achieves the overall result that Kant intended for it. For while his principle that “Only existing entities stand in need of an existence explanation” is all very well as far as it goes, it does not really manage to negate the potential utility of reasoning in the manner of the cosmological argument. The proscription of world totalization may block “Why does the world as a whole exist?” But it does not affect—let alone invalidate—the ontological question in the form originally envisioned by Leibniz: “Why is there something rather than nothing?” And this condition of affairs prevails more widely. Those metaphysical issues that Kant sought to eliminate by rejection of *totalized objects* can generally be reconstituted in ways that avert the illicit reification that is supposedly at issue. But while Kant’s prohibition of unrestricted totalization may not in the end prove to have been

quite as powerful a destroyer of metaphysical issues as he thought, it is and will ever remain a philosophical resource of substantial utility.

Notes

1. To be sure, where the matter is one of picking out items from a *pre-established* group, there need be no difficulty.
2. What is at stake here is a flaw different from the more familiar identification-defect that arises where there is vagueness—a penumbral, “fuzzy” whole where one cannot tell exactly what is within and what is not. For in our present case, the very item at issue is not defined as a meaningful unit because it itself is presupposed by its mode of identification/specification. Accordingly, it is not the purported totality’s membership but its very identity that is questionable.
3. Throughout these formulations, “all” is to be construed as “all but only.” Accordingly, a supertruth (for example) would be said to be a truth that conjoins all members of $\{p: p \text{ is true}\}$, the set of all propositions p such that p is true.
4. On problems of self-reference in general, see R. M. Sainsbury, *Paradoxes* (Cambridge: Cambridge University Press, 1987; 2nd ed. 1995), pp. 121–22 of the 2nd ed.
5. If μ is not *membership* but something different but cognate, such as the part-whole relationship of mereology, all bets on the individual line of argumentation may be off. For while parts collectively *constitute* a whole, they will not in general be needed to *define* it.
6. See Bertrand Russell and A. N. Whitehead, *Principia Mathematica*, vol. I (Cambridge: Cambridge University Press, 1910) [re-issued in paperback in 1967], pp. 31, 37. Elsewhere, Russell puts it as follows: “If provided a certain collection has a total, it would have members only definable in terms of that total, then the said collection has no total” (*American Journal of Mathematics*, vol. 30 (1908), pp. 222–262, see p. 240). Russell went on to explain that “When I say that a collection has no total, I mean that statements about all its members are nonsense” (Ibid.; see also P.M.). He appears to think, however, that the culprit here is “all” instead of “it”!
7. Bertrand Russell, “Mathematical Logic as Based on the Theory of Types” (1908) rptd. in J. van Heijenoort, *From Frege to Gödel* (Cambridge MA: Harvard University Press, 1967), pp. 153–182; see p. 155. Elsewhere, Russell is more cautious: “If, provided [we assume] a certain collection has a total, it would have members only definable in terms of the total, then the said collection has no total” (Bertrand Russell, “Mathematical Logic as Based on the Theory of Types,” *American Journal of Mathematics*, vol. 30 [1908], pp. 222–262; see p. 227; cf. *Principia Mathematica*, vol. I, pp. 31, 36.) The phrase “only definable” helps matters but also introduces problems of its own.
8. See Kant’s discussion of these ideas at CPuR A680-B708 = A689-B717.
9. This ITP might well be grounded in a broader principle to the effect that for successful identification, the items being referred to in the course of the process must themselves already have been identified. This approach, in effect, seems to lie at the basis of Henri Poincaré’s treatment of the paradoxes, a strategy

whose decided difference from Russell's neither he nor Russell seem to have appreciated sufficiently. See the lucid discussion of all these matters in Charles S. Chihara's *Ontology and the Vicious Circle Principle* (Ithaca NY: Cornell University Press, 1973), esp. pp. 138–140.

10. "Thus it is necessary, if we are not to sin against the above negative principle, to construct our logic without mentioning such things as 'all propositions' or 'all properties'—and without ever having to say that we are excluding such things. The exclusion must result naturally and inevitably from our positive doctrines which must make it plain that 'all propositions,' and 'all properties' are meaningless phrases." (Bertrand Russell, *Mathematical Logic as Based on the Theory of Types*, op. cit., p. 277)
11. We thus need not follow Russell in rejecting an ontology of classes and adopting a "no class theory" on grounds that statements about "all classes" are meaningless. There lies open the option of holding, with Gödel, that classes can be abstract objects that "exist independently of our definitions and constructions." For we can merely hold that certain ways of trying to refer to classes are flawed and cannot succeed in effecting such reference. Compare Kurt Gödel, "Russell's Mathematical Logic" in *The Philosophy of Bertrand Russell*, ed. by P. Schilpp (New York: Tudor, 1969), pp. 123–53.

Chapter 13

Mind Questions



1. What Can Minds Do?

Minds can engage in a wide variety of activities. They can affirm (putative) facts, speculate about possibilities, be aware of considerations, engage in deliberations, experience feelings, make evaluations, and much else. The range of thought proceedings open to minds is vast in extent. But what of its depth? We shall return to this critical question, but must first address some others.

2. Are All of Those Capacities Essential to Mind, or Can There Be Minds Lacking One or Another of Them?

An issue of substantial complexity arises here. Because each of those mental operations has relationships with others. The network of interlinkage are extensive. Thus it is clearly possible for there to be a creature capable of cognition without evaluation—able to take note of facts without any capacity for assessing them as positive/negative or welcome/unwelcome. Or again it might be able to experience pro or con feelings about possibilities but not affirm objective facts. Various distinct capacities are at issue, sometimes in separation, sometimes in combination. For there can in theory be minds that are only able to deal with possibilities without ever accepting or endorsing any one of them as correct?

3. What Is the Role of Consciousness in the Operations of Mind?

Three sorts of cognition can be contemplated, namely those that proceed:

- sub-consciously
- consciously
- self-consciously

We often perform various habitual, automatic actions with only subconscious awareness—scratching an itch for instance. We read the newspaper consciously yet without any heed of our eye movements in the proceeding. N of what we do is done without overt awareness where nothing ever rises to the level of awareness, we could not really speak of a mind at all. The potential for

consciousness is requisite for mind. But self-consciousness is not. When there are conscious proceedings—even in the absence of self-consciousness—it still would be correct to acknowledge the presence of a mind.

4. What Is Required for Consciousness?

How does “unconscious awareness” differ from an “authentic consciousness” of something? This can only be elucidated by an analogy. The difference is like that between the center of our visual field and its unheeded periphery.

It is clear that such tacit, unwitting knowledge is a possibility—that we know many things that are not consciously explicit in our mind possibly ever and certainly not at any given point. (What these very words mean, for example.) But the possibility of acknowledgment is always there. Whatever we accept—and whatever we know—is something we can particularly become aware of. The conscious is that which figure within the range of attention. The subconscious may be available for future reference but currently shines only at the far side of the attention’s moon.

5. Can Thought Be Mechanized?

Machines can certainly simulate the management of information—its formulation (encoding) and the assessment of its relationship (via compatibility and inference.) Whether we should say that in such matters the machine’s operators manage the information via the machines or whether to credit this to the machines themselves is problematic.

Those machines can do many things which, if people did them, would be accounted as thought, even as snowballs can do things, which, if animals did them, would be accounted as growth. But whether it is appropriate to biomorphize here is questionable. It instead be best simply to describe what is going on and let it go at that with further judgement suspended.

6. Can a Mind Controls its Own Operation?

Can a mind decide to remember, to forget, to enjoy, to dream, to misunderstand? Clearly not. There is a wide spectrum of things that minds can be said to do that they cannot—and often would rather not—do under voluntary self-control.

7. When are Minds the Same? What is the Identity Criterion for Minds?

Minds are specifically intensified by the particularity of what they do. And so it would be appropriate to maintain that when their thought-content is exactly and comprehensively the same, the minds in question are the same. And thought-content incorporates thought *process*, since not only actual items but their considerate relationships and linkages (if-then, therefore, and so) are encompassed within the range of thought.

8. Can One Contemplate Merely Possible Minds Over and Above the Actual Ones?

Yes, of course. There is no reason of principle why every possible constitution of thought—every complex of content—should be actualized.

9. Are There Limitations to What Minds Can Achieve—Mentaleque Proceedings that Minds Cannot Ever Actually Manage to Achieve?

As long as minds do their conscious deliberations in language—a resource whose functioning is inherently recursive and thus enumerable—our cognitive

efforts will never be able to accommodate the larger range of a trans-enumerable complex reality.

10. Could There be Omniscient Minds?

Minds cannot acknowledge the totality of all facts as long as overt acknowledgment is something verbal. For verbal formulations are only countably infinite while facts are transdenumerable. An omniscient mind is bound to transact its cognitive affairs in ways incomprehensible to humans.

11. How are Minds related to Brains?

Electronic brain activity correlates with mind activity much as words correlate with physical inscriptions. In theory words can also be produced in other ways (e.g. speech). And in theory thought can also be produced in other ways—say by chemical process place of electromagnetic activity. But in the contingent arrangements of this world, it seems that only brain activity correlates with thought in humans.

12. The Existence of Minds Clearly Brings an Entirely Different Dimension of Being into the World Viz. Irreality

Without minds there is no irreality—no mistake, error, or misconception about things—and in general no non-being. Minds alone provide for unrealized possibilities and thereby and are then way beyond reality—to irreality and fiction.

13. Do Minds Have Different Degrees of Power

Of course they do—both as to what they can do and how much of it they can accomplish. For mental power has two domains: *Range* as with the spectrum of different substantive issues they can successfully address, and *Depth* as with the extent of complexity and difficulty of the issues they can successfully address within a given range.

14. How Will Minds of a Lesser Power Regard Those of a Greater?

With bafflement. To a mind of lesser power the capacities of more powerful minds will meet with incomprehension and seem like magic. To someone who can just barely manage the multiplication table the feats of a Srinivasa Ramanujan are utterly unintelligible. Like other works of genius, it lies beyond the comprehension of ordinary mortals.

Chapter 14

Intuition and Mathematical Idealism



14.1 Recourse to Intuition

Since Plato's day, mathematicians have been continually attracted to the belief that the objects of their field—numbers, conceptual structures, geometric configurations in specific—should be thought of in idealistic terms as constituting a sense-inaccessible realm of ideal objects. (Peculiarly, this theory of an immaterial realm of ideal objects is generally characterized as Platonic *realism*—i.e., realism of ideal objects, which sounds rather odd in the context of a realism/idealism contrast).¹

Mathematical idealism crucially depends on intuition since the objects and facts of mathematical deliberation are neither physical (like rocks and their properties) nor mere fictions (like elves and fairies) but abstractions. As such, they are cognized neither by sensation nor by imagination, as the cognitive factuality by which one reaches them is something distinctive and different—namely *intuition*.²

The human mind has two critical cognitive powers: the power to envision possibilities and the power to adjudge actualities. And the second of these, the determination of facts, proceeds—sensation apart—either by reasoning or by intuition. Reasoning looks to evidentiality and substantiation—to a consideration of evidence and indicators of probability or plausibility. Intuition is immediate and proceeds without reference to substantive intermediation. Our present deliberations will focus on this more problematic and less examined cognitive recourse.

Intuitive apprehension is not necessarily a matter of being evident. For what is evident and obvious can become so as the fruit of learning from extensive experience. That *D*, *O*, and *G* are letters of the alphabet, and that D-O-G means *dog* are evident and obvious facts but not ones we arrive at by intuition.

Conversely, intuition need not yield only what is evident and obvious. Its findings may require training, experience, and skill (e.g., in photointerpretation). A good deal of preparatory groundwork may be needed before something becomes intuitively clear to someone.

An intuitively self-evident fact need not be utterly obvious and informatively trivial. A good deal of effort at understanding may be required for its comprehension—grasping that point may be far from easy because the point is far from simple. The crux is that once the point is grasped, agreement can no longer be rationally withheld. Adequate comprehension presupposes acceptance. An evident fact need not be obvious to anyone and everyone but only to those who adequately understand the matter at issue. Intuitive facts will not be available to everyone but may be accessed only by those able “to see the point.” Access to them will not be universal. (That a polygon with n sides must also have n angles is intuitively self-evident but is neither trivial nor cognitively obvious).

Exactly *how* this faculty works is little elucidated by theoreticians. And in the final analysis it is not crucial, seeing that what is crucial is simply *that* it works: product not process is of the essence here. For theoretical purposes, what intuition is defined by what it *does*, or is supposed to do—namely to provide us with correct information regarding a realm of abstract, observation-inaccessible realities.

Intuition comes into play whenever the relevant rules of procedure do not suffice to resolve a significant issue.

The philosophical area in which intuition has traditionally been attached the most prominent role is that of ethics and morality. A large sector of what is good and bad, just and unjust, fair and unfair, proper and improper, has been deemed as ultimately intuitive.

We have a moral obligation to be charitable, but which charity to support and to what extent is left open. We have an ethical obligation to develop our talents, but just how to go about it is left open. We have a social obligation to support our community, but this obligation too is open-ended and indefinite. The existence of such indefinite obligations raises significant questions. Just how are they to be met—what is required to meet them? And how is this to be determined?

In effect, things are what Kant calls “imperfect duties”: duties whose specific nature fails to determine the extent, frequency, or manner of their performance; duties, that is to say, involving indeterminacy of performatory satisfaction. For the classic principles of ethics don’t afford much real help here. Take the utilitarian “greatest good for the greatest number.” If I have a talent for chemistry and also for gardening, am I obligated to develop the former because more people are likely to benefit? And am I obligated to devote every available moment to its cultivation? Nor does the Kantian generalization principle of “Do as you would have everyone else do” help all that much. Even when my greatest talent is for mathematics, I could not necessarily want every potential mathematician to become one. Nor yet does an Epicurean “Just follow your heart” do the needful. For it faces the risk of serious social imbalance in the face of transient fashion.

All such ethically important but nevertheless principle-irreducible matters have to be managed by the “good judgment” affordable by a duly cultivated ethical intuition.

And such intuition is a pervasive requisite. Consider the classical “Paradox of the Heap.” Six grains of sand will not consist a heap; six hundred will. When is the transition between too few and too many in such matters? The best and most we can do is to have recourse to an intuitive resolution.

It would have been better if German philosophers had coined the term *Einschauung* instead of using *Anschauung* for “intuition.” The “insight” at issue in the intuitive apprehension of matters of fact is only roughly analogous to ordinary sight. The work it does is not a matter of physical (or intellectual) inspection: it is more like “getting a joke.” As a faculty for coming to a realization of certain facts, intuition is less a matter of inspection than a special sort of apprehension—one based on a recognition solely by properly understanding the matter at issue (See Display 14.1).

There is no reason to hold that intuition is a particular sort of faculty that functions uniformly and works alike with respect to matters of abstract theory and human phenomena. There is no reason to think that its unreasoned judgments in ethical, social, or psychological matters cannot be unreasonable. But nothing here counter-indicates the reliability of intuition. What is at issue in intuition is not a matter of affectively rooted psychological dispositions to believe but of an intellectual insight into certain objectively mind-independent facts, e.g., that circles can always be emplaced in triangles.

Display 14.1

HOW INTUITION WORKS

I. AN INTUITIVELY ACQUIRED FACT MUST BE:

- self-substantiating: factual, actually true, and assured as such by the very meaning of the matters (terms, concepts, ideas) at issue, without requiring external substantiation or evidentiality
- nondiscursively apprehended, without receiving further external substantiation or evidentiality

II. AN INTUITIVELY ACQUIRED FACT MAY, BUT NOT NEED, BE:

- trivial, tautologous
 - self-evident and obvious
 - basic and rudimentary
 - acquirable without special training
 - relating to the apprehender’s self
-

With an intuitively accessed claim, the information needed to determine its meaning already suffices to ensure its truth so that the claim is, as it were, self-validating. (Of course, not everyone to whom such a claim is available will avail themselves to it: intuitive knowledge need not and will not be universal.) What makes something intuitively comprehensible is that what is needed for understanding

it adequately also suffices to furnish decisive evidence for its truth. Questioning it betokens misunderstanding because the requisites for apprehending its meaning encompass a realization of its truth. Intuitively assumed facts need not, however, be obvious. For obvious facts can be learned rather than intuited; trivial facts can be intuited by some and not others. Knowledgeable people have a broader intuition range.

There are thus two aspects to intuitively acquired information: one relating to the information itself, and the other to the mode of its acquisition. Display 5 provides a round-up of the overall situation.

Only those familiar with the Latin alphabet will realize intuitively that *A* is its first letter. Only those who have the rudiments of arithmetic at their disposal will realize intuitively that $2 \times 2 = 4$. Only those who know you will intuitively know your name. Only those who know some English will realize that C-A-T means *cat*. Only you yourself can say really intuitively that you are amused by something. An intuitively accessible idea of information may actually be accessed by (and sometimes even only accessible to) particular individuals.

The range of intuitively acquired information is indicated in Display 6. The claims throughout are facts acknowledged without evidence or reasoning beyond what is required for understanding.

Intuition has been at the center of modern philosophy from the start. Regarding Descartes' *cogito ergo sum* ("I think and therefore am"), scholars debate whether the transit from *cogito* to *sum*—from "I am aware that I think" to "I am a thinking being"—is discursive or intuitive. But, of course, the former itself is available only by intuition (See Display 14.2).

There is no procedural account for how we obtain intuition of mathematical facts any more than there can be a procedural account of how we remember past experiences or how we understand language. Throughout the area of such cognitive proceedings we realize *that* we can do those things without understanding *how* we manage it. To complain about the absence of how-to-do-it instruction in such matters is to lament the inability of doing the impossible.

14.2 Mathematical Intuition

Mathematical intuition differs from ethical or grammatical intuition not insofar as it is a matter, mode, or matter of idealization, but only insofar as its subject motto is involved through its relating to quality, shape, or structure. And it is through idealism with such abstract matters that mathematical intuition relates to idealization.

 Display 14.2

 TYPES OF INTUITIVE INFORMATION
 (Facts one can “see” with the mind’s eye)

- I. Self-regarding observations and convictions*
 - I am under the impression I see a book
 - I feel (that I am) being chilled by a breeze
 - I believe that it is raining
 - I am feeling exhausted (i.e., that my body is weakened)
 - I have forgotten the date
- II. Learned but (ultimately) immediate realizations
 - M and Y are letters of the alphabet
 - D-O-G spells *dog* (in English)
- III. Nondiscursively immediate realizations
 - $2 \times 2 = 4$
 - triangles have sides
 - knowers have minds
 - some circles are larger than others
 - facts cannot be false

* The feelings here at issue relate to facts not states. In locutions like “I feel X ” the X has to be a propositional (e.g. “that a storm is impending”) rather than adjectival (e.g., “tired” or “sad.”)

Mathematical intuition of this sort has been called upon to fulfill several crucial theoretical functions. As Aristotle has already taught, knowledge cannot always be validated by demonstration because demonstration requires premisses, and these themselves must ultimately be indemonstrable—that is, “primary” premisses.³ And Aristotle has it that these basic or axiomatic premisses will be either definitions or stipulations (i.e., postulates or, as Aristotle called them, *hypotheses*). Definitions, to be sure, are no real problem, seeing that they are merely communicative conventions. But what is it that validates otherwise acknowledged basics? What puts us into a position to stake those basic and undesirable basic substantive claims? This clearly poses problems. For such claims must provide knowledge of the highest quality for

“Since the axiomatic premisses are the basis of our knowledge—our conviction—it must be that we know them better, and are more firmly convinced of them, than their consequences.”⁴ Such evident (*dêlos*) truths have to be endorsed directly, without mediation or demonstration, and just this is to be a key task of “intuition.”

An omniscient being would have little use for the discursive reasoning of demonstration and proof by being able to resolve all matters directly.

The Indian mathematician Srinivasa Ramanujan was the human with the most highly developed arithmetical intuition. He saw little point in troubling with proofs, seeing that he could answer questions directly by immediate insight. It caused his mentor, the mathematician G. H. Hardy, no little trouble to persuade him that lesser mortals were committed to proofs.

With mathematics as a paradigm, the range of systematized fact can be constructively divided into two sectors:

1. The *basic or fundamental* that is to be immediately evident in being validated directly by intuition, which by tradition is to include all of the axioms of the system.
2. The *discursively provable* that is to be validated by the mediation of logical demonstration.

Intuition, so tradition has it, is our standard resource for substantiating the facts that obtain at the basic level, where discursive validation through demonstration is not (as yet) applicable.

Mathematical intuition has acquired a special significance in the light Kurt Gödel’s 1930s demonstration of the provability-incompleteness of axiomatic arithmetic—a monumental achievement in mathematical logic which marked him as “one of the most significant logicians in history.”⁵

Thus, Gödel wrote in 1966 in the wake of Paul J. Cohen’s results on the continuum problem: “I don’t think [mathematical] realists need expect any permanent ramifications [sc. obstacles] [to an intuition-based Platonism] as long as they [i.e., the reasonings] are grounded in the choice of axioms by mathematical intuition and by other [systemic] criteria of rationality.”⁶ For as long as proof and demonstration prove to be insufficient for the rational substantiation of mathematical truth, then where save to intuition can we look for a validation of acceptance? Mathematical intuitionists would not find Kurt Gödel’s provability-incompleteness of axiomatic arithmetic all that shocking. After all, it shows no more than the insufficiency of discursive reasoning and axiomatic formalization, and thus the need for invoking intuition as an access way into an idealized domain.

Nevertheless, from its very origin in classical antiquity this view has faced a problem. For if—or rather since—the objects of this ideal realm are inaccessible to our sensory experience, then how are we to establish cognitive contact with them—by what means are we to acquire the knowledge we purport to have about them? And from the start, the standard answer has been that such cognition, not provided by physical *sensation*, is instead provided by a purely intellectual *intuition*. Such insight—rather than sensory sight—has standardly been invoked to validate the requisite mode of cognition. For this intuition is not just a matter of instinct to accept certain claims;

it is a faculty that provides the rational basis for seeing these claims as true and their acceptance as rationally justified.

To be sure, there yet remains the key question of *modus operandi*. For it is one thing to claim *that* intellectual intuition provides informative insight into abstract entities and yet another to know *how* it does so. But that is a different issue for another day⁷. It is certainly true that mathematical theoreticians (such as Kant, Hilbert, and Gödel) conceive of intuition (German *Anschauung*) in somewhat different ways.⁸ But virtually all agree in that its definitive task is to validate what is noninferentially immediate and in no way probatively discursive evidential.⁹

Accordingly, a cognitive faculty of intuition has come to be seen as the traditional recourse for validating as self-evident the *basics* of a cognitive system—a term used here to encompass generally the axioms, postulates, and definitions of a cognitive system that is the totality of its indemonstrable fundamentals. This is a proceeding that was adumbrated by Aristotle, systematized by Leibniz, and further refined by Husserl and Gödel.

14.3 The Problem of Overload

But consider what this standard approach asks of mathematical intuition in relation to matters of systemic proof and demonstration. On its basis, the axiomatic theses that intuition is invoked to validate must be not only (1) self-evident (obvious, self-explanatory), but also (2) highly informative (able in conjunction to provide a deductive basis for the entire field). The problem is that these requirements stand in inherent tension and conflict.

And just here is the difficulty with the standard view of intuition's role, seeing that what is being demanded of it is to provide the totally secure informative basis for the entire system at issue. It is no wonder that there should be difficulties here! Considerations of fundamental principles accordingly indicate the ultimate need for a different approach that does not ask the impossible of intuition, and a variant non-classical view of the mission of mathematical intuition is called for.

14.4 An Alternative Strategy

The central problem of idealism is: How can subjective, personal experiences yield objective and impersonal information? There are two possible answers: In the face of suitable subjective experience, objective information can be *given* or else *taken*. Experience can serve as the vehicle by which objective facts are *conveyed* by reality, or else that experience can provide the occasion for the individual's *postulating* that reality has the correlative features. On the former view, experience services an occasion for *reception* of a reality-based transmission; on the latter, it provides the occasion for the postulation of a putative reality.

The one view (taking) is a subjective idealism, and the other (receiving) is an objective idealism. (There is also a mixed view: that of a reality-authorized (i.e., conveyed) taking—a taking whose authenticity is validated by successful application and implementation of the taken information. This constitutes a pragmatic idealism). Classical mathematical intuitionism allies itself to objective idealism; its variant Brouwerian form of constructive intuitionism is a subject idealism.¹⁰

On the traditional approach, the procedure is for intuition to establish a set of self-evident, demonstration-foregoing axiomatic theses and then to move on to exfoliate that body of theorematic findings that this axiomatic basis authorizes by deductive inference. But this is problematic—if only because it imposes a heavy burden on intuition in regard to the presumptive adequacy of its deliverances.

Fortunately, an alternative procedure is also available. As this is now envisioned, the procedure is to begin by identifying a representative sampling of the self-evident truths of the field and then provide a set of axioms for this (limited) body of contentions. So, intuition is then no longer asked to authenticate all the axioms directly.

For the sake of an example think of schoolroom arithmetic. What are to be the basic rules of calculation here? To answer this question on the presently envisioned approach we begin by seeking out the rules that suffice throughout the comparatively smallish range of intuitively evident fact spanned by informal (“mental arithmetic”) calculation (for example $a + b = b + a$). And we then move on to postulate that these apply to the entire field at large. We thus secure our own axiomatization of the whole by adopting what merely suffices for an intuitively manageable part. Accordingly, on the variant approach now at issue, the axioms are validated not immediately and directly, but discursively through their capacity to provide an inferential basis for a representative submanifold of what is intuitively acceptable.

Intuition once again provides the basis for axiomatization, but now does so in a way very different from and far less demanding than the traditional approach. Rather than being themselves self-evident, the axioms are validated retrospectively, on the basis of the sorts of consequences that they produce. In point of intuitiveness, axiomatization is now a derivative rather than a primary status—earned on the basis of productivity in yielding the “right” theorems. Intuition would no longer be required to do the entire job. And so, in placing a diminished burden on intuition, this more “realistic” approach has obvious advantages.

In effect, we now turn the usual mode of proceeding on its head. Classically, we are assured of the right theorems because they issue by derivation from intuitively acceptable axioms. Alternatively, now we are assured of the right axioms because they yield the intuitively acceptable theorems. Strange though it may sound, axiomatization will now be consequent upon consequences, seeing that we now class as axioms those of theses that effectively provide for the appropriate theorems.

Whether or not those axioms themselves qualify as “evident” is now immaterial.

And so, on the revised *modus operandi* now in contemplation, one does not call on intuition to validate the axiomatic thesis of a system directly. All that one asks of it is to validate a representative sampling of the facts. One then looks for a suitably efficient axiomatization able to yield these facts as theorems. One thereby achieves

that axiomatization via intuition, alright, but not by intuition directly. Rather one does so obliquely because these axioms are effective in yielding that representative manifold of intricately acceptable contentions.

Approached in this way, mathematics comes to look somewhat akin to an empirical science. For its development begins with data, facts available as deliverances of intuition—the fruit of intellectual apprehension rather than sensory observation. And much as scientific systematization spins a web of natural laws to provide an explanatory basis for explaining the observational data, so mathematics spins a web of axiomatic basics to provide a deductive basis for the intuitive data.

In discussing the search for an approximate axiomatization for set theory, Gödel himself came close to this view, characterizing this project as quasi-empirical not in that set theory is an empirical science, but because the methods of empirical science could sometimes serve in the development of an a priori science such as set theory. And so the possibility of combining ontology with an empiricism in methodology now arises, mediated by a concern for purposive efficacy that can itself be characterized as a version of pragmatism.

Here a reliance on intuition remains in place. The axioms themselves need not be intuitively obvious as long as they efficiently yield the requisite manifold of intuitive fact. Intuition still controls the determination of axiomaticity—but without pervasive immediacy. Yet on either sort of approach—be it direct or oblique—intuition provides our ultimate access to the domain of mathematical idealities.

14.5 Idealistic Retrospect

But can mathematical Platonism survive such a shift to a substantially inductive approach? Where does it leave the ideal status of the objects of mathematics when we abandon the idea of the *direct* and immediate apprehension of their rational validity?

Platonic idealism in mathematics pivots on the distinction between what is made and what is found: what is created through by the mind's conventions and postulations, and what obtains in an objective, mind-independent manner so as to be *discovered* by the mind rather than *created* by it. And the fact is that the present variant approach does not really touch this issue. It gives up on directness of intuitive insight into axiomaticity, but not on the objectivity of the facts at issue. It does not abandon the end result, but merely substantiates a different, more oblique way of getting there. Mathematical Platonism as such can still remain intact when less is asked of intuition.

Why should this be deemed an asset? What is it that speaks for a Platonic idealism here? What are its advantages? They are principally two-fold:

1. It authorizes us to regard those mathematical facts as informative truths. Conventions or postulations are just that—they are not true or false and

convey no objective information, but do no more than provide instruments of communication.

2. It authorizes us to see learned facts as objective and intersubjectively valid. They constitute matters of impersonal uniformity, unlike conventions which are group coordinated. Conventions invite differentiation and cacophony, while objectivity makes for consonance and agreement regarding matters of fact.

The key mission of an intuition-based idealism in regard to the abstract objects, such as concepts, numbers, shapes, and structures, at issue in logic and mathematics lies in its affording a natural pathway to the impersonal uniformity of cognitive access to the objects at issue.¹¹

And so, to summarize. Mathematical idealism requires intuition, which alone can provide for adequate knowledge of those idealities. Conversely, mathematical intuition invites—but does not require—a coordinate idealism. But it could avert this only by seeing intuition's role as productive/creative rather than descriptive/reportive. But such a constructive idealism faces the problem that its idealities are a product of the artifice that affords no assurance of generality and uniformity.

In the end, the Platonic idealism—like most other philosophical theories—must be appraised on the basis of its contribution to our understanding of the relevant issues. There are primarily four:

1. Its service in explaining the intensity of our (personal) conviction in the truths of mathematics and/or acceptance of their inescapable necessity.
2. Its service in explaining the (interpersonal) uniformity of people's conviction in adjudging the truths of mathematics.
3. Its service in accounting for the systemic harmony and elegance of the overall domain of mathematical facts.
4. Its service in helping to account for the otherwise unexpected utility of mathematics in the description and explanation of the phenomena of nature.

Mathematical idealism as developed since Plato has a great deal to be said on its behalf. And since intuition is an indispensable *sine qua non* for such an idealism, it too cannot easily be dismissed.

The question of why the idealities of mathematics are so effective in the characterization of the natural world we inhabit is almost universally regarded as a mystery.

As long as people thought of the world as the product of the creative activity of mathematicizing intelligence—as the work of a creator who proceeds more mathematico in designing nature—the issue is wholly unproblematic. God endows nature with a mathematically intelligible order and mind with a duly consonant mathematicizing intelligence. There is thus no problem about how the two get together—God simply arranged it that way. But, of course, if this is to be the canonical rationale for the mind's grasp on nature's laws, then when we forego explanatory recourse to God, we also—to all appearances—lose our grip on the intelligibility of nature. Some of the deepest intellects of the day accordingly think that this possibility is gone forever, confidently affirming that there is no way to solve this puzzle of how it

is that nature is intelligible in a mathematically lawful manner. Erwin Schrodinger characterizes the circumstance that man can discover the laws of nature as “a miracle that may well be beyond human understanding.”¹² Eugene Wigner asserts that “the enormous usefulness of mathematics in the natural sciences is something bordering on the mysterious, and there is no rational explanation for it”¹³ and he goes on to wax surprisingly lyrical in maintaining that “The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve.”¹⁴ Even Albert Einstein stood in awe before this problem. In a letter written in 1952 to an old friend of his Berne days, Maurice Solovine, he wrote:

You find it curious that I regard the intelligibility of the world (in the measure that we are authorized to speak of such as intelligibility) as a miracle or an external mystery. Well, a priori one should expect that the world can be rendered lawful only to the extent that we intervene with our ordering intelligence ... [But] the kind of order, on the contrary, created, for example by Newton’s theory of gravitation, is of an altogether different character. Even if the axioms of the theory are set by men, the success of such an endeavor presupposes in the objective world a high degree of order that we were a priori in no way authorized to expect. This is the “miracle” that is strengthened more and more with the development of our knowledge ... The curious thing is that we have to content ourselves with recognizing the “miracle” without having a legitimate way of going beyond it ...¹⁵

According to all these eminent physicists, we are confronted with a profound mystery. They take the line that we have to acknowledge that nature is intelligible, but have no prospect of understanding why this is so. The problem of nature’s intelligibility by means of our mathematical resources is seen as intractable, unresolvable, hopeless. All three of these distinguished Nobel laureates in physics unblushingly employ the word “miracle” in this connection.

But, of course, expounding the role of an ideal order to which our intuition provides insight is one thing, and explaining our commitment to its *existence* yet another¹⁶. But in this latter regard, evolutionary consideration can manage to do yeoman work.

Notes

1. Given that philosophers invariably envision Plato’s general position as an idealism, it is ironic that Plato’s view of mathematics goes by the name of realism. For the sake of accuracy mathematical Platonism is here characterized as an idealism rather than as the more common Platonic realism.
2. Historically, intuition has been called upon to validate theses throughout the realm of inquiry—philosophy, psychology, religion, aesthetics, and so on. Books on the subject include: Joel Pust, *Intuition as Evidence* (New York: Garland/Routledge, 2000), Hermann Cappelen, *Philosophy Without Intuitions* (New York: Oxford University Press, 2012), and M. R. DePaul and Wm. Ramsey (eds.), *Rethinking Intuition* (Lanham, MD: Rowman & Littlefield, 1998). The *Stanford Encyclopedia of Philosophy* (available on the internet) contains an informative discussion of the subject. The literature on mathematical intuition in particular is substantial, seeing that theorists from Kant to Hilbert have maintained that the basic principles of the subject must be *anschaulich*. It is,

however, discussable—although to my knowledge has never been discussed—whether mathematical intuition is simply one particular form of a human intuitive capacity or is something *sui generis* and functioning quite apart from unreasoned conclusions in other areas of deliberation.

3. See Aristotle, *Posterior Analytics*, Book I.
4. Aristotle, *Posterior Analytics*, 72b30–33. Aristotle himself does not have a term of *intuitive knowledge*. The closest he comes is to characterize it as the insight at issue with nous/noêsis. See *Posterior Analytics*, 100b5–17.
5. *Wikipedia*. Relevant data are given in the chapter “On the Philosophical Development of Kurt Gödel” by Mark van Atten and Juliette Kennedy in S. Feferman et al. (eds.), *Kurt Gödel: Essays for his Centennial* (Cambridge: Cambridge University Press, 2019), pp. 275–325.
6. Mark van Atten, *Essays in Gödel’s Reception of Leibniz, Husserl and Brouwer* (Cham: Springer, 2015), p. 138.
7. In the final analysis a claim that *X* produces *Y* cannot always be provided with a mechanism. For whenever *Z* is an intervening condition we have *X* produce *Y* by way of *Z*. And this simply opens up the problem of moving from *X* to *Z*. So in the final analysis the quest for a mediating *how* for that-production has to be abandoned as impracticable.
8. See W. W. Tait, “Gödel on Intuition and on Hilbert’s Finitism” in S. Feferman et al., *Kurt Gödel: Essays for his Centennial* (Cambridge: Cambridge University Press, 2010), pp. 88–108.
9. The concept of intuition roots in the neoplatonic distinction between knowledge arrived at discursively by justificatory reasoning and that achieved by immediate insight (*epibolê*). Boethius here introduced the Latinate terminology of *intuitus*. It formed the basis of Spinoza’s conception of *scientia intuitiva*, and Leibniz explained it via his conception of innate ideas. However, intuition became the target of C. S. Peirce’s teaching that this idea is deeply erroneous because all knowledge is to some extent discursive.
10. On these issues, see Mark van Atten, *Essays in Gödel’s Reception of Leibniz, Husserl and Brouwer* (Cham: Springer, 2015).
11. Immanuel Kant sought to achieve the generality and uniformity of immediate knowledge in “the faculty structure of the human mind.” But he had no way to assure that this is something uniform across its multiple realizations in different people. Only a post-Kantian recourse to evolution could rescue his position.
12. Erwin Schroedinger, *What is Life?* (Cambridge, 1945), p. 31.
13. Eugene P. Wigner, “The Unreasonable Effectiveness of Mathematics in the Natural Sciences,” *Communications on Pure and Applied Mathematics*, vol. 13 (1960), pp. 1–14 (see p. 2).
14. *Ibid.*, p. 14.
15. Albert Einstein, *Lettres à Maurice Solovine* (Paris, 1956), pp. 114–115.
16. On the former issue, see the author’s *A Useful Inheritance* (Savage MD: Roman & Littlefield, 1990).

Chapter 15

Outlandish Hypotheses and the Limits of Thought Experimentation



15.1 Far-Fetched Hypotheses and Diminishing Returns

Thought experimentation has a long reach. Even an impossibility can be thought about, talked about, discussed, and be considered in ways that treat it merely as an object of thought. And so while impossibilities cannot be given a place in the realm of being as a part of reality, they nevertheless can indeed function in the realm of thought or discussion.

Many situations are impossible given the way things work in the world. “Suppose Julius Caesar were alive today” (but people just can’t live for thousands of years!) “Suppose lead floated on water” (but gravity pulls heavier things down!). It is not that impossibilities cannot be supposed, but rather that we just do not know what to make of them *in vacuo*, in the absence of adequate explanatory stagesetting. However, when a suitable supplementary context is provided then matters may be remedied. Consider “If Julius Caesar were alive today, he would find Rome substantially changed.” No problems there—we have simply found a picturesque way of saying that the Rome of today is very different from the Rome of Caesar’s day. “If lead floated on water, then this object would float on water.” Again, no big problem. We have simply found an odd way of saying “This object is made of lead.” There is nothing impossible about supposing the impossible—it is something one can perfectly well come to reasonable terms with, if one is careful. For example, in the sort of thought experimentation involved in *per impossible* reasoning can involve recognizedly incoherent suppositions. For consider:

- Suppose (per impossible) that Cicero had successfully squared the circle. Then he would not only be a famous statesman and writer, but a famous mathematician as well.⁶

An “impossible supposition” is one thing and a “supposition OF the impossible” another. Even as there need be nothing foolish about a supposition that

projects someone's doing something foolish, so there is nothing impossible about a supposition that supposes some impossible thing or situation.

It is sometimes suggested that thought experimentation is impracticable when the projecting suppositions at issue are too unrealistic: too bizarre and overly hypothetical. But this idea has to be rejected, lest in abandoning strongly counterfactual suppositions we also abandon *reductio ad absurdum* let alone of *per impossible* reasoning. Nothing is sacrosanct in thought experimentation. Not facts, not laws of nature—and not even the so-called necessities of logic and mathematics—are beyond the scale of counter-supposition. It was long thought that there could be no such thing as infinite numbers. Ultimately Georg Cantor developed a workable theory of infinite real and ordinal numbers. But we have it as crux of a perfectly workable thought experiment that “If Cantor had demonstrated that infinite numbers cannot exist, then the history of mathematics in the 20th century would have been very different.” The point is that if such suppositions are to make sense they must be fitted out with a context that enables them to accomplish useful communicative work.

Impossibilities are not of a piece. The locution that suppositions can be more or less bizarre and “far out” affords a useful metaphor. Of course the things we know or believe to be true we need not suppose—they, by definition, we are prepared to affirm. But moving beyond that we have the situation of Display 1. Here those non-proximate suppositions fall into three groups, according as their falsity inheres in factual, reality-descriptive (merely contingent) considerations; in nomic, necessary, reality-characterizing considerations; or in semantical (meaning-rooted) considerations. And the prospect of profitable thought experimentation will depend on how “far out” along this road experimental supposition happens to be. The more outlandish a supposition is in doing violence to the supposer's entrenched commitments, the narrower the range of reasoning where it can accomplish useful work.

There is nothing wrong with far-fetched thought experiments as such. If somebody can dream up an instructive lesson from some weird supposition regarding the modus operandi of humans (reincarnation, say, or psychokinesis or levitation or brain transplants) then so be it. We are, after all, inured to extraordinary eventuations by such oddities of nature as hypnosis, acupuncture, spike-sitting and fire-walking. But plunging into meaninglessness is going too far. Thus if humans—as we do and must think of them—indeed are rational animals, then we cannot suppose these to be human trees. To be sure there are many things that trees and humans can do in common: stand still, get blown over by high winds, take in nourishment, and (conceivably) react to music. But being human and thus functioning as intelligent rational agents is something that trees just cannot manage—not for reasons of operational difficulty but for reasons of conceptual nihilism. Since the very idea at issue with humans is one of animals of some sort, the idea of human plants is not just impracticable but descends into meaningless. And so there is need for caution. For while violations of the usual course of things should not faze us, nevertheless what cannot and must not be asked of thought experiments is doing something whose characterization plunges us into meaninglessness (See Display 15.1).

Display 15.1

SUPPOSITIONAL PROXIMITY/REMOTENESS

EPISTEMICALLY PROXIMATE SUPPOSITIONS

Possibilities: Those with respect to whose truth-states we are agnostic, but which may be true “for aught that we know” seeing that they are compatible with what we take ourselves to know.

EPISTEMICALLY DISTANT SUPPOSITIONS

(Mere) Falsities: Those that we deem to be false (albeit on substantive rather than demonstrative grounds).

EPISTEMICALLY REMOTE SUPPOSITIONS

Impossibilities: Those we see as demonstrably false.

Incomprehensibilities: Those that we take to violate the principles of cogent meaningfulness.

Various counterfactual theorists see remoteness as a matter of what is “conceivable” or “imaginable.”¹ But the psychologism of “conceivability” is decidedly unhelpful in the present context; by making matters needlessly subjective and contingent it serves to beckoned rather than clarify the issues. In this context one must distinguish among:

- *conceivability*: the capacity of an intelligent being (of a certain kind, be it human or alien) to configure concepts that characterize some state of affairs as a viable object of thought.
- *(logical) possibility*: the condition of internal logico-conceptual coherence and consistency of the propositions characterizing a potentially realization state of affairs.
- *intelligibility*: the condition of internal well-definedness of the concepts at issue in a proposition.

It is important to note: (1) an intelligible state of affairs need not be conceivable: a type of being (ourselves included) may, simply for reasons of cognitive imoverishment be unable to come to terms with something that is, in and of itself, intelligible. (2) Nor, for analogous reasons, need an inherently possible state of affairs be conceivable: it may just be too complex for our minds to get a secure grip on it. (3) Moreover, something that is conceivable need not be possible: impossibilities can in principle be considered and entertained in thought. (4) However, whatever is either conceivable or possible must be intelligible, seeing that being understandable is something objective, contrary to being understood by people. It is decidedly preferable to see the issue of remoteness in terms of compatibility and consistency rather than psychological difficulty in conceiving or imagining. For unlike possibility, which is a logico-conceptual matter, conceivably is something psychological and subjectively variable with the diverse cognitive capabilities and aptitudes of different individuals. Nor is it even true that the supposition at issue in a cogent thought experiment must be logical-conceptually coherent. What is at issue is a matter of logical compatibility with

relevant fact, in laws of nature, and logico-conceptual principles—with sequential incompatibility indicating increasing remoteness.

One thought-experiment theoretician maintains that “stipulation is free: if you want to stipulate something ever so grotesque, who’s to prevent you?”² But the obvious response here is that while it is indeed true nobody can prevent you, you should be given pause by the consideration that a principle of diminishing returns is at work. For the prospect of profitable thought experimentation will depend on how “far out” experimental supposition happens to be. As suppositions grow increasingly distant this task becomes increasingly troublesome. The more outlandish a supposition, the narrower the range of reasoning where we can usefully put it to work. Here the non-proximate suppositions fall into three groups according as we see their falsity as inhering in factual, reality-describing (merely contingent) circumstances, in matters of lawful necessity, or in semantical (meaning-rooted) considerations. And the more a supposition is “far out” in its increasingly drastic violation of the principles of good epistemic sense the more limited will be range of its appropriate employment in matters of hypothetical reasoning. What all this specifically means is apparent from the specifications of Display 15.2.

Display 15.2	
LIMITS OF SUPPOSITION	
<u>If a supposition is</u>	<u>Then its employment is appropriate</u>
Strongly meaningless	never
Weakly meaningless	only for <i>per impossible</i> reasoning
Demonstrably false	for <i>reductio ad impossibile</i> or for <i>per impossible</i> reasoning
Belief contravening (albeit possible)	for counterfactual reasoning
Belief compatible	for hypothetical reasoning at large

The price paid by thought experiments as their supposition becomes increasingly far out is simply that the sort of lesson that can be drawn becomes restricted to an increasingly narrower, more impoverished range. A mere physical impossibility (such as traveling faster than the speed of light) still allows for many instructive lessons regarding nature’s ways. But a thought experiment whose launching supposition is *logically* impossible (let alone one that verges on the meaninglessness of *semantical* impossibility) is something from which only modest miniscule instruction can be derived. Far-fetched thought experiments are not impossible but they are only very modestly helpful.

The cardinal rule of thought experimentation is based on the principle that the utility-range of a supposition is directly proportional to its plausibility. And, inversely, this principle means that the more far-fetched a supposition is the smaller the scope of

its serviceable employment. It is thus not that a thought experiment based supposition involving self-contradiction is impossible or impracticable, but rather that the only thing that can be done with such a thought experiment is a *reductio ad absurdum* refutation of that supposition.

Overall, what this means becomes apparent from the specifications of Display 3. As this tabulation shows, the more a supposition is “far out” in its increasingly drastic violation of the principles of good epistemic sense the more limited will be range of its appropriate employment in matters of hypothetical reasoning. So the key point is that while thought experiments are possible throughout the range of “far out” suppositions—up to but excluding the strongly meaningless ones—the range of their useful applications becomes increasingly narrower as the suppositions at issue become increasingly far-fetched.

One thought-experiment theoretician asks: “stipulation is free; if you want to stipulate something ever so grotesque, who’s to prevent you?”³ The answer here is of course that while there is indeed nobody to prevent you, you should be given serious pause by the consideration that a principle of diminishing returns is at work.

With regard to radically “far out” thought experiments, Katherin Wilkes has argued that “these ‘experiments’ take us too far out from the actual world, and from the ‘other things’ that hold roughly ‘equal’ here.”⁴ But in fact what causes difficulty here is not a lack of normalcy and other-things-equal failure, but a lack of meaningfulness. It is because the terms of reference at work in the discussion are designed for and predicated upon real-world conditions and thus presuppose the realism of the usual run of that an abrogation of these preconditions plunges the discussion into a morass of unmeaning where we no longer have a communicative grips on what is being said. And so what renders increasingly remote suppositions problematic is not their inconceivability but their increasingly dysfunctional nature. A principle of diminishing returns is at work. The more far-out a thought experiment is, the less it is able to do for us in the line of useful work (See Display 15.3).

Display 15.3	
LIMITS OF SUPPOSITION	
<u>If a supposition is</u>	<u>Then its employment is appropriate</u>
Strongly meaningless (gibberish)	never
Weakly meaningless (incoherence)	only for <i>per impossible</i> reasoning
Demonstrably false (impossibility)	for <i>reductio ad impossible</i> or <i>per Impossible</i> reasoning
Belief contravening albeit possible	for counterfactual reasoning
Belief compatible	for hypothetical reasoning at large

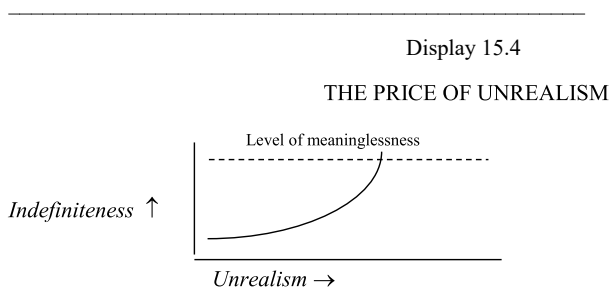
The argumentation presently at issue is emphatically not a case against far-fetched thought experiments as such but rather the issue of the sort of use for which they are appropriate. Thus in contemplating mind body interchanges we are (on the present analysis) not in a position to draw conclusions about the nature of personal identity. But we certainly do not come away empty handed, for the very fact that such a conclusion cannot be drawn is itself an instructive lesson that we derive from a consideration of this thought experiment.

15.2 Meaninglessness

Two sorts of considerations are at issue here, considerations which, though qualitative in nature, vary so in degree that they can be conceptualized quantitatively as well. They are:

- *unrealism*: remoteness from reality, “far-out”-ness, hyperbolic fancifulness.
- *indefiniteness*: communicative imprecision, equocality, openness to alternative constructs.

And these are related in such a way that as the unrealism of a supposition increases (linearly) its indefiniteness increases exponentially (See Display 15.4). The further we get away from reality, the wider the range of alternative possibility proliferates so that what confronts us is more and more indefinite.



A matter of deep-rooted general principle is at issue here which can be clarified as follows. Let it be that exactly one X is inscribed in this box:



Accordingly what is so inscribed is correctly describable as:

- a consonant of the Latin alphabet
- a letter of the Latin alphabet
- a symbol of some sort

And now consider the supposition that there is something other than an x . We can of course still retain those several descriptions. But the further out we go away from realistic specificity, and the more our suppositions cast the details of reality to the winds, the less they can do for us in relation to informativeness. For the range of possibilities grows ever larger, and the larger the range of possibilities the more we are cognitively at a loss. A basic principle of logic is at issue here. The larger the range of possibilities the less definite and meaningful the scope of what is being affirmed is bound to be. With increasing unrealism there is an ever larger proliferation of alternatives for their interpretation.

With a world without anthropods we are in difficulty. (How would evolution proceed without them?) And with a world without physical substances of different types we are in ever deeper difficulty. But with a world of talking flowers we hover on the precipice of meaninglessness, and with a world with round squares we have fallen off the edge. No matter how we struggle with telling a long story, we are not going to be able to provide an adequate account of this. There just is no satisfactory way to resolve the questions that arise here.

Making sense of this sort of thing becomes an ongoingly more elaborate business: there are ever more questions to be answered in trying to pin things down to making sense. An ever more elaborate story has to be provided to secure an informative footing. And as this complexification proceeds, eventually the problem field that has to be addressed becomes so vast as to become effectively unmanageable. So eventually we move from bafflement to unmeaning. The communicative utility of the assumption at issue is compromised beyond repair. A supposition that falls afoul of the meaning of its own terms is simply absurd.

Accordingly, the proper pathway to rejecting a thought experiment proceeds not via mere strangeness but via actual meaninglessness. Consider, for example, the following passage from G. W. Leibniz's fifth letter to Samuel Clarke:

As to the question whether God could have created the world sooner, it is necessary here to understand each other rightly. . . . It is manifest that if anyone should say that this same world which has been actually created might have been created sooner without any other change, he would say nothing that is intelligible. For there is no mark or difference whereby it would be possible to know that this world was created sooner. And, therefore (as I have already said), to suppose that God created the same world sooner is supposing a chimerical thing. It is making time a thing absolute, independent upon God, whereas time must coexist with creatures and is only conceived by the order and quantity of their changes.⁵

As Leibniz saw it, since time by its very nature relates to the sequence of events within the world's history, the very idea of creating the world-as-a-whole sooner or later becomes meaningless.

Another example of a supposition that is meaningless in just this way is

- Suppose that Aristotle knew that $22 \times 22 = 488$.

For whatever deserves acknowledgement as *knowledge* must be veridical—that's just how the concept works. Now there is no thematical impediment to

- Suppose that Aristotle *thought* that $22 \times 22 = 488$.

After all, even smart people can be mistaken in mathematical matters. But it lies in the very nature of knowledge as such that one can only *know* what is true: one cannot know something that is false—let alone something that is self-contradictory or even meaningless. We can perfectly well suppose that $22 \times 22 = 488$ —say for the purposes of a *reductio ad absurdum*. But we cannot meaningfully suppose that somebody *knows* this absurdity.

Some things are physically impossible (jumping to the moon), some mathematically impossible (squaring the circle), and some are semantically impossible (being a round square), and these last take the plunge into meaninglessness. This fundamental fact has significant implications.

15.3 Suppositions that Go Too Far: Limits of Meaningfulness

In embarking on a thought experiment that proceeds from a recognizably inconsistent supposition we are not necessarily doing something that is inappropriate and pointless—let alone “meaningless.” After all, the very point of the exercise may well be to reveal an inconsistency, as in *reductio ad absurdum*, or to make a larger point, as can happen in *per impossible* reasoning. (“If—per impossible—the end justified the means then it would take a better and superior end to justify a worse and inferior means.”) For as G. W. Leibniz already noted in the 1670's:

We certainly sometimes think about impossible things and we even construct demonstrations from them. For example, Descartes holds that squaring the circle is impossible, and yet we still think about it and draw consequences about what would happen if it were given. The motion having the greatest speed is impossible in any body whatsoever, because, for example, if we assumed it in a circle, then another circle concentric to the former circle, surrounding it and firmly attached to it, would move with a speed still greater than the former, which, consequently, would not be of the greatest speed, in contradiction to what we had assumed. In spite of all that, we think about this greatest speed, something that has no idea since it is impossible. Similarly, the greatest circle of all is an impossible thing, and the number of all possible units is no less so since there is we have a demonstration of this.

So there is nothing wrong with far-fetched thought experiments as such. If somebody can draw an instructive inference from some weird supposition regarding the doings of humans (reincarnation, say, or psychokinesis or levitation or brain transplants) then so be it. We are, after all, inured to extraordinary eventuations by such oddities

as hypnosis, acupuncture, spike-sitting, and fire-walking. But launching into outright meaninglessness is going too far.

One is free to suppose anything one pleases; supposition is a limitless resource—one can suppose one's way to a free lunch. There are no restrictions to the rationality of assumption and supposition—save for one exception alone, namely that of meaningfulness. One can only suppose to be the case that which it is possible to indicate meaningfully as being so. And that, of course, rules gibberish out and puts incomprehensibly meaningless suppositions outside the pale.

Once forks are defined as eating implements that have tynes then we cannot coherently suppose a tyneless fork. (To be sure, no difficulty is posed by damaged forks whose tynes are broken off, but these of course have ipso facto ceased to be actual forks.) A supposition cannot remain meaningful and viable that disproves something of this feature which must characterize it in the definitional and conceptual order of things. Such ultra-far-fetched suppositions step outside the realm of what is meaningful and coherent. But plunging into meaninglessness is going too far.

Again, if humans—as we do and must think of them—indeed are rational animals, then we cannot suppose these to be human trees. To be sure there are many things that trees and humans can do in common: stand still, get blown over by high winds, take in nourishment, and (conceivably) react to music. But being human is something that trees just cannot manage—not for reasons of operational difficulty but for reasons of conceptual coherence. Since the very idea at issue with humans is one of animals of some sort, the idea of human plants is not just impracticable but utterly meaningless.

The trouble with Meinong's *round square* is not just a matter of fortuitous inconsistency—of a logical incompatibility between two perfectly meaningful ideas or claims, such as a writer's assertion in one chapter that his protagonist was born in January and in a latter chapter putting his birthday in June (or claiming in one place that a dovecote is round and in another that it is square). Rather it is to project an idea that is internally incoherent and literally senseless. When something is represented in these terms nothing intelligible is conveyed. The stipulation that something is a round square conveys no meaningful supposition. Suppositions cannot conjoin what our very concepts hold apart, nor can they disconnect what our very concepts conjoin.

A supposition of human plants or round squares or green integers is inappropriate not through its oddity but rather its meaninglessness. Such a supposition is not just "crazy" but *meaningless*. Squares are (by conception/definition) not the sort of thing that can take on further shapes; integers are (by conception/definition) not the sort of object that can be colored. What these suppositions violate is not just common sense but the meaning of terms. They commit a category violation through postulating for something on the agenda of consideration a feature which, in the conceptual/definitional nature of things, that sort of item is in principle unable to exhibit. And the limits of meaning set limits to the viability of suppositions. For the limits of meaning set limits to the viability—indeed the very meaningfulness—of our suppositions.

To be sure, meaningful talk about round squares is possible (for example in the way of denying that such things exist). We cannot regard suppositions about them as

totally unintelligible. Accordingly, we must acknowledge that there are two significantly different senses of meaning/meaningless, namely the strong sense in which a meaningless statement is one that is *unintelligible gibberish*, and a weaker state in which there is merely a matter of *conceptual incongruity or incoherence*, as per a round-square.

Consider the question: “What is the largest integer?” To pose a question coherently we must presuppose that it indeed has an answer—that there is a fact of the matter one way or another. But where this presupposition fails, the question is deconstructed into meaninglessness—and so is any thought experiment in which this question is involucrated. And this will happen in particular whenever meaninglessness afflicts the terms of reference in which the issue on which a thought experiment is posed.

The salient point here is that the more grotesque your supposition the narrower is the range of instructive uses at your disposal, until in the end there indeed is something that *should* prevent you, namely your own good sense.

After all, suppositions at odds with the very concepts at issue in their formulation are conceptually incoherent (“meaningless” in one of its senses) will undermine the viability of supposed thought experiment. The philosophical theologians of the middle ages already insisted on substantially this point when they dismissed the supposition that a God whose very concept involves necessary existence might chose to destroy himself or that a God whose nature is omnibenevolent might do something evil, or again that a God whose uniqueness is of his very essence might reduplicate himself.

And so there is need for caution in thought experimentation. For while violations of the usual course of things should not faze us in matters of supposition, nevertheless what cannot and must not be asked of thought experiments is doing something that is conceptually impossible. The limits of meaningfulness serve as limits for meaningful hypothesizing, supposing, and thought experimentation.

John Stuart Mill rejected any theory of substance that contemplates a non-sensible *substrate* of sensation through reasoning as follows:

If there be such a substratum, then suppose it is at this instant miraculously annihilated, and let the sensations continue in the same order. How would the *substratum* be missed? By what signs should we be able to observe that its existence had been terminated? Should we not have as much reason to believe that it still existed as we now have? And if we should not then be warranted in believing it, how can we be so now?

But note that Mill’s thought-experiment turns on our supposing that *it [the substratum] is . . . annihilated and . . . [the] sensations continue [unchanged and] in the same order*. And this supposition is, on the face of it, absurd. If the non-sensible substrate of sensation indeed is what it is by hypothesis supposed to be in its very nature—namely that which accounts for the substance and the ordering of our sensations—then the hypothesis we are being invited to make is simply self-contradictory: it makes no sense to suppose the phenomenon in the absence of that which—by hypothesis—supposedly produces it. (It would be like imagining sunlight in the absence of the sun.) If—as is indeed the case—our standard view of the world is *de facto* a causal one, so that our sensations are taken to have physical causes,

then the prospect of dismissing this non-sensuous causal basis without thereby annihilating its sensuous results is simply absurd—much like supposing that the lake's water is drained off but the ripples in its surface remain.

15.4 How Outlandish Hypotheses Pose Problems

Analytic philosophers, who do and ex officio must put a high premium terminological clarity, are particularly given to thought experiments, and constantly press questions of the format: “But what would one say if - - .” But special dangers threaten, and here as elsewhere, self-destruction is something to be avoided. A philosophical thought experiment must take care not to saw off the very limb on which its suppositions hang. After all, since it is we who projected thought experiments, they should certainly be intelligible to us.

Our concepts generally develop against the background of an understanding of how things work in the world (or better, are taken by us to work); they are tied to a view of the realities of nature and to the empirical detail of actually existing practices. Anyone genuinely concerned for the philosophical elucidation concepts as we actually use them, must accordingly bear in mind that the thought-framework that is their native habitat is not an abstract logicians' tool for dealing with the endless ramifications of an infinite spectrum of theoretical possibilities, but an historically developed product arising within the experimental framework of a specific and concretely real cognitive setting. In consequence, these concepts are such that their viability is linked indissolubly to the experienced realities of this actual world.⁷

However, philosophy's interests being largely theoretical and abstract, the standards of precision, generality, exactness, and the like, that are used by philosophers have traditionally been far higher than those of the practical people concerned with everyday affairs. Accordingly, philosophers have generally looked upon language as imperfect and inadequate—in need of tidying up, clarification, and supplementation (if not outright replacement). But everyday concepts do not admit of this improvement without revision—and thus without abandonment. They are made for everyday use in real-world situations and cannot survive unaltered in the more stressful theoretical atmosphere of strongly hypothetical concerns. In projecting wildly fanciful hypotheses, philosophers bent on theoretical tidiness often end up annihilating the very concepts they are to clarify.

For the sake of an example, when thought experiments are projected in the context of ethical deliberations regarding personal behavior or social practices, one has to be mindful that the ethical norms of conduct are instrumentalities designed for use in the conditions and circumstances of this world's general and ordinary course of things. The rules and regulations of ethics and morality are everyday guidelines designed to function in everyday circumstances and conditions. Accordingly, hypotheses that project extreme and altogether extra-ordinary conditions will not be relevantly serviceable for the task in hand. And this is not just because suppositions

that contemplate considerations of extreme scarcity (e.g., killing one healthy individual to use his organs and body parts for saving many) are too outlandish and far fetched for contemplation but because the course of their development has designed our ethical concepts and principles with very different sorts of situations in view. These far fetched hypothesis in that specific deliberative setting are inappropriate and will be unable to meet the dialectical needs of the situation.⁸

Again, consider the thought experiment, proposed by Hilary Putnam; of supposing that all the animals we regard as domestic cats are actually ingeniously contrived (and presumably self-replicating) robots inserted on the world's stage by some malicious but vastly clever deceiver, so that what we generally regard as member of the (organic) species *Felix domesticus* are actually robots controlled by Martian receiving and sending messages via a transmitter in the "cat's" pineal gland. Are these robots cats? In the face of this supposition, two considerations come into potential conflict

- (1) To be a domestic cat is to be a member of the (organic) species *felix leo*. That is, cats are what they are in view of their zoological standing and biological condition. "Cat" is at bottom a *biological* category.
- (2) To be a domestic cat is to look and behave in the way that cats do. "Cat" is at bottom a *behavioral* category.

Which of these alternative ways does the concept of *cat* actually go? The answer is: both and neither. In the ordinary run of things, biology and behavior run together. What Putnam's robot hypothesis manages to do is to pry the two apart and to free us to a decision as to the cooperative precedence and priority of (1) versus (2). And this of course is something that we cannot comfortably accomplish—nor for that matter can Putnam. The hypothesis tears apart what experience—and our experientially based concept in its wake—bounds together. If the circumstance of the hypothesis were realized, and in fact the items we deem cats were robots were just would not know what to say—all bets would be off. What we would then have to do is to *abandon* our concept of cat and trade it in for something substantially new and different. This thought experiment is simply infeasible given the conception of "cats" as the term actually functions in the language. It is unable to address the concept of *cat* as we actually have it.

Again, consider the thought experiment proposed by Bernard Williams⁹ of a devise that can interchange people's psychic resources (memories, affirmatives, dispositions, skills, character). And now suppose that you are put into a situation where (1) you know your psyche will be exchanged with X's, and (2) you have the choice between your old body being tortured (post operatively) or your new. Since it is "your" present mind that will experience the pain if you so choose, no doubt you—and most of us as well—would prefer to have it be the old body that undergoes torture. Accordingly, the Williams takes his thought experiment to indicate that—as per Locke's changeling (see pp. xyz)—bodily continuity is less crucial for personal continuity than psychic continuity is. He concludes that it is with the mind rather than the body that personhood is most decidedly identified—that "in the final analysis" it is psychic rather than bodily continuity that is crucial for personal identity.

But it is very questionable whether this prizing apart of mind and body helps with clarifying the concept of a person as we have it. For in various other contexts, where the balance of benefit runs the other way—people would incline to a different situation. For instance, it seems likely that if your grandfather’s vast future—which would otherwise come to you as your father’s son—were to go to someone else descended from somebody with whom you father had undergone this psychic exchange you would change your line on to how being “the same person” is to be determined. It is precisely because our concept of personal identity is an analogue it transpires that single-circumstance resolutions are indecisive. So William’s example winds up unable to bear the weight he proposes to impose upon it.

The contemporary literature of the philosophy of mind is full of robots whose communicative behavior is remarkably anthropoidal (are they “conscious” or not?) and of personality exchanges between people (which one is “the same person?”). But all such proceedings are intrinsically problematic. The assumptions at issue call for the suppositional severing of what normally goes together—and do so in circumstances where the concepts we use are predicated upon a certain background of “normality.” No supposedly clarificatory hypothesis should arbitrarily cut asunder what the basic facts of this world have joined together—at any rate not where elucidating those concepts whose life-blood is drawn from the source of fact is concerned. If we abrogate or abolish this factual framework by projecting some contrary-to-fact supposition—however well-intended to clarify the issues—we thereby destroy the undergirding basis that is essential to the applicability and viability of these concepts. We no longer connect the discussion with the concepts supposedly at issue, but change the subject.

15.5 Use and Usage

Nothing but incomprehension will result when we tear our common concepts out of the context of everyday normality in which their usage has developed and their correlative meaning established. By launching into the issue of who’s who with such fanciful hypotheses as

- the property interchanges between two different individuals in the wake of some magical transformation.¹⁰
- the psychological interchange between two individuals in the wake of brain transplantation.¹¹

We enter into a situation where “all bets are off.” In resorting to such suppositions we are not *clarifying* the established concept of personal identity but *abrogating* it (and thereby creating a need for a new and very different conception of personhood). For in endeavoring to impose an impracticable precision on the inherently imprecise concepts we employ for the communicative purposes of everyday life, the philosopher, thought experiments all too often create confusion rather than clarity. And while

concept clarification is indeed a desideratum, pursuing it through concept destruction makes no sense.

A classic instance of the futility of trying to render precise the meaning of inherently imprecise concepts is afforded by the classic Paradox of the Heap, to which we have already alluded. It runs as follows:

A single grain of sand is certainly not a heap. Nor is the addition of a single grain of sand enough to transform a non-heap into a heap: when we have a collection of grains of sand that is not a heap, then adding but one single grain will not create a heap. And so by adding successive grains, moving from 1 to 2 to 3 and so on, we will *never* arrive at a heap. And yet we know full well that a collection of 1,000,000 grains of sand is a heap, even if not an enormous one.¹²

The difficulty here is clearly that the fit between the resources of language (the heap terminology) and the existing states of affairs (the actual collection of so many grains of sand) is not exact. There comes a point where the correct decision as between “heap” and “no heap” becomes uncertain—we no longer know just what to say. Moreover, there is no neat boundary here. For just as there is no precise demarcation between “heap” and “no heap,” so also there is none between “clear heap” and “uncertain heap.” Different regions are indeed involved, but their boundary is shadowy and imprecise. To be sure, the facts themselves will be clear: there are exactly N sand grains. But their linguistic characterization in terms of heapness is uncertain. The concept at issue is coherently resistant to precisification.¹³ If you “exactify” the heap idea by specifying 32 grains of sand to be IN and fewer than that to be OUT you are dealing with something that just does not answer to the concept of a *heap* of sand grains.

This little example conveys a large lesson. Our everyday-life conceptions are all too often subject to an inherent imprecision rendered acceptable by the ordinary course of things. (In ordinary life, the eccentric cases that cause problems just don’t arise often enough to be an obstacle—and, anyhow, can be dealt with by other means should they arise.) Any attempt to render the concept unwarrantedly precise by projecting them into extreme conditions would plunge us into perplexity and paradox.

No doubt, clarity is a key philosophical virtue. But when the thought experimentation in which philosophers engage unravel the very concepts they are endeavoring to clarify we are impelled into chaos and confusion. Those extreme hypotheses, while intended to serve the interest of clarity, end up in accomplishing the reverse.

Various theorists attached to the “radical” idea/thought experimentation nevertheless quite rightly feel deep misgivings about the process. Richard Rorty, for example, sees thought experimentation as effectively circular because our belief determines what we deem imaginable and plausible. Gilbert Harman disdains thought experimentations merely clarifying what we believe rather than informing us about the nature of reality. And J. L. Mackie rejects thought experimentation in ethics as a mere “illustration of illusion seeded with moral language.”¹⁴

In just this vein, W. V. Quine protested as follows against the outlandish hypotheses in a debate between Sydney Shoemaker and David Wiggins over personal identity:

Later, [Shoemaker] examines Wiggins on personal identity, where the reasoning veers off in familiar fashion into speculation on what we might say in absurd situations of cloning

and transplanting. The method of science fiction has its uses in philosophy, but at points in the Shoemaker-Wiggins exchange and elsewhere I wonder whether the limits of the method are properly heeded. To seek what is “logically required” for sameness of person under unprecedented circumstances is to suggest that words have some logical force beyond what our past needs have invested them with.¹⁵

And here, Quine is essentially correct.

In defending far-fetched thought experiments in philosophy, Derek Parfit writes as follows:

Criticism might be justified if when considering such imagined cases we had no [instructive] reactions. But these cases [often] arouse in most of us strong beliefs . . . beliefs not about our words, but about ourselves . . . We discover [that] our beliefs are revealed most clearly when we consider imaginary cases, [and indeed] these beliefs also cover actual cases, and our own lives.¹⁶

This may be true up to a point—but that point is well just when the far-fetched assumptions at issue deconstruct our terminology and turn what is being said into gibberish.

15.6 The Shipwreck of Conjectural Analysis in Philosophy

The salient lesson of the present deliberations is that the limits of thought experimentation in philosophy are set by the limits of meaning bound to the *modus operandi* of our concepts. Suppositions that unravel and demolish the very concepts with which they are concerned are self-destructive. And this sort of self-destructiveness invalidates the proceedings at issue as part of a useful cognitive venture. No constructive clarifactory or explanatory purpose can possibly be served by a supposition that creates conceptual anarchy. For in philosophy, as in life, our experientially based concepts are made into viably integrated units only by the factual arrangements of the world in which they evolved. These concepts represent internally diversified *combinations* of logically separable elements that are held together by the glue of a substantive view of the empirical facts. Their integrity as viable conceptual units rests on a factual rather than theoretical basis: they hinge upon an empirically based, fact-laden vision of how things work in the world. They lack the abstract integrity of purely theoretical coherence that alone could enable them to accommodate the demands of purely theoretical, fact-abstractive precision. And when the very meaning of a concept presupposes certain facts, its explication and analysis clearly cannot—in the nature of the case—suppose that this basis is simply abrogated.

When one introduces fanciful hypotheses to abrogate these “underlying realities” on which our very concepts and predicated, then the foothold for our concepts dissolves and the relevant sector of our conceptual scheme simply dissolves with it. The clarification of such issues cannot be pressed beyond the cohesive force of the factual considerations that unify the operative concepts and thereby underwrite their applicability. The limits of meaning set limits to potentially instructive thought experimentation.

A widespread programmatic attitude towards philosophical thought experimentation accordingly demands reformation. Many practitioners of conceptual clarification assume a disdainful attitude towards the “mere matters of fact” that determine the meanings of words. Seeing themselves as concerned with the abstract “logic of concepts” in a purely, aprioristic manner, they are determined to uncover formal relationships holding *sub specie aeternitatis*, and insist upon addressing conceptual issues quite in the abstract, independently of any factual considerations. But insofar as our portrayal of the philosophically pivotal role of fact-coordinating concepts has merit, this approach cannot be maintained. It becomes necessary to abandon the view—so prominent in some quarters—that one automatically ceases “to do philosophy” once one begins to take account of empirical considerations.

To be sure, some philosophers see the aim of the discipline in terms of an abstract exploration of theoretical possibilities. With Christian Wolff (who defined philosophy as the study of possibility as such), they take the line that natural science deals with the real world, while speculative philosophy deals with the realm of the possible. The present position is not totally at odds with such a view. It does, however, require one to heed the distinction between *realistic* possibilities that do not abrogate our understanding of the normal course of things and keep their speculative flights within the limits of the plausible, and *fanciful* possibilities that kick over the traces of our understanding of the ways in which the world works. In subjecting our normality-geared conceptions to the impact of suppositions of the latter sort, we come to be at a loss for what to say. Genuine conceptual innovation now becomes necessary, and there is no way of predicting its outcome. To the question “What would you say if . . . ?” we would in such cases have to reply: “We just wouldn’t know what to say . . . We’ll just have to cross that bridge when we get there.”¹⁷ For when we embark on a radical hypothesis that violates the conditions of normality, then our normalcy-predicated concepts cannot be brought to bear at all. We have no ready answer to the question “What would you say if . . . (if worst came to worst—e.g., if flowers started talking like people)?” When the hypothetical upheaval is so extreme, cannot avert bafflement—we would have to go through the agonizingly innovative process of rebuilding part of our conceptual scheme from the ground up.

We have only one guide in effecting readjustment of beliefs that is necessitated by a contrary-to-fact hypothesis—the grasp on the world’s normal course of things. For we must strive here to create the least *possible* disturbance in the fabric of envisioned reality. But if the hypothesis at issue is sufficiently wild (“Suppose that bees spoke English”) then this guidance is lost to us. If normality is violated too radically, then we just do not have enough to go on in making sense of counterfactual hypotheses. Bafflement and confusion is now inevitable. When too much damage is done to the fabric of fact on which our concepts are predicated, then we literally “just don’t know what to say.”

The key consideration here is that our experientially based concepts are—and must be—inherently geared to the world’s contingent *modus operandi*. They are made into viably integrated units only by the factual arrangements of the world in which they have evolved. Accordingly they are held together by the glue of a substantive view of the empirical facts. Such fact-based concepts have an inner structure in which

theoretically separable factors are conjoined in coordinated juxtaposition. They lack the abstract integrity of purely theoretical coherence which alone could enable them to accommodate the demands of fact-abstractive precision. And so when the very meaning of a concept presupposes certain facts, its explication and analysis clearly cannot—in the nature of the case—suppose that this basis is simply abrogated. The operation of such concepts cannot be pressed beyond the cohesive force of the factual considerations that synthesize them into meaningful units. Once we project a fact-contradicting supposition that abrogates the factual basis on which such a concept is predicted, then, its meaningful employment is automatically precluded. The limits of meaningfulness accordingly set limits to viable supposition.

The point of these observations is not to advocate an unbudging conservatism in conceptual matters. There might conceivably be various advantages to be gained by giving up some of our concepts in favor of others. But the hermeneutical gain of elucidation and issue-clarification in philosophy is not among them. For pressing our philosophical concepts beyond the limits of the realities that make them viable is bound to result not in clarification but in mystification.¹⁸

Notes

1. See for example Unger 1983 and 1984, and Sorensen 1992 (pp. 35–41), as well as the extensive recourse to these factors in Haggqvist 1996.
2. Haggqvist 1996, p. 146.
3. Haggqvist 1996, p. 146.
4. Wilkes 1988, p. 45.
5. G. W. Leibniz, *Philosophical Papers and Letters*, 2nd ed. by L. E. Loemker (Dordrecht: Reidel, 1969), pp. 706–7.
6. G. W. Leibniz, *Philosophische Schriften*, ed. C. I. Gerhardt, Vol. IV (Berlin: Weidmann, 18xy), pp. 290–96. (See p. 294.)
7. Analytic philosophers have often stressed the empirically laden nature of our ordinary concepts—although they have done so from very different points of view (witness Wittgenstein, Carnap, and Quine). And they have realized that this has substantially negative implications for conducting philosophy along traditional lines. However, they have drawn a dire and drastic conclusion from this state of affairs—namely that the problems and projects of traditional philosophizing should be abandoned. The present far less radical prospect of construing philosophical generalizations in a more modest, standardistic way has not struck them—perhaps because their positivistic inclinations made them so eager to be rid of philosophical chaff that they were willing to discard the wheat as well.
8. See Hilary Putnam, “It Ain’t Necessarily so,” *The Journal of Philosophy*, vol. 59 (1962), pp. 658–71. Conduct Putnam’s variant view of the matter in “The Meaning of ‘Meaning’”, and in Keith Bunderson (ed.), *Language, Mind, and Knowing* (Minneapolis, University of Minnesota Press, 1975).
9. Bernard Williams, *Problems of the self* (Cambridge: Cambridge University Press, 1973), pp. 46–63.
10. See Chisholm 1989.

11. See S. Shoemaker 19xy.
12. On this paradox see the author's *Paradoxes* (Chicago & LaSalle: Open Court, 2001), pp. 76–83.
13. The fact that certain “what would you say if” questions are not readily answered is often not due to a shortfall of contextual information (as Quine 1972 seems to suggest), but rather to the realities of the way the language works. When I am baffled as to whether twelve grains of sand suffice to make a “heap” it is certainly not because I lack contextual information (say as to the shape of those grains, etc.).
14. Thought-experiment rejectionism along these lines is discussed at some length in Sorensen 1992.
15. W. V. Quine, “Review of Identity and Individualism,” *The Journal of Philosophy*, vol. 69 (1972), pp. 489–90. Quine's worries are echoed (at length) in Kathleen Wilkes 1988 (NR: see Brown, p. 28).
16. Derek Parfit, *Reasons and Persons* (Oxford: Clarendon Press, 1984), p. 200. The author is clearly in the grips of the widespread conviction that psychological self-examination is the pure aim of thought experimentation.
17. A position substantially along the same lines is ably defended from a variant line of approach in Fodor 1964.
18. Some of the issues of this chapter are also treated in the author's *Philosophical Standardism* (Pittsburgh: University of Pittsburgh Press, 1994).

Chapter 16

Limitations and the World Beyond



(Co-authored with Patrick Grim)

16.1 Introduction

The present topic is that of limits—the metamathematical limits of axiomatic systems, epistemic limits of explanation, linguistic limitations of expression, conceptual limits of conceivability, and, ultimately, questions of ontological and metaphysical limits as well. The limitations of axiomatic demonstration and of mechanical computation are clear from the Turing and Gödelian traditions. Section 16.2 pursues extensions and analogies to limitations intrinsic in the structure of explanation, restrictive on a Principle of Sufficient Reason (PSR). In Sect. 16.3 we consider the limitations on expression entailed by recursive linguistic structure, extending the argument from single languages to sets of possible languages and showing that even the properties of languages inevitably outstrip the properties expressible within those languages. Section 16.4 considers epistemic implications, extending the discussion beyond language to incompleteness of any body of *conceivable* truths in the face of a demonstrably larger realm of fact. Notwithstanding this amorphousness, something can nevertheless be shown regarding its general character that merits further consideration.

Sections 16.5 and 16.6 thus try to go further metaphysically and ontologically, for a glimpse of the world beyond our limits. The attempt itself sounds paradoxical, and it is in fact a paradox that we take as the key. The world as the totality of fact lies inevitably beyond our limitations—explanatory, expressive, and conceptual. But we propose we can nonetheless know something of its general character. The world as the totality of fact must form a *plenum* with implications we set out here to explore.¹

Plato's *Timaeus* launched the pivotal belief of ancient Neo-Platonism that Reality reflects the operations of Reason and accordingly constitutes a rationally intelligible manifold. In consequence, man, the rational animal, is able to get a reason-engendered cognitive grip on Reality's key features. This fundamental idea was to become one

of the mainstays of Western philosophy. But no-one, then or since, maintained that human reason's grip on Reality was complete or completable—that human cognition and speculation could exhaust the unbounded vastness of ontological possibility and plumb the bottomless depths of its relationship to the real—a task which, if achievable at all, required an intelligence of supra- and super-human capacity. But just where can we find clear signs of the limits of human intellection and pinpoint some of the issues that lie beyond the horizons of our cognitive reach? No doubt this is a difficult question but there are some things that can plausibly be said on the problem and hopefully some of them will be said here.

The limitations we track are characteristically not some boundary imposed from without but intrinsic limitations of reach from within an entire method of axiomatization, explanation, expression, or comprehension. The problematic clearly traces to Kant for whom human cognition has limits by way of limitations (*Grenzen*) but not boundaries (*Schranken*), there being no wall or fence that somehow ontologizes those limits. For us, those limits lie not, as with Kant, in the faculty structure of the human intellect, but in the nature of the conceptual resources characteristic of our cognition, or perhaps of any cognition.

16.2 Limits from Axiomatization to Explanation

The limitations of axiomatization are well known. No formal system adequate for basic arithmetic can be both consistent and complete. No axiomatic system can contain as theorems both all and only the truths expressible in the formal language of the system. We cannot hope to grasp all of mathematical truth—restricted even to the mathematical truth we have the means to express—with the techniques of axiomatization.

It is a short step from Gödel to Turing, from formal systems to mechanical algorithms. By the same token and in much the same way, no mechanical algorithm can give us all and only correct answers to some easily expressible questions about the function of mechanical algorithms. In both the Gödel and Turing results, it is the system itself—by a particular power of embedding—that reveals its own limitations. It is because a system for number theory can represent (or echo) any mechanism of axiomatic deduction that any axiomatic system will be provably incomplete. It is because Turing machines can echo and embed any algorithmic mechanism that there can be no faultless algorithmic mechanism for any of a range of basic questions regarding them all.

We will return to this explanation and the Principle of Sufficient Reason (PSR) at a number of points. Here, we start with a particularly simple version:

(PSR-T) For every truth there is some other, epistemically distinct, truth that provides a cogent explanation for it.

If we take “explanation” to demand a deductively valid accounting, PSR-T will be untenable for precisely Gödelian reasons. Any deductive system adequate for

scientific explanation will have to be adequate for arithmetic. But in any deductive system adequate for arithmetic, there will be truths expressible in the system which will not be deducible as theorems. These will be truths in violation of PSR-T.

We can take the result further, and make it more pressing, by *replacing* the concept of deduction in the Gödel result with a concept of explanation instead. Mathematical exploration through the last century, eloquently expressed by Hilbert, was a vision of some distant but attainably completed mathematics. That vision died with Gödel's proof. A vision of a completed explanatory *science* has spurred scientific exploration in much the same way. That vision of scientific explanation is as impossible as the correlate vision of mathematical explanation, and for precisely the same reasons.

Suppose a science which contains (a) a complete set of basic facts, and (b) a complete set of "explanatory consequence" principles whereby further facts follow from others. It is clear that any such system must also contain the mechanisms of any system adequate for arithmetic. Among its "basic facts" must be the axioms and among its "explanatory consequences" principles must be the rules of inference which are required for basic arithmetic. It then follows that there will be true statements in the language of such a science for which our "completed science" will be unable to offer a scientific explanation.

There is an older and simpler problem with PSR-T, of course. The explanatory project confronts us with the prospect of basic explanatory elements analogous to axioms which, by hypothesis, cannot be derived from anything else. Further forms of the principle of sufficient reason, correlate to even wider limitations on explanation, reappear later in our discussion.

16.3 Intrinsic Limits of Language and Truth

We humans conduct our cognitive business by means of language, broadly conceived to include all processes of symbolic communication. Linguistic articulation, both in human communicative reality and in its formal representation, is fundamentally recursive. Beginning with a finite vocabulary, it elaborates meaningful statements by means of a finite number of grammatical rules of combination. The result is a potentially infinite number of meaningful statements in any such language, but those statements will be enumerable and thereby denumerable in number. And, of course, if the meaningful statements (the well-formed formulae as a whole) can be enumerated (and thus be denumerable in number), this will also have to hold for the subset of them that are true. The truths expressible in any language, in sum, form a denumerable manifold.

At this point, a distinction between truths and facts becomes critical. We take truths to be linguistically articulated claims—specifically, those that are correct. We take facts to be something else again: states of affairs that obtain and do so independently of any articulation by linguistic means.

- A. We begin with the simplest formal case, which is also closest to the reality of human languages. Consider a language with a finite number of basic symbols and a finite number of recursive rules for combination. Such a language will afford us with a countably infinite number of formulae. At best, the expressible truths for such a language will be countably infinite

It's clear that there will be more than countably infinite facts, a point provable using the example of this language alone. The formulae of any such language L form a countably infinite set. But by the basic mechanisms of Cantor's Theorem, there will be more elements of the power set of any set than elements of that set itself. Consider then the power set PL of the set of formulae of this initial language. For each set element of PL there will be a distinct fact: the fact that a specific formula does or does not belong to that set, for example. Even this small corner of a world of fact—facts about the language L —will have facts inexpressible in L itself. The facts *about* such a language inevitably outstrip the truths it can express.

What are we to make of there being infinitely more actual facts than articulable truths? With human knowledge functioning linguistically by way of a recognition and acknowledgment of truths, does this disparity between facts and truths not entail the existence of an unknowable truth?

Here, it is instructive to begin with a simple analogy: that of Musical Chairs. Where there are more players than chairs it is inevitable that some will be left unseated when the music stops. So, the existence of *unseated* players is inescapable. But this, of course, does not itself mean that any players are *unseatable* so that it is in principle impossible for such a player to be seated. The prospect of seating cannot be denied to any of them.² When this situation is analogized to the truth/fact situation, we will have it that the inevitability of *unknown* facts does not of itself establish the existence of *unknowable* ones. All we can maintain at this point is that there are bound to be *unknown* facts: that there are unknowable ones does not follow. That *not every fact can be known* does not of itself enjoin that *some fact cannot possibly be known*. The quantitative disparity between formulable truths and objective facts does not immediately establish the existence of unknowable facts.

- B. What of the truths expressible by any *possible* language of this simple formal and very human form, involving finitely many basic symbols and finitely many recursive rules of combination? We begin by supposing that each possible language takes its basic symbols from some single but countably infinite reservoir of possible symbols, awash with as many basic symbols as there are numbers 1, 2, 3... On that assumption, the basic symbol sets of the full set of our possible languages will be enumerable: there will be only a countably infinite number of basic symbol sets

Because those finite sets of symbols can simply be appended as the first of the countably infinite formulae generable using them, within our basic assumptions we can envisage an enumeration of all formulae of all possible languages of this form as an infinite series of infinite arrays. Using $s1_{L1}$ through sn_{L1} to represent the finitely

many basic symbols of language 1 and $f1_{L1}, f2_{L1}, f3_{L1} \dots$ to represent its infinitely many compound formulae, such an array might take this form:

Language 1 $s1_{L1}, s2_{L1}, s3_{L1}, \dots sn_{L1}, f1_{L1}, f2_{L1}, f3_{L1}, f4_{L1}, \dots$

Language 2 $s1_{L2}, s2_{L2}, s3_{L2}, \dots sn_{L2}, f1_{L2}, f2_{L2}, f3_{L2}, f4_{L2}, \dots$

Language 3 $s1_{L3}, s2_{L3}, s3_{L3}, \dots sn_{L3}, f1_{L3}, f2_{L3}, f3_{L3}, f4_{L3}, \dots$

Language 4 $s1_{L4}, s2_{L4}, s3_{L4}, \dots sn_{L4}, f1_{L4}, f2_{L4}, f3_{L4}, f4_{L4}, \dots$

As in Cantor's proof for the countability of the rationals, however, we can introduce the individuated, circuitous but systematic enumeration of every item in that array as well:

Language 1	$s1_{L1},$	$s2_{L1},$	$s3_{L1}, \dots$	$sn_{L1},$	$f1_{L1},$	$f2_{L1},$	$f3_{L1},$	$f4_{L1}, \dots$
Language 2	$s1_{L2},$	$s2_{L2},$	$s3_{L2}, \dots$	$sn_{L2},$	$f1_{L2},$	$f2_{L2},$	$f3_{L2},$	$f4_{L2}, \dots$
Language 3	$s1_{L3},$	$s2_{L3},$	$s3_{L3}, \dots$	$sn_{L3},$	$f1_{L3},$	$f2_{L3},$	$f3_{L3},$	$f4_{L3}, \dots$
Language 4	$s1_{L4},$	$s2_{L4},$	$s3_{L4}, \dots$	$sn_{L4},$	$f1_{L4},$	$f2_{L4},$	$f3_{L4},$	$f4_{L4}, \dots$

On the assumption of a countable reservoir of basic symbols, then, there will be only countably many truths expressible in all *possible* languages of this basic form. We know the facts of even one of those languages form more than a countable set, and thus the facts regarding even one of these possible languages outstrip the truths expressible in all such possible languages.

- C. But perhaps we've sold linguistic possibilities short. We can expand our conception of formal languages, recognizing as we do so that we are leaving the limitations of human languages behind

Limitations like those above are demonstrable for even some superhuman languages. Let us start by allowing a language to contain more than a finite number of basic symbols. It is indeed standard in outlining formal systems to envisage a countably infinite number of basic formulae $p1, p2, p3 \dots$. That change alone won't alter the results for single languages. The countably infinite basic symbols of such a language can be interwoven with the countably infinite formulae that can be recursively generated from those formulae, giving us no more than countably infinite formulae overall. The cardinality of our formulae, the factual limitations of truths, will remain.

As long as our basic symbols are drawn from a countably infinite pool, the same will hold for all *possible* languages of such a form. For each language we can envisage

an enumeration that interweaves the countable series of basic symbols with the countable series of recursively combinatorial formulae:

Language 1 $s_{1L1}, f_{1L1}, s_{2L1}, f_{2L1}, s_{3L1}, f_{3L1}, \dots$

Language 2 $s_{1L2}, f_{1L2}, s_{2L2}, f_{2L2}, s_{3L2}, f_{3L2}, \dots$

Language 3 $s_{1L3}, f_{1L3}, s_{2L3}, f_{2L3}, s_{3L3}, f_{3L3}, \dots$

Language 4 $s_{1L4}, f_{1L4}, s_{2L4}, f_{2L4}, s_{3L4}, f_{3L4}, \dots$

All formulae in all languages can be enumerated as before:

Language 1	$s_{1L1},$	$f_{1L1},$	$s_{2L1},$	$f_{2L1},$	$s_{3L1},$	$f_{3L1},$	$\dots$
Language 2	$s_{1L2},$	$f_{1L2},$	$s_{2L2},$	$f_{2L2},$	$s_{3L2},$	$f_{3L2},$	$\dots$
Language 3	$s_{1L3},$	$f_{1L3},$	$s_{2L3},$	$f_{2L3},$	$s_{3L3},$	$f_{3L3},$	$\dots$
Language 4	$s_{1L4},$	$f_{1L4},$	$s_{2L4},$	$f_{2L4},$	$s_{3L4},$	$f_{3L4},$	$\dots$

The formulae of all possible languages based on countably infinite symbols from a countably infinite pool will still form merely a countable set. The truths expressible in all possible languages of such a form will be merely countable.

- D. The situation changes if we further broaden assumptions, leaving human capabilities even further behind. Consider the possibility of a larger reservoir from which a language might draw its basic symbols: a reservoir that has as many basic symbols not merely as the rationals, for example, but as many as the reals

Any language that has either a finite number of basic symbols drawn from such a pool or a countably infinite number of such symbols will be limited, as above, to a countably infinite number of formulae. But the conclusions drawn so far will not hold for all *possible* languages of this expanded form. A very simple way of seeing this is to envisage those languages that have merely one basic symbol. Since that symbol can be any of a collection as large as the reals, we will not be able to enumerate all of those languages, prohibiting the countable list of languages used on the left axis in the arrays above. For languages with basic symbols drawn from a set the size of the reals, then, formulae of *each* language will be countable but formulae of all *possible* such languages will not.

Limitations of countably many formulae are obviously lifted for even *single* languages if we allow a language to have as many simple formulae as the reals. Somewhat less obviously, limitation to the countably infinite is lifted for a single language

with countably many basic formulae and infinite combinations: infinite conjunctions or disjunctions, for example. We might list conjunctions in such a language by using 0 or 1 to indicate whether they include symbol 1, symbol 2, symbol 3, and so on:

Conjunction contain: $s_1 s_2 s_3 s_4 s_5 s_6 s_7 s_8 \dots$

Conjunction 1 : 0 1 1 0 1 0 0 1 ...

Conjunction 2 : 1 0 0 0 1 0 1 0 ...

Conjunction 3 : 0 0 0 1 0 1 0 0 ...

It is clear that every infinite series of 1 s and 0 s will be represented by some conjunction in such a system. But these correspond to the infinite decimals between 0 and 1, which correspond to the reals. Cantor's proof that there are nondenumerably many reals may be performed quite directly on any proposed enumeration of these conjunctions. We can produce a conjunction *not* on the list by exchanging 1 s and 0 s on the diagonal.

- E. If we weaken assumptions and stretch possibilities for languages far enough, then, we can have sets of possible languages and even single languages that transcend the limits of a countable infinity of expressible truths. In a very real sense, however, such languages bring us no closer to the world of facts

No matter how large the set of formulae expressible in any of these languages, the power set of that set will be larger than the set itself. For every set element of that power set, there will be a fact: the fact that a given formulae is or is not an element of that set, for example. There will still be more facts expressible in any given language.

Given any set of specifications for a form of language, there will be a set of formulae and thus a set of truths expressible in all *possible* languages of that form. The power set of *that* set of all possible formulae or expressible truths will be larger still, and thus the facts even about sets of truths expressible in all possible languages of a specific form will transcend the truths so expressible. Like the individual languages within them, ranges of possible languages embody more facts than they can possibly express.

- F. All of the arguments presented to this point have been written in terms of syntax: numbers of formulae generable within a given language. But languages in the sense we are after are perhaps better conceived of semantically, such that formulae are *about* certain things, using predicates to express properties *of* certain things. A more semantic and in that sense more philosophical form of the argument makes the point in its most general form

With any language there will be those things that it can say things about: what we might term the *linguistic objects* of a language. Things in general, linguistically reachable within such a language or not, we can term *factual objects*. Any language will also have those things that it can say *about* things: its *predicates*. Factual properties that actually hold of things, linguistically bound or not, we will simply term *properties*.

On this simple outline, it's clear that the predicates of any language will themselves be factual objects. By an analog of Cantor's Theorem, we know that the sets of those objects outnumber the objects themselves. But for each such set, there is a unique property, indeed an extensional property: the property of belonging to such a set, for example. There are therefore more properties of factual objects than there are predicates available in any language to express those properties. Indeed, there are more properties of the predicates of any language than there are predicates in the language. The facts of properties inevitably outstrip truths expressible by predicates.

What holds for a single language holds for all possible languages. If we consider the predicates applicable in any possible language, of whatever form, we are considering a set of factual objects. But there will be more sets of such objects, and thus there will be more factual properties than there are predicates applicable in any possible language.

- G. Does this entail that there is any specific inexpressible truth? One can hardly ask for an example. To this point, considering languages both syntactically and semantically, the image of Musical Chairs still holds: each language will leave out some fact, but nothing yet identifies a specific fact that will be left out

Languages are more than syntactic structures, more even than syntactic structures with correspondences to objects and properties. Languages are means of managing information. Information is packaged in the form of expressions, unpacked by means of derivation. It is in terms of information that we can begin to see some specifics regarding linguistic limits: for any language L , a *specific* body of information beyond it.

We have termed truths those linguistic elements that correspond to facts. For any language there will be those truths expressible in the language. Each truth will embody some information, reflecting some fact. But there is one body of information that will inevitably escape a language, in one way or another: that body of information that is represented in *all* of its truths combined. For any language L , we will term that megafact M_L . There is no single truth in L that can capture this megafact: totalistic self-representation cannot be internalized declaratively.

Suppose any language L , and all truths expressible in L . Consider moreover a truth-preserving set of rules of derivation R employed in L which allows one to squeeze out as consequent truths the information contained in a given truth. Finally, consider M_L , the information contained in all the truths of L . M_L will either be L -inexpressible or R -inaccessible, at least in part. M_L will be inexpressible in L in any way in which all the information in M_L will be derivable by R .

Our discussion started with Gödel and Turing as a foundation, moving from there to considerations that were largely Cantorian. Here, the argument turns on Gödel once again. Were M_L both L -expressible and fully R -accessible, there would be an axiomatic system with R as its rules and M_L as an axiom from which all truths expressible in L were derivable. By Gödel, there can be no such axiom system.

The result will clearly hold for all the kinds of languages to which Gödel applies: all those satisfying the minimal requirements of an L and R adequate for arithmetic. It is also possible to generalize the result beyond those specific requirements.³ Given any rules of derivation R , a language that can represent R -derivability we will call R -expressive. A language that can take any of its own expressions as objects we will term expressibility-reflective. For any expressibility-reflective language L that is R -expressive, for any truth-preserving R , the megafact M_L for that language will either be L -inexpressible or R -inaccessible, at least in part: either M_L will be inexpressible in the language of L , or there will be information in M_L that will be underivable by R .

For any language within these minimal constraints there will be a particular fact that proves inaccessible for it: the megafact M_L that represents the totality of information in the facts that it does represent. Note that M_L doesn't have to extend to all facts. It is specified relative to a language and encapsulates merely the information expressible in the facts captured in that language. Even that smaller language-relative totality of facts escapes the nets of language and derivability.

H. Here, again, our reflections impact the principle of sufficient reason

Anything rationale offered as an explanation, in any language, will be a set of expressions within that language. The available rationales for any language will therefore be limited by the available expressions. For a standard language L with countably many expressions, for example, there will be only countably many possible finite rationales.

It's clear from the pattern of argument above that for any L there will be not only more facts than linguistically expressible truths, but more facts than there are available rationales. Using "explanation _{L} " to indicate rationales in language L , then, the following version of the PSR will fail for any L :

(PSR-F) Every distinct fact has a distinct explanation _{L}

The lesson will extend to the languages of non-standard forms considered above. It will also extend to explanation in any or all possible languages. If we consider the rationales expressible in any possible language, of whatever form, we are considering a set of factual objects. But there will be more sets of rationales than rationales themselves. For each of those sets there will be a distinct fact. There will therefore be more distinct facts than distinct rationales in any possible language. Generalizing "explanation" from "explanation in L " to "explanation in *any* possible language," this more encompassing version of the PSR will fail as well:

(PSR'-F) Every distinct fact has a distinct explanation.

Throughout these deliberations the crucial fact that it is possible to know *that* something of a certain sort obtains without knowing of any particular item of the sort that it obtains is a control background consideration. There is accordingly a salient difference between knowing facts generally and knowing them individually and distinctly.

16.4 Epistemic Reflections and Conceivability

What does this disparity between linguistic truth and trans-linguistic fact mean for our knowledge? To what extent do the limitations of language extend to limits of conceivability?

- A. At first glance, axiomatization as a model of a distinction between explicit and implicit knowledge might seem to offer some hope

By the Cantorian argument, the expressible truths of any language will be outnumbered by the facts. But there are two ways of affirming or claiming a fact. One is to state it explicitly and specifically, in the form, for example, of a corresponding truth. Another is to affirm it obliquely and implicitly by stating other facts from which it follows. In that sense, a single statement—the conjunction of the axioms of a system, for example—can be seen as implicitly containing the full information of all theorems of the system. It lies in the logic of things that one truth can informatively encompass a vast—indeed a potentially infinite—multitude of other distinct claims.

One true claim, such as a conjunction of the axioms of plane geometry, can informationally encompass the entire field. Finite access to claims does not itself therefore entail finitude in knowledge. Given the distinction between explicit expression and implicit deducibility on the model of axioms, the quantitative disparity between truth and fact might not seem all that portentous.

We might, then, distinguish two basic questions:

- Q1. Can the totality of the facts in the domain at issue be stated and acknowledged explicitly in terms of coordinate truths?
- Q2. Can the totality of the facts in the domain at issue be substantiated at least obliquely and implicitly by way of inferential axiomatization?

The force of the Cantorian argument—there are more facts than truths with which to express them—is that the answer to Q1 is a clear “No.” But for standard systems, at least, a Cantorian argument shows that the answer to Q2 must be “No” as well.

Standard systems will have only a countable number of theorems. Even implicitly, therefore, their axioms will contain only a countable number of truths. Implicit knowledge amounts to deductive closure: we implicitly know whatever can be derived from what we explicitly know. Derivation is a recursive process. It begins with

premisses and applies stepwise to any of a finite register of inferential rules. A body of explicit axioms, then, be it finite or countably infinite, can never represent more than a countable body of implicit knowledge. In the previous section we envisaged systems beyond standard systems. But even these will have only some limited cardinality of implicit theorems—a cardinality that will be provably exceeded by the range of fact... even the range of fact about those theorems.

If our model of implicit knowledge is axiomatic, it must be recognized that the power of an axiomatic system cannot exceed that of the language in which it is expressed. Since the above-mentioned findings hold for all languages, they hold for the implicit knowledge contained in any axioms written within those languages as well. Any hope for conceivability beyond linguistic limits must appeal to something beyond implicit knowledge, or at least implicit knowledge conceived on the model of axiomatization.

B. Given the distinction between facts and linguistic truths employed throughout, there is another question close to those above. Here, the question is again one of implicit as opposed to explicit knowledge, but limited merely to the facts expressible in a language:

Q3. Can the totality of *truth* in the domain at issue be claimed and affirmed at least obliquely and implicitly on the model of inference from axioms?

This question demands something more like a Gödelian than a Cantorian analysis. Here, again, in ways allied with considerations above, the answer will be “No.”

For any system adequate for arithmetic and therefore, of course, for realms of truth and fact at large, there will be *truths* expressible in the language that are not deducible from the axioms. If even expressible truths within a language outstrip the implicit information of any axiom set, the implicit knowledge contained in axioms does not seem to offer an escape.

The question of implicitly knowing M_L is particularly instructive. M_L is too “large” to be seen as a consequence of some other truth in the system: it contains by definition all information of all truths in the system. Nor can it function as an axiom which implicitly contains all other information, as long as “implicitly” is taken on the model of inference from a consequence function R . By the results above, no system can express an M_L from which all information is recoverable by inference.

Any appeal to implicit knowledge in the hopes of overcoming the limits we’ve documented above must appeal to implicit knowledge conceived on some model other than that of axiomatic containment or logical inference. The distinction between implicit and explicit knowledge remains an intriguing one, and one that will reoccur in thinking about conceivability and reference to a world beyond.

C. Does essential limitation of knowledge doom us to error?

The numerical discrepancy between truth and fact means that our knowledge of a world of fact is bound to be imperfect. Specifically, it means this knowledge is incomplete. Does it also mean that it is incorrect—that it contains not only gaps

but errors? After all, suppose that you are otherwise fully informed about swans in general but are totally unaware that some Australian swans are black. One is then bound to arrive at the erroneous conclusion that all swans are white.

The incompleteness of our knowledge does not, of course, *ensure* its incorrectness—after all, even a single isolated belief can represent a truth. But it does strongly *invite* it. For if our information about some object is incomplete, then it is bound to be unrepresentative of the objective make-up as a whole so that a judgment regarding that object is liable to be false. The situation is akin to that depicted in John Godfrey Saxe's *The Blind Men and the Elephant* which tells the story of certain blind sages who variously read incomplete evidence as indicating a creature like a wall, a spear, a snake, a fan, or a rope. "Each was partly right," Saxe concludes, "And all were in the wrong."

The lesson is clear. The incompleteness of object-descriptive statements certainly does not entail their incorrectness: incomplete information does not ensure false belief with categorical necessity. But it does ensure inadequate understanding since at the level of generality there will be too many gaps that need filling in. There are just too many alternative ways in which reality can round out an incomplete account to warrant confidence in the exclusion of error.

This vulnerability of our putative knowledge of the world in the face of potential error is rather *exhibited* than *refuted* in our scientific knowledge. For this is by no means as secure and absolute as we like to think. We cannot but recognize in our heart of hearts that our putative truth, in fact, incorporates a great deal of error. There is every reason to believe that where scientific knowledge is concerned, further knowledge does not just supplement but generally corrects our knowledge in hand, so that the incompleteness of our in-formation implies its presumptive incorrectness as well.

- D. To this point we have concentrated on the disparity between the limited world of linguistic truth and the larger world of fact beyond, but the range of these deliberations can be extended yet further. It is not merely in language that we manage our attempts at grasping facts, but in conceptualization and thought. Although neither speculation nor conceptualization need be recursively conceived or recursively limited, the same quantitative disparity between epistemic thinkability and ontological actuality will obtain in these contexts as well

Is there reason to think that the realm of fact must outstrip pure conceivability? We have seen the limitations of language, and a long philosophical tradition insists that the limitations of language are necessarily the limitations of conceivability and therefore of knowledge as well. If we conduct the business of conception and knowledge via language, the limitations we've already noted, essential to any language, will be limitations of conceivability and knowability as well.

But limitations will still face us even if we abandon the assumption that conceivability and knowledge are tied to language. Let us assume a notion of conceivable propositions beyond the limits of linguistic expression: the conceptual parallel to facts rather than truths, perhaps. Consider all the propositions you have entertained in the course of reading this chapter, or all the propositions that have come to mind

throughout the day. Consider all the propositions you have ever entertained, or all the propositions which you will in fact entertain throughout your lifetime.

The world of fact will necessarily outstrip any such set of propositions. There will be more subsets of propositions than there are propositions themselves. For each of these, there will be a specific fact: that a given proposition *P* is or is not a member of that set, for example. There will then be more factual propositions than those that you conceive in a day or indeed that all humans conceive in the course of human history. The world of fact will necessarily outstrip the realm of propositions conceived, and thus, of course, of things known.

The argument takes us even further. For consider not merely the propositions that have or will be conceived, but the propositions it is in any way possible to conceive: not merely the conceived but the *conceivable* propositions. For even these a numerical argument will apply: there will be more subsets of propositions, and thus more facts, than there are *conceivable* propositions.

The implication is that there are facts that are not even *conceivable*. That conclusion, of course, is one that holds on the level of generality. We cannot meaningfully claim to know—or even conceive—of any of them. The claim that there are inconceivable facts is in that regard like the claim that there are facts that I do not, in fact, know. I can conceive of there being inconceivable facts, of course, without being able to conceive of any of the specifics, just as I can know there are facts I don't know, of course, without knowing any of those specific facts.

Unlike the image of Musical Chairs, the inconceivable facts would have to be specific inconceivable propositions. The realm of what is actually conceived, by a person on a day, in a lifetime, across all human history, or by all creatures capable of entertaining propositions, might have been different. But the realm of what is *conceivable* in any of these categories would seem to be metaphysically fixed. If there are more facts than there are conceivable propositions, there must be *specific* facts beyond the range of propositional conceivability.

- E. There is an air of paradox at this point: in conceiving of inconceivable facts, have we not somehow made them conceivable after all? Hints of paradox do mark any attempt to glimpse the world beyond, but there are several relevant considerations here

Here, as before, we might appeal to a distinction between explicit and implicit conception, direct or indirect, full or weakly oblique. In a full sense, a proposition is conceived only when it is entertained in full content and with genuine understanding. In a far weaker sense, a proposition may be conceived *of* in any of a number of indirect ways—as the core propositions that a speaker will be arguing for, for example, but that I have not yet heard. We can thus think of the numerical argument as leading us to the weaker conception of propositions that are beyond conceivability in the full sense.

We can perhaps press the paradoxical character of the argument, however, by explicitly considering all facts that might be conceived *of*. In a similar fashion, we might consider all the facts that might be referred to in any way, either directly or

obliquely. Given the basic Cantorian argument, there will be more facts than can be conceived of, and more facts that can be referred to in any way. If the “however possible” defines a fixed set, there will be specific facts that cannot even be conceived of, and which cannot be referred to in any way. But have we not just conceived of those? Have we not just referred to them?

There is an escape clause here that we will return to below and that we will in fact use as a window to the world beyond. For now, let us note that the core argument, like its predecessors, relies on essential assumptions of number: the assumption of fixed collectivities with a given cardinality.

Applied to conceivable or referable facts, the argument takes the same form as that used earlier to show that the truths within any language will be outstripped by the facts of a world beyond. In that case, the character of languages does indeed commit us to a fixed collectivity of expressions and thus of expressible truths that has a specific cardinality. When it comes to facts conceivable in any sense, to facts referable in any sense, or to all the facts themselves, it will be these assumptions of collectivities bound by the familiar principle of number that we will have to leave behind.

The present proposal is not to treat the reasoning that leads to such a point as somehow illegitimate and in need of a “solution.” Instead, the idea is to let the logic of the matter lead into a genuine, though radically unfamiliar, realm beyond.

16.5 Facing Facts

Any world of fact must extend beyond language and beyond explanation. In at least some sense, it must extend beyond conceivability as well. Is any glimpse of the character of such a world simply impossible?

Surely not. For what the present discussion attempts is just exactly a glimpse into that world beyond.

The previous considerations provide a warning that the full world of fact will not be conceived in standard terms. Some of our familiar ways of approaching things must be compromised. Interestingly, they may be compromised in any of several ways.

If there is a world of fact, we will propose, its collectivity must be conceived as a *plenum*. Plena are supra-numerical collectivities that violate at least one of several standard logical assumptions. Among such supra-numerical collectivities are the totality of all things, of all abstract objects, of all propositions. Like these, we propose, the world of fact constitutes a plenum.

- A. Consider a Cantorian argument applied directly to the totality of facts. Given any such totality, there will be more sub-collections of the totality than there are members. But for each of those sub-collections there will be a distinct fact: that a given fact *f* is or is not a member of that sub-collection, for example. There will then be more facts than contained in the totality of facts

Something has to give. The argument can be perspicuously rendered as an aporetic triad:

1. The Cantorian assumption: There will be more sub-collectivities of any collectivity than there are members of that collectivity.
2. The Factual assumption: For any sub-collectivity of any collectivity there will be a distinct fact.
3. The Totality assumption: there is a collectivity that contains all facts.

Given (1), there will be more sub-collectivities of the collectivity assumed in (3) than there are members of (3). Given (2), there will be more facts than there are members of (3). Given (3), there will be more facts than are contained in a collectivity that contains all facts.

In this form the aporia is clearly one of number: any supposed totality of facts will have more members than it has members. Whatever *number* it contains, it must contain more than that number. Our exploration will involve digging beneath that concept of number. We begin, however, by surveying possible options.

One option is to deny (3). Despite appearances, despite deep intuitions, and perhaps despite our apparent ability to quantify over facts in general, there simply is no totality of facts. The world of facts is essentially incomplete: facts refuse to form a whole. The universe, on such an approach, is incomplete. It is this option that one of us has argued for in earlier work.⁴ Aristotle, Kant, and Russell can be seen as precursors.⁵ “Indefinite extensibility” approaches, in denying a completed totality, can also be seen in this tradition.⁶

Another option is to deny (2). Despite appearances and despite deep intuitions, there are things regarding which there are no facts. The things are there, they are what they are, but there is no fact regarding them. However difficult to believe, such an approach has also been attempted.⁷

The third option, which we will pursue, is to deny (1). There are collectivities for which Cantorian assumptions do not hold: collectivities beyond standard principles of number.

These collectivities will in fact be *defined* as having a unique member for each of their sub-collectivities. For any conception of their contents at any moment of thought—for any snapshot of membership at any conceptual moment—these collectivities will contain more. These collectivities, beyond standard assumptions of either sets or any collectivities like them, are *plena*.

We can construct a graphic example if we think of patterns of one or more patches on a two-dimensional plane, where each patch of a pattern must have an area. A pattern in our sense consists of a collection of patches that need not be contiguous, and indeed that might overlap. Graphically portrayed, one might think of a patch within another patch distinguished by a different color. For completeness, we include a completely blank plane as a pattern as a well.

Given this concept of patterns, it is clear that both any sub-pattern of a pattern and any collectivity of patterns will themselves constitute a pattern. The totality of all patterns will constitute a plenum, since every collectivity of elements of that

totality—analogue to the elements of the power set of a set—will also constitute an element of the totality.

If propositions are understood as claims to facticity in the abstract, beyond any linguistic limits of mere statements, the totality of all propositions will constitute a plenum. For every collectivity of propositions, there will be a distinct proposition—that a favored proposition p is included in that collectivity, for example (whether true or not)—and thus the totality of propositions will contain as many propositions as there are collections of propositions. The totality of things will constitute a plenum, if “things” is broad enough to include collections. Every collectivity of things will constitute a thing in its own right. The totality of abstract objects will constitute a plenum for similar reasons.

Moreover, facts taken as a whole will form a plenum as well. There indeed is a world beyond language, sets, and systems. This, to be specific, is the plenum constituted by the world of facts.

There are several approaches to the aporetic triad that have points in common with the approach we take here, though we regard these as mere points of contact, short of the full metaphysical vision of a trans-numeric world of fact that we propose. In an attempt to understand truth, Hans Herzberger, Anil Gupta, and Nuel Belnap envisage truth as a concept that forces its own revision, much in the way that any attempt to conceive of the contents of a plenum as a fixed collectivity forces a revised vision of its further extent.⁸ In the same light, an approach in terms of “indefinite extensibility” has points of contact with our own. Graham Priest urges us to welcome any inconsistency in the aporetic triad for its own sake, opening dialectic arms to “true contradictions.”⁹ We will not knowingly embrace contradiction. There is nonetheless a way of reading some of Priest’s conclusions—that totalities at issue are both complete and not—that does resonate to some extent with the vision of plena we wish to present.

- B. As expressed above, the aporetic triad turns on a concept of number that is buried within the Cantorian assumption: “There will be *more* sub-collectivities of any collectivity than there are members of that collectivity.” On a Cantorian conception of number, the claim that a collectivity Y contains more than another collectivity X means simply that any line-up of the two, such that every member of X is assigned a distinct member of Y , will leave out some member of Y : the “more” that Y contains

Cantor’s theorem is that the subsets of any set S —elements of its power set PS —will necessarily outnumber the elements of S . The proof is that there can be no mapping M of elements of S onto distinct elements of PS that doesn’t leave some element of PS out. For any proposed M , the proof offers a specific element of PS that must be left out. Here, two points are of particular note. The first is that the “specific element of PS ” or subset of S that is necessarily excluded from the mapping M is itself specified in terms of M and a specific relation R . The second is that the “necessary exclusion” of that element is exclusion on pain of contradiction. Derivation of the contradiction demands exclusive and exhaustive alternatives regarding an element of PS and that element of S mapped to it by M . S must either stand in relation R to its corresponding

element or not. At its foundations, then, the “more” of our aporetic triad is a matter of contradiction given exclusive alternatives and a peculiar reflexivity involving a mapping M and relation R .¹⁰

Although our target is collectivities well beyond mere sets, it is worthwhile to review the general mechanisms of the familiar set-theoretic proof. We assume any mapping M designed to assign each member of S to a unique member of its power set PS. The relationship R is set-membership, a crisp, binary relationship fully obtaining or failing to obtain between any two candidates. We then consider a particular subset of our original set, specified in terms of M and R : the set D (for diagonal) of precisely those members of S which are not members of the subset to which they are assigned by our mapping M . If M fulfilled the conditions of “same number,” giving us a one-to-one correspondence onto all elements of PS, it would assign some member s of our original set to D . But given either of two exclusive and exhaustive alternatives regarding membership, any such assignment leads to contradiction. If s of S is a member of D , it will by specification of D *not* be a member: D is to contain *only* those elements of S that are not members of the subset assigned by M . If s is not a member of D , it will by specification of D be a member of D : D is to contain *all* those elements of S that are not members of their corresponding subset.

The power set of any set must be larger than the set itself. In the context of classical set theory, the obvious next question has always been: “And what of the set of all sets?” By virtue of containing all sets, it must contain the elements of its own power set. But won’t we then be forced to conclude that it is larger than itself?

With an eye to possible exportation to the aporia regarding all facts, consider standard responses to the strictly set-theoretic issue of a set of all sets. The standard line, despite appearances, despite intuitions, and perhaps despite our apparent ability to quantify over sets in general, is to deny the existence of a set of all sets. One move here, kicking the problem upstairs, is to create a new department of “classes,” to one of which all sets (but, of course, not all classes) are assigned.¹¹ Another move is to deny or restrict the power set axiom, required in standard axiomatization to give us PS for arbitrary sets S to begin with.¹² A third move, echoing a theory of types, is to attempt to restrict the specifications of subsets so as to exclude the specification required to give us D .

- C. None of the standard options for dealing with a set of all sets can be said to be intuitive. All look like cheating. All carry an atmosphere of the ad hoc. Parallels to those options become even less intuitive when we attempt to export them to the issue of a totality of facts

For every collectivity of facts there will be a distinct fact: that a chosen fact is an element of that collectivity, for example, or that it is not. That a chosen fact is entailed by the collectivity, or that it is not. That the collectivity is finite, for example, or that it is not. That some of its elements entail other elements, or that all elements of that collectivity are logically distinct. Consider any of these “collectivity facts” regarding the facts of a specific collectivity.

Consider now (a) the elements of a collectivity of all facts and (b) facts regarding collectivities of these, of any of the forms above: facts as to the facts they contain, facts regarding the facts they entail, the finitude or infinitude of the collectivities at issue, or the like. We can think of the facts falling within the collectivity of a collectivity fact (b) as facts within its domain. Somewhat more informally, but to the same point, we might think of the facts within the domain of a collectivity fact as facts it is *about*.¹³

Take any one-to-one mapping M from the facts of (a) to the collectivity facts of (b). Any such mapping must leave some element of (b) out. Consider in particular all those facts on the left that do not fall within the domain of their associated collectivity fact. There will be a fact df about precisely that collectivity: that it entails a chosen fact f or that it does not, that it is finite or infinite, and the like. But there can be no element f^* of (a) mapped to fact df . If f^* falls within the domain of df , it cannot, by specification of df in terms of our mapping M . If f^* does not fall within the domain of df , it must, again by specification of df .

In the context of the argument targeted to facts, the option of denying the existence of a set of all sets would be paralleled by a denial of any totality of all facts: denial of (3) in our aporetic triad above. On that line there is no world of all facts: the factual world refuses to form a coherent whole.¹⁴ Such a route seems to violate the concept of a world.

The option of avoiding a set-theoretic diagonal set D by denying all sets within a power set PS can be paralleled here by avoiding df , denying that any collectivity of facts is something about which there will be a fact. This amounts to a denial of (2) above. This route seems to violate the very concept of facts.

Neither of these options allows us a world of facts. One offers us a totality of something short of the ubiquity of facts. One offers us facts without a totality. On either approach, on pain of contradiction, we are again forced to conclude that there are too “many” Cantorian facts to form a world.

One might choose simply to revel in contradiction. We take the result more seriously than that, as an invitation to explore a realm beyond. In the present line of inquiry, we assume a genuine world of fact. We ask what results such as these have to show us about the possible character of that world, however strange.

What we explore is what must follow if we deny (1) of the aporetic triad. The world of facts, we propose, lies beyond a number of the Cantorian assumptions. The world of facts forms a plenum.

16.6 The World of Fact as Plenum

We define a *plenum* as a collectivity that contains distinct elements corresponding to each of its sub-collectivities, where sub-collectivities follow the same pattern as subsets: something qualifies as a sub-collectivity of a collectivity C just in case each of its members is a member of C .

In a membership plenum, such as a collectivity of all collectivities A , each sub-collectivity is itself a member of A . In other plena, such as the collectivity of all facts F , there is a fact regarding each sub-collectivity of F that is itself a member of F . Membership plena contain their own power collectivities. Other forms of plena contain members that map onto their power collectivities.

We take such plena to exist, with the world of fact as an example so intuitive as to be undeniable. The question for us, then, is not whether there is a world of fact but what such a world must be like.

- A. We assume both (2) and (3) of the aporetic triad above. For anything that exists—and thus for any sub-collectivity of any collectivity—there will be a distinct fact. There is moreover a world of all facts. What we must deny, then, is the Cantorian core in (1): the claim that there will be more sub-collectivities of any collectivity than there are members of that collectivity

The key to the Cantorian argument is that crucial concept of number: the claim that there will be *more* sub-collectivities of any collectivity than there are members of that collectivity. That “more” amounts to the thesis that there can be no one-to-one mapping M from elements of a collectivity C to elements of its power-collectivity PC or some collectivity FPC which contains distinct members for each element of PC .

If we are to embrace plena as collectivities with members for each sub-collectivity, we must deny that there will be “more” of the latter. We must hold that the PC be a mapping M from C to PC or FPC which leaves no element of the latter out.

In doing so we have to find the loophole in the Cantorian argument that attempts to show there can be no such M . That argument rests on specification of a particular element D of PC or FPC which stands in relation R to all and only those elements of C to which their corresponding M -correlate does *not* stand in relation R . Our assumed mapping, in assigning an element of C to every element of PC or FPC , must assign an element d to D .

- B. The crucial step in the argument is the dilemma step. Does d stand in relation R to D , or not? If not, by specification of D in terms of M , d must stand in relation R to D . But if it does, again by specification of D , it cannot

The lesson, we believe, is that for any plenum there will be inherent indeterminacy in R . For any M , any R , and any D definable in terms of M and R , the M -correlate to that D neither will nor will not stand in relation R to D . In the case of a simple membership plenum C , for every way M of assigning elements of C to elements PC one-to-one, the element d of the plenum assigned to that D by M neither will nor will not be a member of D . In at least some cases, the Law of Excluded Middle (LEM) will fail for the membership relation within plena. For some items x within a plenum P , it will be neither the case that $x \in P$ nor $x \notin P$. In that sense, some of the borders of plena will be imperfect, imprecise, or indeterminate.¹⁵

The lesson regarding a world of all facts is clear as well. The Cantorian argument regarding facts relies on “collectivity facts”: facts regarding whether a specific

collectivity of facts contains or entails a specific fact, for example, or is finite or infinite. The crucial question of that argument is whether a specific fact lies within the domain of such a fact: somewhat informally, whether it is one of the facts that collectivity fact is about. Because the world of facts is a plenum, the relevant relationship—that a fact lies within the domain of another, is one of the facts it is about, or is one of the facts for which the collectivity fact holds—must in at least some cases be indeterminate. It is not always the case that a fact either is an element of a specified collectivity of facts or is not. It is not always the case that a fact either is one of the facts another fact is true of or is not. It is not always the case that one fact subsumes another, or is about another, or is not.

That, we suggest, is the lesson to be drawn from the clear existence of a world of fact. Given a total world of fact, various facts about the world will have to be indefinite, indeterminist, or undefined. Corresponding to a multitude of collectivity-defining characteristics Y , there will be a multitude of factual theses of the form “It is not always the case—it is not always itself a fact—that a particular fact f is either Y or not Y .” What might be called alethic indeterminacy—indeterminacy of fact—will pervade the world of fact.

Our reflections have brought us to alethic indeterminacy from consideration of a fact’s membership in a given collectivity of facts, or having a characteristic shared by certain facts. In that train of thought, it appears to be on the meta-level of facts about facts that is crucial.

At this point, both the substance and form of the result are reminiscent of Gödel, though with an enlarged perspective. Gödel showed that any consistent systematization of arithmetic will be incomplete, leaving the provable truth or falsity of certain arithmetical truths undetermined. The proof involves the technique of Gödel numbering, allowing statements of the base language to correspond to or “encode” second-order statements regarding theoremhood within the system. Our conclusion also involves reflexivity, though it applies in the metaphysical realm well beyond logical systems: any totalization of fact is going to leave the status of certain factuality-claims indeterminate. Given the structural similarities, resonant results in this enlargement of perspective should perhaps not be entirely surprising.¹⁶

It should be emphasized that the denial of LEM at issue throughout is a *strong* denial, rather than invocation of either a third alternative or any number of additional alternatives. Were we to think in terms of three exhaustive categories—that a fact (i) falls within the domain of another, (ii) does not, or (iii) neither does nor does not—we could construct a relation R in terms of the second two that would be sufficient for resurrection of the basic argument. Were there *any* totality of exhaustive categories, we could do precisely the same. The strong denial of LEM is a denial that there is *any* set of exhaustive categories regarding the relationships between facts and collectivity facts at issue.¹⁷ The lesson to be drawn from the clear existence of a world of fact is that a prime characteristic of some facts—that they take others as part of their subject collectivity—does not hold in terms of any set of exhaustive categories regarding all pairs of facts. In that sense, the lesson of a world of fact is that certain characteristics of facts themselves are not what we might have taken them to be.

The argument may well generalize to other characteristics of facts. It is worthy of note, however, that it will not generalize to all. The Cantorian argument cannot be plausibly constructed in terms of just any relation R .

Consider an attempt to construct the argument in terms of logical entailment, for example. Some facts and some sets of facts logically entail others. For any M , from facts to elements of the power set of a set of all facts, we might then envisage D as all those facts which are not entailed by the elements of the power set to which M assigns them. M must assign a fact d to that D .

But what then is the crucial question required for a Cantorian dilemma? We might first phrase the question as one of membership: Will d be a member of D or not? If it *is* a member, it will not be entailed by its corresponding set D . Interestingly, we cannot maintain that option: if d is a member of D , D certainly will entail d . But we can maintain that d is *not* a member of D . It follows that D will entail d without containing it, but that does not give us contradiction. A set of propositions may entail many that it does not strictly contain.

We might alternatively ask whether d will be logically entailed by D . If it is not, it is an element of C not entailed by its M -correlate, and so will be a member of D . But as a member of D , of course, it will be logically entailed by D . The hypothesis that d will not be logically entailed by D is inconsistent. But the hypothesis that d *will* be logically entailed by D is not. In that case, d , though not a member of D , will be entailed by D . Once again, a set of propositions may entail many that it does not strictly contain.

Given a world of facts, some relations—whether one fact falls within the collectivity addressed by another, for example—must be indeterminate. Logical entailment, on the other hand, need not be.

Though short of contradiction, there is a strange consequence of the argument phrased in terms of logical entailment. Because it can be run for any proposed one-to-one correspondence M from facts to collectivities of facts, the diagonal construction D for *every* such M will entail whatever d is assigned to it.

- C. We have defined plena as collectivities which take as members either their own subsets or elements such as facts mapped onto their subsets. Any world of fact would necessarily meet that criterion

There are, we think, four options regarding plena:

1. Using standard logical principles, we might insist on Cantorian grounds that plena do not and cannot exist.
2. We might hold that plena do exist, but that the law of excluded middle fails to hold for all cases for membership and crucial relations R .
3. We might hold that they do exist, but that the law of non-contradiction (NC) fails to hold in all cases for membership and crucial relations R .
4. We might hold that plena do exist, with every element of their power set as or corresponding to a member, and with power sets that are indeed larger than they are

On the assumption of a world of all facts, (1) must be rejected. We have outlined (2) as a favored option, tracking some of its implications for the nature of facts. We consider (3) and (4) more radical options, but include consideration of these as well.

- D. The dilemma at the core of the Cantorian argument takes the form “Does d stand in relation R to D or not?” That dilemma assumes that its options are exhaustive—precisely the assumption denied in putting aside the law of excluded middle for such a case. That dilemma also assumes, however, that its options and their consequences are exclusive: that something cannot both stand in relation R to D and not. The force of the argument can be broken at that point if we simply shrug and accept both options

The implication would be that for *plena*, issues of membership can be both “yes” and “no”: in some cases, collectivity c can both be a member of another collectivity c' and not be a member. In some cases, a fact f can both fall within the domain of another fact f' and not fall within that domain.

Here, consequences are roughly the dual of those outlined above. On denial of LEM, membership and whether a fact is among those another fact applies to are indeterminate in some cases. On denial of the law of NC, these will be overdeterminate in some cases. In one case, it is exhaustiveness of alternatives that is denied—that a fact is either among the collectivity to which another applies or that it is not. In another case, it is exclusiveness of alternatives that is denied—that a fact cannot be both.

Our tendency, as noted, is to go for indeterminacy and the LEM. Another tack, however, would be to derive a disjunctive lesson. For *plena*, membership must either be indeterminate or overdeterminate in some cases. For facts, whether one fact falls within the domain of another must be either indeterminate or overdeterminate in some cases.

- E. A last option, though the most radical, also has its attractions. Could there be a one-to-one mapping from a plenum to its subsets? From facts to sub-collectivities of facts? The answer from (2) and (3) is that there could be such a mapping. *Plena* need not be larger than themselves

The last option is to accept the conclusion of the Cantorian argument. There can be no exhaustive mapping from a plenum to its subsets. Its power set is larger than it is, in that sense. But every one of its subsets appears as a member. It is therefore larger than itself. On this approach we maintain both the law of non-contradiction and the law of excluded middle. All the assumptions of the Cantorian argument stand, as does its conclusion.

Such an approach has some aesthetically pleasing elements. The idea that *plena* will be larger than themselves has an intuitive resonance with feelings one gets when thinking about a totality of fact, for example: having thought one had them all, one finds they are more. *Plena* seem to expand under our gaze.

There is also something pleasing in thinking of plena as the third step in size conception of collectivities. Finite sets are collectivities such that all proper sub-collectivities are smaller than the collectivity itself. Infinite collectivities are those such that some proper sub-collectivities are as large as the collectivity itself. Plena are collectivities such that some proper sub-collectivities are larger than the collectivity itself.

There is, however, a major sacrifice here as well. On such an approach there will be no one-to-one mapping from plena onto themselves. If there were, there would be a mapping onto their power set, violating the conclusion of the Cantorian argument.

The non-existence of a one-to-one mapping for plena would mean that there is no relation that holds one-to-one between members of a plenum. That would seem to force us to the most radically contentious option of all: to hold that items of a plenum will even fail to map onto themselves by way of a relation of identity. Even self-identity will fail for at least some items of a plenum.

For collectivities, this would appear to mean that whether something is identical to another—is the same collectivity as another—would in some cases be indeterminate. For facts, this would mean that whether something is the same fact as another would be indeterminate. On such a view we would have individual facts, we would have a totality of all facts as a plenum, but the concept of “the same fact” would lose its grip. Here, perhaps, is the most complete sense in which we would lose the concept of number: we would lose the concept of distinct entities involved in the counting.

We cannot say that we recommend such a route: after all, “everything is what it is, and not another thing.” Were one to take such an approach, however, we think the appropriate route would be to emphasize the extent to which the concept of identity in general becomes problematic at this juncture. Classically, identity is detailed in terms of features or properties: $x = y$ for $(\forall F)(Fx \equiv Fy)$. If having certain properties itself becomes problematic for elements of plena, the applicability of identity so understood may become problematic as well. It should also be noted that such a route, however radically contentious, is not without precedent: Peirce denies identity for elements of a continuum, which has a number of points of contact with plena as considered here.¹⁸

F. With the concept of plena in hand, we can return to some of the issues raised in previous sections

It is clear by Cantorian argument that there will be more facts than there are propositions conceived of in the course of human history. There are more sets of those propositions than there are those propositions themselves. But for each such set there will be a distinct fact. The world of fact will outstrip the world of human conception.

That alone may not seem surprising. Extending the argument in Sect. 16.3, however, seemed to lead us into paradox. A Cantorian argument can be run not merely on all propositions that have or will be conceived, but the propositions it is in any way possible to conceive: a collectivity of all *conceivable* propositions. On such an argument it appears that there will be propositions that cannot in any way be

conceived. But doesn't our grasp of the argument itself demonstrate that we have in some way conceived of them? Similar paradoxes accompany Cantorian arguments regarding all facts that might be referred to in any way, either explicitly or indirectly. There will be more facts than these; but are we not at this point referring to those facts supposedly beyond reference?

In Sect. 16.3 we alluded to an escape clause, pointing out that each of these relies on the essential assumption of fixed collectivities with a given cardinality. That assumption is the Cantorian assumption (1) that we have abandoned in favor of plena in exploring a world of facts. An escape from paradox by way of a similar denial seems called for in these cases as well.

These apparent paradoxes, we propose, like the question of a totality of facts, point to the existence of plena. The realm of conceivable propositions, conceivable facts, and facts to which we might at least obliquely refer may all form plena: collectivities for which every sub-collectivity corresponds to a member. If all of this holds for actual facts, it will clearly hold for the still richer realm of possibilities: these will all the more emphatically constitute a plenum.

On the assumption of such plena, cashed out in any of the ways we've outlined—by strong denial of the law of excluded middle, exceptions to the law of non-contradiction, or a vagueness of identity—the Cantorian argument falls short. When broadly construed so as to include oblique conception and reference, we can see the realm of possible reference and conception—like the world of fact itself—as forming a plenum.

We should remind ourselves that in a familiar range of more restricted considerations all the classical principles can still be maintained. It is only when we reach for a grasp of totalities such as the world of all facts that we turn the page, forcing us to resort to new devices. Are compromises in familiar principles such as the law of excluded middle too high a price to pay for recognizing the existence of plena? Here, the simplest answer, we think, is that we have no choice: it seems inescapable that there must be a world of fact as a whole. If so, here as elsewhere, it is our thinking we must mold to the world rather than the other way around.

Newtonian physicists confronted modern science with a physically infinite astronomical cosmos, the contemplation of whose vastness filled Pascal with vertiginous fright. Cantorian set theory confronted modern mathematics with a qualitatively infinite numerical realm of numberless quantities.

The present deliberations confront modern philosophy with an epistemically infinite manifold of fact. Modernity is replete with challenges of coming to terms with the many guises of infinitude. Our discussion here is simply another instance of this larger phenomenon.

16.7 Lessons

It is clearly demonstrable, from a number of sources and in a number of ways, that we face major limitations in the face of a world beyond the accustomed horizons

of thought.¹⁹ Our axiomatics imposes limits on formalization, with corresponding limits on explanation and the principle of sufficient reason. Gödelian arguments show that demonstrable fact cannot exhaust fact.

Our language imposes limits on expressibility, limits that extend even to all possible languages. We argue that even expressible fact cannot exhaust fact. Beyond these, even conceivability faces inherent limits: the world of facts necessarily outstrips the world as we conceive it.

Despite those limitations, we propose that we can get a glimpse of the world of fact beyond. We can limn its general shape as that of a plenum: a collectivity that includes elements corresponding to all sub-collectivities.

Recognition of that fact, however, also forces us to recognize that such a world is unfamiliar in at least one of several ways. There is indeed a world of fact. But certain relations of facts to facts that might be assumed unproblematic—such as the question of whether one fact falls in the subject domain of another—will have weaker logical properties than we might have assumed. We have to conclude that whether one fact is about another may be indeterminate, in the sense of a strong denial of the law of excluded middle, or overdeterminate, in the sense of a violation of the law of non-contradiction. A third alternative is that both of these hold, but hold for a range of things that are themselves less determinate than we might have taken them to be. On the third alternative, it is a principle of identity that fails to hold in all cases: “the same fact” loses its grip.

Language is a purposive instrument. Ordinary language has evolved for everyday use. Logico-Mathematical language is primarily for logico-mathematical purposes. But beyond those familiar purposive horizons there lies the realm of abstract deliberation—a conceptual Wild West outside the pale of familiar logical law. Here, the very questions one asks tend to be non-standard. When you ask extraordinary questions, we propose, you must expect extraordinary answers.

The reality beyond our conceptual horizons is a world that instantiates a plenum regarding whose *being* we can reasonably say something but regarding whose *nature* we do and can know effectively nothing. Our acknowledgment of this world is an instructive reminder to being honest and humble. It is the epistemic equivalent of the Roman functionary whose task was to give the emperor an ongoing reminder: “Remember that thou art but mortal.”²⁰

Notes

1. Nicholas Rescher and Patrick Grim, “Plenum Theory,” *Noûs* 42 (2008): 422, Rescher and Grim, *Beyond Sets: A Venture in Collection-Theoretic Revisionism* (Frankfurt: Ontos Verlag, 2011).
2. In logical notation, the different at issue is that between $\Box(\exists x) \sim Sx$ and $(\exists x)\Box \sim Sx$.
3. Here, the generalization of Gödel follows roughly the lines of Chapter 3 of Patrick Grim, *The Incomplete Universe* (Cambridge, MA: MIT Press, 1991).
4. Grim, *The Incomplete Universe*.
5. Graham Priest, *Beyond the Limits of Thought* (New York: Oxford Univ. Press, 2002), p. 229.

6. See Stewart Shapiro and Crispin Wright, "All Things Indefinitely Extensible," in Agustín Rayo and Gabriel Uzquiano, eds., *Absolute Generality* (Oxford: Oxford Univ. Press, 2006), p. 255.
7. Keith Simmons, "On An Argument Against Omniscience," American Philosophical Association, New Orleans, April 1989.
8. Hans Herzberger, "Notes on Naïve Semantics," *Journal of Philosophical Logic* 11 (1982): 61; Anil Gupta and Nuel Belnap, *The Revision Theory of Truth* (Cambridge, MA: MIT Press, 1993).
9. Priest, *Beyond the Limits of Thought*.
10. Patrick Grim and Nicholas Rescher, *Reflexivity: From Paradox to Consciousness* (Frankfurt: Ontos Verlag, 2012).
11. The further sorrows of class theory are documented in Grim, *The Incomplete Universe* and Priest, *Beyond the Limits of Thought*.
12. Christopher Menzel, "On Set Theoretic Possible Worlds," *Analysis*, vol. 46 (1986), p. 68. See also Menzel, "Sets and Worlds Again," *Analysis*, vol. 72 (2012), p. 304.
13. The difficulties of pinning down the concept of aboutness in even the context of linguistic statements, making free use of the concept of designating expressions, became evident long ago in an exchange between Rescher and Goodman (Goodman, "About", *Mind* 70 (1961): 1; Rescher, "A Note on 'About'," *Mind* 72 (1963): 268). The current deliberations extend beyond language, targeting a relation of aboutness between facts. In the context of facts, we'll argue, the concept of aboutness is not merely difficult to define but indeterminate in application.
14. As in Grim, *The Incomplete Universe*.
15. This indeterminism bespeaks a curious parallelism between the realm of the theoretically very large—plena—and the physically very small—quanta.
16. See also Grim and Rescher, *Reflexivity*.
17. See Rescher and Grim, *Beyond Sets*, Chapter 6.
18. See Wayne C. Myrvold, "Peirce on Cantor's Paradox and the Continuum," *Transactions of the Charles S. Peirce Society* 30, no. 3 (1995): 508; Fernando Zalamea, *Peirce's Logic of Continuity* (Boston MA: Docent Press, 2012); and Benjamin Lee Buckley, *The Continuity Debate: Dedekind, Cantor, du Bois-Raymond, and Peirce on Continuity and Infinitesimals* (Boston, MA: Docent Press, 2012).
19. Although such a phrase and much of the spirit of our piece echo Graham Priest's title for *Beyond the Limits of Thought*, it should be clear that his acceptance of contradictions is just one of the approaches outlined here.
20. This chapter originally appeared in *Logos and Epistemology*, vol. VIII (2017), pp. 425-454.

Part IV
Issues of Philosophizing

Chapter 17

Philosophical Confrontations



17.1 Philosophical Conflict

In common practice, material that is in doubt and where there are things to be said pro and con is said to be “discussable.” For better and for worse just about every issue within philosophy’s domain merits this characterization. But only rather rarely does actual discussion take place. Usually philosophers toil in isolation—often taking account of what is said by others (albeit generally only those who are long dead). Actual confrontation by over interaction is a rarity.

Philosophers are usually loners toiling away in self-imposed isolation. So, almost is there collaboration; and despite extensive disagreement even overt conflict is rare.

The present discussion will consider a few exceptional and thereby exceptionally striking episodes of philosophical confrontation. The conflicts at issue here did not, to be sure, involve an exchange of physical blows—with philosophers it fortunately seems not to come to that! Rather the matter has been one of an exchange of words, of letters, and (in one case) of books.

17.2 St. Paul Versus The Greek Philosophers (Athens, ca. 50 A.D.)

A good place to begin with is St. Paul’s account of his encounter in Athens ca. 50 AD with “certain philosophers among the Epicureans and the Stoics” (*Acts*. 17: 18 ff.). Upon hearing reports about his sermons, they inquired regarding “strange things” being maintained in the “new doctrines” that he was presenting on various and sundry occasions.

Paul’s response pointed out that since the Athens already had an altar dedicated TO THE UNKNOWN GOD his doctrine was not really all that new and strange. This conception would surely encompass the idea of an immortal and not localizable “lord

of heaven and earth” who “dwells not in temples made by man,” but is the creative agent giving reality and life to all things and through whose agency “we live, and move, and have our being.” However, so Paul continued, this God has appointed someone whom, though he died, was revived to life and will return on an appointed day “to judge the world with righteousness.” At this point the Athenians closed their ears and minds—the idea of a resurrection from the dead was just too much for the philosophers. And there is no indication that it made much of a favorable impression in the bystanders. So all in all, the exchange between St. Paul and the philosophers of Athens did not go well because they were not prepared to take one of the key aspects of his message all that seriously. His doctrine were just too far outside the range of these accustomed ideas.

St. Paul’s interchange with the philosophers of Athens typifies the ever-present duality in human thinking between the commonplace familiarities of current experience and the transcendental idealities of speculative thinking. For the Greeks, the gods were simply humans made larger—replicated on a greater scale of power, agency, and duration. For them, the idea of a divine being’s death—and even more that of a posthumous resurrection—was something that asked too much of them by way of a suspension of disbelief. And just here we find one of the prime difficulties of disposition. For when—as is only normal and natural the participants conduct their thought and reasoning with the framework of their established beliefs, they are likely to find it somewhere between difficult and impossible to wrap their minds around the conception of their opponents.

17.3 Las Casas Versus Sepulveda (Valladolid, 1550 A.D.)

As Spain was colonizing the New World in the time of Charles V (1516–1556) there arose a bitter discord between the lucre-hungry conquistadors and the pious friars who, on orders of the king, always accompanied their explorations. The object of dispute was the status of the local natives, the Amerindians. Were they—as the friars maintained—human beings with souls to be saved and lives to be integrated into the community of the Church? Or were they—as the conquistadors preferred to think—like more than the larger hominids of Africa, sophisticated mammals available as slavers for labor in the gold and silver mines? Were they actually fellow humans or were they to be seen as an inferior sort of being?

As the friars resisted the exploitation of the Amerindians and insisted on pressing their position, the king referred the matter to some of the best-available experts of the day—the cream of the crop among the theologians and academics of Spain. They assembled in 1550–55 to address the issue in a scholastic debate at the *Dominican Colegio de San Gregorio* in Valladolid, whose focus was, in effect, the following proposition: *The indigenous natives of the New World are rational and ensouled beings who, as such, deserve the protection of king and church.*

The Salamanca-trained Dominican friar and bishop, Bartholomeo de las Casas (1484–1566)—ever after dubbed “the Apostle of the Indies”—pleaded the friars’

case with great eloquence and cogency. The principal spokesman for the opposition was the formidable Renaissance scholar and philosopher Juan Ginés de Sepulveda (1494–1573).

Against Sepulveda's degrading those Amerindians as practitioners of sodomy, ideology and con-ion Las Casas argued that the conception of "natural slavery" in Book I of Aristotle's *Politics* as fit for barbarians deficient in reason, morality, and justice fails to apply to the Amerindians.

The Valladolid debate issued quantifiably in favor of Las Casas but made a deep imprint in the sense of morality and fosters on matters of colonization in 16th century Spain to an extent for which other colonial powers had to await the passage of centuries. Regrettably, however, this only made rather small impact on the actual treatment of the Amerindians. When theories clash with interests, their implementation is all too frequently abandoned.

In point of fundamentals, however, the controversy posed large issues of transcended philosophical. How to decide whether or not a creature seemingly capable of intelligent action—not obviously human and possibly even alien or android in nature—is or is not a fellow rational being? Is the matter to be addressed entirely in terms of analogies such as those at issue with the plea of Shylock in Shakespeare's *Merchant of Venice*:

Hath not a Jew eyes? Hath not a Jew hands, organs, dimensions, senses, affections, passions? Fed with the same food, hurt with the same weapons, subject to the same diseases, healed by the same means, warmed and cooled by the same winter and summer, as a Christian is? If you prick us, do we not bleed? If you tickle us, do we not laugh? If you poison us, do we not die? And if you wrong us, shall we not revenge? If we are like you in the rest, we will resemble you in that.¹

Many deep questions arise here. Does being human pivot on a close scrutiny of the extent of such analogies? Or is the operative factor simply a benefit of doubt as long as there is reasonable room for it? Does the weight of such determinations rest on the factual or on the ethical balance of the scale? Should it be necessary to press the analogy of *modus operandi* ever onwards into greater detail—or should even a little of it suffice to settle matters by bringing the principle of Christian charity to bear? The Valladolid episode provides much food for thought along these lines, inviting reflection about just what it is takes to qualify creatures as actually human.²

17.4 Leibniz Versus Clarke (Hannover/London, 1714–15)

For a wide complex of theoretical, practical, and personal reasons G. W. Leibniz, the great German philosopher-mathematician-polymath was interested in launching into controversial interaction with theorists in the larger and influential circle around Isaac Newton. Geographic separation dictated that any such controversy would have to be conducted by post. In the face of Newton's proverbial inaccessibility, Leibniz employed a well-tried stage for launching a debate. He picked an imminent and influential intermediary (in this case his old friend the Princess of Wales).

Leibniz inaugurated the correspondence with a bold provocation of little more than a dozen sentences, accusing Newton and his followers of a crude materialism. Newton, he charged, saw space as “an origin to perceive thought by” and viewed the physical world as an imperfect machine that needed occasional repair to function properly. (This later point was based on the apparent need in Newton’s physics for giving the earth an occasional push to keep its orbit from spiraling into the sun under the impetus of gravitation.) Newton’s philosophy of nature, so Leibniz charged, could not be squared with the general convictions of educated people. And so the two correspondents soon locked horns on such matters as:

- the structure of the physical universe and the questions of absolute vs. relative space and time
- The nature of causality and the prospect of action at a distance
- the principle of sufficient reason and the universal rationale explicability of nature’s proceedings
- the sufficiency of the laws of nature for explaining the phenomena without recourse to God

The interesting aspect of the epistolary exchange that followed is its exponential growth. As the dialectical interchange of objection and replay went on, each successive discussion was twice as long as its predecessor, and the deliberations entered into ever greater detail. Had the exchange continued much beyond its fifth interaction it would have taken up a whole library.³

Running throughout the Leibniz-Clarke Debate like a leitmotiv of an opera is the idea of the difference between a *planned* universe and a *managed* one, between a transient and an imminent role for the Creator. For Leibniz world history unfolds according to a rational plan designed to optimize the realization of certain objectives. Clarke, by contrast, envisions an actively managed universe. Big theories are at stake—God, the universe, and man’s place in the scheme of things.

It is noteworthy that in the course of the Leibniz-Clarke exchange—and its aftermath—no-one on either side yielded an inch of ground relating to their initial position. As the participant saw it, the other party should have seen their view as obviously correct if only they had understood it properly.

17.5 De Bois Reymond Versus Haeckel (Berlin, 1882–99)

In the 1880s, the German physiologist, philosophers, and historian of science Emil du Bois-Reymond published a widely discussed lecture on *The Seven Riddles of the Universe* (*Die Sieben Welträtsel*).⁴ In it, he maintained that some of the most fundamental problems about the workings of the world were insoluble. A rigorous mechanist, he argued that the limit of our secure knowledge of the world is confined to the range where purely mechanical principles can be applied. Regarding anything else, we not only *do not* have but *cannot* in principle obtain reliable knowledge. Under the banner of the slogan *ignoramus et ignorabimus* (“we *do not* know and

shall never know”), du Bois-Reymond maintained a skeptically agnostic position with respect to various foundational issues in physics (the nature of matter and force, and the ultimate source of motion) and psychology (the origin of sensation and of consciousness). These basic issues are simply explanatory *insolubilia* that altogether transcend man’s scientific capabilities. Certain fundamental biological problems he regarded as unsolved but perhaps in principle soluble (though very difficult): the origin of life, the adaptiveness of organisms, and the development of language and reason. And as regards his seventh riddle—the problem of freedom of the will—he was undecided.

The position of du Bois-Reymond was soon sharply contested by the zoologist Ernest Haeckel. (*Die Welträtsel*), published in 1889,⁵ he maintained that far from being intractable or even insoluble, those riddles of du Bois-Reymond had all virtually been solved. Dismissing the problem of free will as a pseudo-problem—since free will “is a pure dogma [which] rests on mere illusion and in reality does not exist at all”—Haeckel turned with relish to the remaining riddles. Problems of the origin of life, of sensation, and of consciousness, Haeckel regarded as solved—or solvable—by appeal to the theory of evolution. Questions of the nature of matter and force, he regarded as solved by modern physics except for one residue: the problem (perhaps less scientific than metaphysical) of the ultimate origin of matter and its laws. This “problem of substance” was the only riddle recognized by Haeckel, but was downgraded by him as not really a problem for science. In discovering the “fundamental law of the conservation of matter and force,” science had done pretty much what it could do with respect to this problem; all that remained was metaphysics, with which the scientist has no proper concern. Haeckel summarized his position as follows:

The number of world-riddles has been continually diminishing in the course of the nineteenth century through the aforesaid progress of a true knowledge of nature. Only one comprehensive riddle of the universe now remains—the problem of substance.... [But now] we have the great, comprehensive “law of substance,” the fundamental law of the constancy of matter and force. The fact that substance is everywhere subject to eternal movement and transformation gives it the character also of the universal law of evolution. As this supreme law has been firmly established, and all others are subordinate to it, we arrive at a conviction of the universal unity of nature and the eternal validity of its laws. From the gloomy *problem* of substance we have evolved the clear *law* of substance.⁶

The basic structure of Haeckel’s position is clear: science is rapidly nearing a state in which all big problems admit of solution—substantially including those “insolubilia” of du Bois-Reymond. All considered, natural science had pretty much accomplished its mission—reaching a state in which all scientifically legitimate problems were substantially resolved.

And the historical record gives support to this point of view. After all, the annals of science are replete with achievements which, before the fact, most theoreticians had insisted could not possibly be accomplished. Charles S. Peirce has put the key point trenchantly:

For my part, I cannot admit the proposition of Kant—that there are certain impassable bounds to human knowledge.... The history of science affords illustrations enough of the folly of

saying that this, that, or the other can never be found out. Auguste Comte said that it was clearly impossible for man ever to learn anything of the chemical constitution of the fixed stars, but before his book had reached its readers the discovery which he had announced as impossible had been made. Legendre said of a certain proposition in the theory of numbers that, while it appeared to be true, it was most likely beyond the powers of the human mind to prove it; yet the next writer on the subject gave six independent demonstrations of the theorem.⁷

The course of historical experience runs counter to the idea that there are any identifiable questions about the world (in a meaningful sense of these terms) that do in principle lie beyond the reach of science. It is always risky to say *never*, and particularly so with respect to the prospects of knowledge. Never is a long time, and in this context “never say never” is a more sensible motto than its paradoxical appearance might indicate.

And yet the idea of the completion of scientific inquiry remains problematic. The phenomenon of the new questions was first emphasized by Immanuel Kant, who saw the development of natural science in terms of a continually evolving cycle of questions and answers, where “*every answer given on principle of experience begets a fresh question, which likewise requires its answer* and thereby clearly shows the insufficiency of all scientific modes of explanation to satisfy reason.”⁸ Kant’s claim suggests the following Principle of Question Propagation (Kant’s Principle): “The answering of our factual (scientific) questions always paves the way to further yet unanswered questions.”⁹

The debate over the thesis that the “Natural science can and eventually will answer every meaningful question about how things work in the universe” continues among scientists and philosophers now as ever with unabated touchiness.

And in the end there yet remains a question already put on the agenda by Leibniz. For no matter what the extent science resolves our questions about nature, the fact remains that it does so by means of the laws of nature, so that the issue ever remains of addressing the problem of why these are as is. This, to all appearance, is the supreme Welträthsel.

17.6 Cassirer Versus Heidegger (Davos, 1929)

In March of 1929 the Swiss health-resort village of Davos was the site of a dramatic philosophical event. The organizers of an occasional “International Seminar” had arranged a dialogue on “What is Man?” and invited the participation of two outstanding German humanists of the day, the philosopher of culture Ernst Cassirer and the philosopher of man Martin Heidegger as rivals for a debate.

Shorn of the technicalities of their own characteristic vocabulary of exposition the difference between the two philosophical gladiators related to four main issues.

1. *Man and world*: For Cassirer it is critical that we humans live in and have to come to terms with a physical domain (the actual world) within which we use theory

and technology to construct a social and cultural environment for ourselves. For Heidegger only our social-cultural context matters. Our prime concern is with the human interaction—Nature, the stage of what the human drama takes place need be of no more concern to us than is the physical stage for a performance “Hamlet.”

2. *Philosophy's Mission.* For Cassirer philosophy's mission is synoptic. The aim is to create a framework of thought in which every aspect of human endeavor—science, history, literature can find a proper role. An encyclopedic vision. For Heidegger philosophical thought that takes a limited and financial perspective in the present realities of the human situation. Focus on the issues and problems of the here and now.
3. *The Problem of Progress.* Cassirer was an heir to the era of enlightenment which has that a key goal of human endeavor is to argue the circumstances and conditions of human life—to use knowledge for the promotion of happiness. Heidegger was a child of WWI disillusionment that sees the utilitarian ideal of human happiness as a delusion, life improvement as a mirage, and coming to terms with the realities of the moment as ineffectual.
4. *Value Renovation.* Cassirer endorsed the traditional standards of human inquiry and action and devoted the bourgeois standards of family, society, intellectual cultivation and physical well-being as appropriate. Heidegger viewed such values them in a Nietzschean perspective as outmoded and unsuited to the needs and opportunities of the day. He argued the value of a more basic and fundamental—more primitive if you will—basis for the conduct of life, deeming traditional civilities as superficial, pointless, and hypocritical.

Both revivals took their debate very seriously—viewing it as a prime opportunity to give expression to their long concerned and elaborated articulated positives. Cassirer, in this regard, was spokesman for a laudation of intellectual idealism stretching back to Plato and controversy via scholasticism and the enlightenment to German idealism and *Kulturwissenschaft*. Heidegger, by contrast, saw this tradition as bankrupt. Combining the cultural staticism of Nietzsche with the post-WWI disillusionment of Otto Spengler's *Decline of the West* he was attuned to a pre-philosophical primitivism that could plausibly look back to Heraclitus as its model.

There can be no surprise that Heidegger was widely viewed as victorious in this debate as conducted in the fraught ideological decade of the 1930s with its dedication to ideological issues, communism, fascism, nihilism, etc. Only when the course of subsequent history brought to light not just the emptiness but the viciousness of these absolutistic issues, did the pendulum swing back to a more just appreciation of more traditional modes of doctrine and valuation espoused by Ernest Cassirer.¹⁰

One of the main disagreements between Heidegger and Cassirer relates to the significance of cognitive evolution. For Cassirer the earliest stages of man's thinking about his world revolved about a group of mythic conceptions whose occult character had to be overcome in the course of more sophisticated modes of understanding. For

Heidegger, by contrast, the primordial style of cognition is not negatively primitive but something positively primordial and more truly faithful to a reality to which greater sophistication conceals from view through realty-distorting artifice. As Heidegger saw it, the ongoing development of scientific understanding does not bring us closer to the fundamentals of the human condition but ongoingly estranges us from its primordial reality. What we moderns deem as progress is mere distortion: our sophistication is alienation from the fundamental realities of the human condition (much like the sophistications of scholasticism and distinctions from rustic faith.)

For Kant, the world as we know it is *our* word—devised by mental operations functioning in line with “The faculty structure of the human intellect.” For Heidegger this meant that reality’s relation to our view of it is revelatory—that Reality discloses itself aspectively in being experienced by us, and that existence in nature is a matter of being-there (*Dasein*) *for us*. What we experience is not reflective of the nature of reality but only indicative of its being. Cassirer, by contrast, viewed Kant the other way round: Reality is the existential (but not *causal*) ground of reality which our experience is a symbolic representation rather than a causal product. (Remember that a symbol is neither the largest consequence nor yet the causal product that which is symbolized). For both thinkers Kant was a founding father, but of very different enterprise—productive of the real in the case of Heidegger and suggestive of its nature in that of Cassirer.

Fundamental to Heidegger’s thought is the idea of inescapable subjectivity: the fact that whatever I might say or think is subject to the subjectivity-gear consideration that I say or think it to be so (As William James put it) the trail of the human serpent is over everything.”)

But against this interpretation Cassirer urged the potential objectivity of human knowledge. We can after all, separate the thinker from the thought. And moreover, whatever I say or think, however idiosyncratic, remains something that has to be said in a language that is openly public and universally accessible. And so while we humans do not have the capacity to enter into one another’s minds we do have the universal to enter into one another’s thoughts by mastering one another’s unavoidably public language. Universal comprehensibility is not limited to science but holds for culture as well. Culture is universal. None of us humans is cut off from the Bible or for Shakespeare.¹¹

The Cassirer-Heidegger dispute reflects a large and pervasive cleft in Western philosophizing pivoting on the contrast between thought and action as reflected in the Schopenhauerian split between *Vorstellung* and *Wille*. On the one side, that of action there predominant thinkers such as Nietzsche, Marx, William James—and with them Heidegger. On the other stands Hegel, Dilthey, and the Marburg Neo-Kantians. This divide—equal in import with that between the partisans of the natural science on the one side and the human *Geisteswissenschaften* on the other—points toward a Peircean reconciliation of practice as the arbiter of adequate theorizing that leave the matter where it began—in Leibniz’s insistence on a fusion that combines and coordinates theory and praxis.

17.7 Popper Versus Wittgenstein (Cambridge, 1946)

The Moral Science Club at Cambridge University was an active center of philosophical exchange dating from... On one memorable evening in October of 1946 it was the site of a remarkable clash between two Viennese scholars who had found refuge in Britain, both leading figures in 20th Century philosophy—Ludwig Wittgenstein (1889–1951) and Karl Popper (1902–1994). The topic looked to be one of those own-navel contemplating issues to which academics are so often drawn: “Are there Philosophical Problems?”

The topic of controversy was straightforward. As Popper saw it—and was prepared to maintain against all comers:

Notwithstanding the universality efforts of philosophers over many centuries, philosophy remains a serious area of inquiry: it unifies meaningful questions and affords the possibility of providing cogent and plausible answers to them.

For Wittgenstein this view is entirely wrong. The classic problems of philosophy are based on misconceptions and misunderstandings and the only legitimate role for philosophical deliberation is to show why and how they are meaningless and should not really be matters of concern. The proper work of philosophy is a sort of disciplinary suicide: the self-destruction of its traditional concerns

No sooner had Popper began his lecture and by stating his positive position than “all hell broke loose.” But as is all too common, exactly what happened depends on whom among the many witnesses you ask.

There is reasonable agreement on the main sequence of events. Popper began with an explanation. His chosen topic was on the *problems* of philosophy, although—so he said—the secretary’s lecture invitation was to inaugurate discussion by giving “a short paper, or a few opening remarks, stating some philosophical puzzles.” But this very formulation begged the question as he saw it: was philosophizing a matter of addressing fundamental problems or was it simply an exercise in puzzle solving? Wittgenstein at once intervened. The secretary’s formulation was appropriate: the issue was one of legitimacy: were those so-called problems really manifest? Popper responded that Wittgenstein’s view of philosophy was thoughtless and superficial: it never looked beyond surface matters of formulation to probe the real difficulties that lay beneath the surface of language. Wittgenstein responded that this was nonsense: that whatever problems there actually were belonged to the sciences—mathematics, sociology, physics. Philosophy had no problems of its own. Popper countered by asking what of ethics?—were matters of right and wrong resolved by science? Having grown visibly upset, Wittgenstein reached and picked up a poker placed by the fireplace. Holding it aloft in gesticulation he said, in effect, “Oh really? Well just what are these characteristically ethical problems you have in view.” Someone—perhaps Russell—interjected to caution him: “Wittgenstein, put that poker down.” Sensing that the sympathy of the group coming against him Wittgenstein shouted: “You always misunderstand me, Russell.” But Russell’s patience was at an end: “No Wittgenstein, you’re the one who is mixing things up. You always mix things up.” A

furious Wittgenstein turned on his heels, stormed from the room, and slammed the door. Either just before or just after—witnesses differ—Popper closed the exchange with the quip “So Wittgenstein requires an example. Well, here it is. Not to threaten visiting lecturers with a poker.” Cambridge’s most dramatic philosophical debate was at an end.

Who won this dispute? To all appearances, Popper. After all, in actual conflict victory in general helps to him who holds the field when the battle is over. But the matter is not really all that simple. Even in warfare there is such a thing as strategic withdrawal to fight more effectively another day.

Victory in these matters lies largely in the eyes of the beholder. If history is applied judge then Popper certainly prevails. The vast majority of contemporary philosophers think of themselves as dealing with real problems and of their work bearing addressing critical important resolute issues of human knowledge and conduct. But now as then there is a tenacious majority of thinkers who see the philosophical enterprise as a matter of clearing the Angrier stables of incomprehension and misunderstanding. Within philosophy and even about philosophizing itself there are no permanent or unlikely ventures. Philosophical principles are vampire-like, refusing to suffice permanent death they spring back to life in reminiscent and rejuvenated favor.

The fullest and most vivid account of the controversy is *Wittgenstein’s Poker: The Story of a Ten-Minute Argument Between Two Great Philosophers* by David Edwards and John Eidngon. However, this otherwise informative account is seriously diminished by a constant speculation about what was going on in people’s thoughts rather than with events in the observable realm. The book pervasively presents pure conjecture as substantial fact.

17.8 Conclusion

These six controversies are of course far from exhaustive. Philosophers are by nature argumentative and inclined to disagreement. But these six episodes are not only strikingly significant in themselves but continue to illustrate the general nature of the phenomenon and to provide some instructive lessons about it.

What, after all, is the point of philosophical controversy? This is not a simple question and accordingly has no simple answer. If the aim of the venture is taken to be persuasion than it would appear pointless. For virtually never does controversy produce a change of mind in the thoughts of one opponent, nor yet do those exchanges seem to exert all that much influence on the interested bystander so as to enroll them on one side or the other.

Instead what seems to be realized is something neither party actually intends—and very possibly does not really prize—namely, *clarification*.

The clashes at issue seem never to have change minds—certainly not with the contestant, nor yet with their followers and adherents and (perhaps more surprisingly)

not even with bystanders who seem from the outset to stand firmly committed to one chosen side.

A general principle seems to be at work here.

Philosophers never change their mind on the fundamentals of their principle: they just readjust and refine the details of its articulation. And it makes little sense to look on these philosophical clashes as a scholastic debate of sorts, given that there are really no winners and losers—and really no judges. For while it might be tempting to ascribed this role to “posterity” the fact is that posterity just does not take sides: each side has its continued following with ongoing allegiance. Such philosophical disputes are not trials with winners and losers, they are ongoing fissures forming canyon of separation in the ideological landscape.

The lesson that emerges from examining philosophical confrontations is not that philosophical problems are irresolvable (let alone meaningless), but that to solve them one must deploy a point of view—a systemic framework of concepts that enable the issues to be addressed meaningfully. Just as we cannot state or consider facts without employing a language that provides for their formulation, so one cannot address philosophical problems without a frame of reference provides for their conceptualization. And just as what the facts are is ultimately independent of the language that permits their accessibility, so the cogent resolution of a philosophical issue does not depend on those means of conceptualization. As a rose by any other name is yet a rose, so a good argument by any other name is still a cogency, and an error a mistake.

Notes

1. William Shakespeare, *The Merchant of Venice*, Act 3, Scene 1.
2. Bartholomeo de las Casas, “A Brief Account of the Destruction of the Indies” in his *Writings* ed. and tr. by George Sanderlin (Maryknoll: Orbis Books, 1993); William H. Prescott, *The History of the Reign of Philip the Second, King of Spain* (London: Routledge, Warne & Routledge, 1855). Instructive Modes accounts of the debate include Angel Losada, “The Controversy between Sepúlveda and Las Casas in the Junta of Valladolid,” in, Juan Friede and Benjamin Keen (eds.), *Bartolome de Las Casas in History* (DeKalb: Northern Illinois University Press, 1971), pp. 279–308 and Lewis Hanke, *All Mankind is One: a study of the disputation between Bartolomé de Las Casas and Juan Ginés de Sepúlveda* (DeKalb: Northern Illinois University Press. 1974).
3. The English text of the Leibniz-Clarke exchange and some related material is presented in H. G. Alexander, *The Leibniz-Clarke Correspondence* (Manchester: Manchester University Press, 1956). Instructive discussions of the controversy include G. V. Leroy, *Die philosophischen Probleme im Briefwechsel zwischen Leibniz und Clarke* (Mainz: Druck von J. Falk III Söhne, 1893); *Leibniz and Clarke: A Study of their Correspondence* (New York: Oxford University Press, 1998); *Space, Time, and Theology in the Leibniz-Newton Controversy* (Frankfurt: Ontos Verlag, 2006).
4. This work was published together with a famous prior (1872) lecture on the limits of scientific knowledge as *Über die Grenzen des Naturekennens: Die Sieben Welträtsel—Zwei Vorträge*, 11th. ed. (Leipzig: Veit & Co., 1916). The

- earlier lecture has appeared in English translation as “The Limits of Our Knowledge of Nature,” *Popular Science Monthly*, vol. 5 (1874), pp. 17–32. For du Bois-Reymond, see Ernst Cassirer, *Determinism and Indeterminism in Modern Physics: Historical and Systematic Studies of the Problem of Causality* (New Haven: Yale University Press, 1956), Part. I.
5. Emil H. Du Bois Reymond, *Die sieben Welträthsel* in his *Zwei Vorträge* (Leipzig, 1882), trans. by J. McCabe as *The Riddle of the Universe—at the close of the Nineteenth Century* (New York and London: Harper & Bros., 1901). On Haeckel, see the article by Rollo Handy in *The Encyclopedia of Philosophy*, ed. by Paul Edwards, Vol. III (New York: Macmillan, 1967).
 6. Ernst Haeckel, *Die Weltrathsel* (Bonn, 1899), pp. 365–66.
 7. Charles Sanders Peirce, *Collected Papers*, ed. by C. Hartshorne et al., Vol. VI (Cambridge, Mass.: Harvard University Press, 1929), Sect. 6.556.
 8. Immanuel Kant, *prolegomena to any Future Metaphysic* (1783), Sect. 57; *Akad.*, p. 352.
 9. The issues of this controversy are expanded in greater detail in the author’s *The Limits of Science* Berkeley and Los Angeles (University of California Press), 1984. Translated into German as *Grenzen der Wissenschaft*. Dietzingen: Reclam Verlag, 1985. Translated into Spanish as *Las Limites de la Ciencia* (Madrid: Editorial Tecnos, 1994). Translated into Italian as *I Limiti della Scienza* (Rome: Armando Editore, 1990). Second (revised and enlarged) edition (Pittsburgh: University of Pittsburgh Press, 1999).
 10. On the Davis debate see Edward Studelsky, *Ernest Cassirer: The Last Philosopher of Culture* (Princeton: Princeton University Press 2008); Peter E. Gordon, *Continental Divide: Heidegger, Cassirer, Davos* (Cambridge, MA: Harvard University Press, 2010); Michael Friedman, *A Parting of the Ways: Curius, Cassirer, and Heidegger* (La Salle: Open Court, 2000).
 11. The point that there is no private language, which Ludwig Wittgenstein was to make famous via his illustration by means of artificial (block/slab) languages, was fully anticipated by Cassirer with respect to real-life languages.

Chapter 18

The Limits of Philosophy



Philosophy's mission is to answer "the big questions" regarding man's place in the world's scheme of things. The first requisite here is accordingly forming a world view—a picture of nature's nature and man's place on its stage. And then come those big issues of orientation—of values, objectives and priorities afforded us by the opportunities of in itself presence on the world's stage. And at this point limits crucial are upon us—limits of time, of energy and resources, of information and insight. Such limits are themselves self-potentiating and set limits to the prospects of effective philosophizing.

The philosophical project presupposes a science that informs us about the world's nature and humanity's constitution. Only then can philosophy begin its life-orienting work of assessing the implications for us and for evaluating a prioritizing the options and opportunities at our disposal. And this means that the adequacy of our philosophizing is limited by that of an understanding of its factual, pre-philosophical basis.

But why not stop there? Why not espouse the antiphilosophical scientism long advocated by the positivists? Ultimately because science itself does not encourage it. For it is clear that the question "Why is the natural world as science describes it to be?" is one that science itself cannot answer.

Philosophy is a particular sort of human enterprise. As such it has a specific issue agenda—a problem domain that defines the range of its concerns. This by itself sets limits. Ethical issues belong to philosophy, demographic issues do not. All the same, questions about philosophy and philosophizing are themselves philosophical. There is no philosophy-external vantage point from which substantive issues about philosophizing can be deliberated.

Philosophy, accordingly is an enterprise which is limited in the *scope* of its task or mission and which, moreover, is also limited with regard to the *extent* to which that mission can be accomplished.

Philosophy does not address the world's facts as such. Its task is not to describe nature's constituents and their *modus operandi*. The realm of word-descriptive fact

belongs to science. And so, philosophy does not endeavor to resolve factual issues. It cannot tell you how many sheep there are in North Carolina. But while philosophy cannot answer that factual question, it can clarify the considerable nature and epistemic status of that answer. And it can remind you that you must, in the end, acknowledge that your answer is imprecise and is at best an estimate.

The vast sphere of cosmic reality belongs to science. Philosophy, by contrast, claims two other realms:

1. That of speculative possibility, (What would human affairs be like if the lifespan of homo sapiens were ten times what it is?)
2. That of evaluative normativity. (Why should people behave in ethically and morally appropriate ways? Why should they try to live morally good lives?)

Philosophy, accordingly, has a problem domain of its own. And division of labor sets limits that places other branches of inquiry—science in particular—outside the limits of philosophy.

Not that those extra-philosophical scientific issues are irrelevant to philosophy. Quite the reverse! The domain of understanding is an integral whole. Thus consider the patently philosophical question:

Given that the universe we inhabit is as science describes it to be, what opportunities for personally satisfying and communally beneficial lives does this put at our disposal?

Clearly philosophical issue is something we cannot begin to address unless and until the products of scientific inquiry are at our disposal.

Consider the following contention by the philosopher C. G. Hempel:

Why is there anything at all, rather than nothing?. .. But what kind of an answer could be appropriate? What seems to be wanted is an explanatory account which does not assume the existence of something or other. But such an account, I would submit, is a logical impossibility. For generally, the question “Why is it the case that *A*?” is answered by “Because *B* is the case”.. [A]n answer to our riddle which made no assumptions about the existence of anything cannot possibly provide adequate grounds.. .. The riddle has been constructed in a manner that makes an answer logically impossible.. ..¹

However, this plausible problem-rejecting line of argumentation is not without its shortcomings. The most serious of these is that it fails to distinguish appropriately between the *existence of things*, on the one hand, and the *obtaining of facts*,² on the other, and supplementarily also between specifically substantial facts regarding existing things, and nonsubstantial facts regarding *states of affairs* that are not dependent on the inclinations of preexisting things.

We are confronted here with a principle of hypostatization to the effect that the reason for anything must ultimately always inhere in the operations of things. And at this point we come to a prejudgment or prejudice as deep-rooted as any in Western philosophy: the idea that things can only originate from things, that nothing can come from nothing (*ex nihilo nihil fit*), in the sense that no thing can emerge from a thingless condition. Now, this somewhat ambiguous principle is perfectly unproblematic when construed as saying that if the existence of something real has a correct

explanation at all, then this explanation must pivot on something that is really and truly so. Clearly, we cannot explain one fact without involving other *facts* to do the explaining. But the principle becomes highly problematic when construed in the manner of the precept that “*things* must come from *things*,” that *substances* must inevitably be invoked to explain the existence of *substances*. For we then become committed to the thesis that everything in nature has an efficient cause in some other natural thing that is somehow its causal source, its reason for being.

This stance lies at the basis of Hempel’s argument. And it is explicit in much of the philosophical tradition. Hume, for one, insists that there is no feasible way in which an existential conclusion can be obtained from nonexistential premises³. And the principle is also supported by philosophers of a very different ilk on the other side of the channel—including Leibniz himself, who writes: “The sufficient reason [of contingent existence]. .. must be outside this series of contingent things, and *must reside in a substance which is the cause of this series..*”⁴ Such a view amounts to a thesis of genetic homogeneity which says (on analogy with the old but now rather obsolete principle that “life must come from life”) that “things must come from things,” or “stuff must come from stuff,” or “substance must come from substance.”

Is it indeed true that only *things* can engender things? Must substance inevitably arise from *substance*? Even to state such a principle is in effect to challenge its credentials. And this challenge is not easily met. Why must the explanation of facts rest in the operation of *things*? To be sure, fact-explanations must have inputs (*all* explanations must). Facts must root in facts. But why thing-existential ones? To pose these questions is to recognize that a highly problematic bit of metaphysics is involved here. Dogmas about explanatory homogeneity aside, there is no discernible reason why an existential fact cannot be grounded in nonexistential ones, and why the existence of substantial *things* cannot be explained on the basis of some nonsubstantial circumstance or principle whose operations can constrain existence in something of the way in which equations can constrain nonzero solutions. Once we give up this principle of genetic homogeneity and abandon the idea that existing things must originate in existing things, we remove the key prop of the idea that asking for an explanation of things in general is a logically inappropriate demand. The footing for Hempel’s rejectionist approach is gravely undermined.

After all, rejectionism is not a particularly appealing course. Any alternative to rejectionism has the significant merit of retaining for rational inquiry and investigation a question that would otherwise be intractable. The question of “the reason why” behind existence is surely important. If there is any possibility of getting an adequate answer—by hook or by crook—it seems reasonable that we would very much like to have it. There is nothing patently meaningless about this “riddle of existence.” And it does not seem to rest in any obvious way on any particularly problematic presupposition—apart from the epistemically optimistic idea that there are always reasons why things are as they are (the “principle of sufficient reason”). To dismiss the question as improper and illegitimate is fruitless. Try as we will to put the question away, it comes back to haunt us notwithstanding its limitations of the limits of actual existence.⁵

What does it take to resolve or answer a philosophical question? Who or what determines success in this regard? And what is the arbiter of adequacy?

Philosophical adequacy is a matter of judgment and such judging requires the basis for a body of experience. (Conclusions need premisses; and judgements need data). And also some people can have experience and make judgements. But people can enter the picture at three levels:

- as individuals
- as experientially congenial groups (“schools of thought,” “communities of judgement”)
- as the entire totality

Commonality of experience is lacking in the third group and confluence of judgement is accordingly also absent. Overall unitary consensus is not to be expected in philosophical matters. The ability to enlist the agreement of others is the ultimate test of philosophical adequacy. And this can nevertheless—even be achieved to an only limited extent.

Schools of thought are matters of aggregation—and agreement—among like-minded individuals. This is the situation among philosophers. But such aggregate depends on and emerges from the effort of individuals.

The history of philosophy is an indispensable resource for philosophizing. Resolving the issue of *what is to be thought* requires determining the manifold of what can be thought, and this in turn is greatly aided by examining what *is properly to be thought*. But nevertheless, the history of philosophy does not do the job for us. For all it does to provide a history of facts along the lines of *X* thought this and *Y* thought that. And this of itself does not address the key issue of adequacy—of what is appropriately to be thought. This in the final analysis is a matter of what every thinker must resolve for himself. Others cannot live your life for you, nor yet can they do your thinking for you. (They can tell you what they think you ought to think, but that is something else again.)

In the final analysis it is the individual who must make judgements of philosophical adequacy on the basis of his/her personal experience. In philosophy it is, in the final analysis, “every man for himself.” Only when one has a developed picture of one’s own can one seek for the social-solidarity of like-minded congeniality.

And only the individual who is rational—who looks for judgements whose basis of accomplishability can enlist others—can function effectively as a philosopher. But there is no reason to expect philosophy to realize completeness and correctness with respect to the issues on its agenda. We simply cannot expect uniformity and consensus here. The variation of human experience means that there is bound to be substantial disagreement among philosophers—both contemporarily and across the passage of time.

Reasoning requires premisses. Reasonable answers to questions require data. In science their data came from observation. In philosophy they came from experience. Technology puts different ranges of observation at the disposal of the scientist of

different eras. History, geography, and culture puts different ranges of experience at the disposal at different thinkers.

Given its difficult mission, philosophy being a work of reason, cannot reach a definitive resolution. For reasons require premisses, and in philosophy's case, at least, these must be based on the available data. The data of philosophy are, first and foremost, the findings of science, the history, or observation, and of common sense. But all these issues for human experience and the range and scope of human experience as constantly changing. And our reasoning, being based on experience, is limited by what experience affords and is bound to change or evoke of its changes. The limits of our world-view are set by the limits of our experience. The change of circumstance and condition is bound to effect change world-outlook that is at issue in philosophizing.

We must expect different bodies of observation to yield different conclusions in science. And we must exempt different ranges of experience to provide different conclusions in philosophy. Wittgenstein said that the limits of our language fix the limits of our world. Instead we say that the limits of our experience fix the limits of our philosophy.

Instability and variability are the order of the day in philosophy. It is no more reasonable to expect different uniqueness with different bodies of experience to achieve uniform answers on philosophical issues than it would be to expect scientists at different state-of-the-art stages of observational technology to achieve consensus.

To do one's thinking one must to some extent enter into a community. For thought must be conducted in language and language is a communal construct. Some things are by nature interpersonal. If it is to be a bridge, others need be able to cross it, if it is to be a language, others must be able to use it. But not necessarily all. Some people may be too heavy to cross the bridge, some conceptions may be too counter-oriented for the language. Languages are correlative with a linguistic communities, and these are formed by cognitive commonalities. This circumstance forms the basis for yet another mode of doctrinal limitation. (NR—develop).

To what extent can we reasonably expect philosophy to accomplish its mission? For sure we cannot expect it to realize comprehensiveness and definitive correctness with in resolving the questions on its agenda? After all, even science whose data are far more authoritative and whose methods are far more exact, one cannot expect to realize these desiderata.

Philosophizing is a work in progress. (Much like science.) The philosophers of one era do not agree (in matters large and small) with those of other eras. Philosophical inquiry does not admit of a final and definitive state of the art.

And as with science—the present cannot speak for the future. Present day practitioners cannot foresee how future practitioners will look at the issues.

Still, can our philosophizing answer all of the questions of the field? Not really, seeing that as Immanuel Kant already stressed ever answer opens the door to further questions. Our efforts are finite but the problems unending.

But this of course does not means that there are insolubilia: possible and specifiable questions that admit of no answer. The situation is akin to that of the game of Musical Chairs. There are always players who will not be seated, even though no players are

in principle unseatable, No questions can be identified and formulated regarding what we don't in principle admit of a sensible answer. And so while there are always unanswered questions there none can be identified that are in principle unanswerable, (With insolubilia it is again akin to asking for the instancing of facts one does not know.) Philosophy exhibits generic but not specifically identifiable intractability. As with the classic Paradox of the Heap there is no specifiable point that sets a fixed limit, notwithstanding the reality of limitations.

Philosophy affords yet another illustration of our cognitive limitations. For here as elsewhere we can come up against the cognitive limitedness of the realization that we do not and cannot know the exact extent of our unknowing and the fact that we cannot possibly know the detail of what we do not know and that the limits of our ignorance, though real, are unfathomable in that while we realize full well that there is much we do not know, we cannot begin to specify *what* this is.

The crucial different between the *that* and the *what* of instancing gets lost in Frank Ramsey's glib quip that "What can't say can't say, and you can't whistle it either." For like "the sandgrain no one ever sees" we realize full well that since they exist but are totally at a loss to identify them. And the same goes for those philosophical issues that fall outside our region of consideration.

The question: "What good is a discipline that does not achieve universal consensus among its investigators?" must be combined with another question: "What good to me is a discipline that does not accommodate the manifold of my experience?" Surely the limitations of harmonization with one's experience is a crucial standard of philosophical adequacy. A doctrine answering to someone else's experience does me as little good as a suit that fits someone else's body. And a doctrine that fits everybody at once is as much a pie in the sky as is a suit that fits everyone from Tom Thumb to Giant George.

It is impossible—a question of general principle—that gets a firm fix on our cognitive limitations, the exact extent of our unknowing is unfathomable. We realize full well that we are not omniscient and that there are many facts that we do not know. But the challenge "Give me one example of a single fact you do not know" cannot be met. (For, after all, you can only properly claim something to be a fact if you know it to be so).

The ultimate lesson is clear. Philosophy must try to envision our knowledge as a whole, developed systemic comprehensiveness. Philosophy without science is blind, science without philosophy disoriented. We are well advised to adopt the Leibnizian vision of pursuing knowledge in its systemic totality. And the difficulty of the project—the effective impossibility of fully adequate realization—can and should induce a good deal of cognitive humility.

Notes

1. Carl G. Hempel, "Science Unlimited," *Annals of the Japan Association for philosophy of Science*, 14 (1973), 200 (italics added).
2. Note, too, that the question of the existence of facts is a horse of a very different color from that of the existence of things. There being no things is undoubtedly a

possible situation; there being no *facts* is not (since if the situation were realized, this would itself constitute a fact).

3. *Dialogues Concerning Natural Religion* ed. by N. K. Smith (London, 1920), 189.
4. G. W. Leibniz, "Principles de la nature et de la grace," Sect. 8 (*italics added*).
5. For criticisms of ways of avoiding the question "Why is there something rather than nothing?" see Chap. 3 of William Rowe, *The Cosmological Argument* (Princeton: Princeton University Press, 1975).

Chapter 19

Antiphilosophy (Philosophical Negativism)



19.1 Introduction

Aristotle characterized man, *homo sapiens*, as “the rational animal,” and the prime objective of philosophy since in his day has been the consolidation of a coherent view of the world and our place within it through reasoning about the data that human experience puts at our disposal.

However, a Leitmotiv of negativism also runs throughout the subject’s history. From the dawn of Western philosophizing in the Pre-Socratic era an ongoing series of thinkers have maintained an unbroken tradition of opposition to the very enterprise itself. Theorist after theorist has maintained that philosophy is a fruitless and unavailing venture—a project pursuing an unrealistic and unattainable goal. And throughout the 20th Century in particular, philosophers of the most diverse orientation—Jamesean Pragmatists, Carnapean Positivists, Logical Positivists, Heideggerian Negativists, Reichenbachian Scientisticists, Wittgensteinian “Quietists,”—have been in consensual agreement that philosophy of the traditional sort—and metaphysical speculation in particular—were unproductive, unavailing, and illegitimate endeavors. One party rejected philosophy as going *against* the strictures of reason; the other as going *beyond* their limits. But both sides agreed in seeing the project of a rational grasp of fundamental as unrealizable.

The present deliberations will undertake a short survey of this line of contention, followed by an indication of its problems and shortcomings.

19.2 An Historical Survey

19.2.1 *Heraclitean Instability and Cratylean Vacuity*

Even at the dawn of philosophy in pre-Socratic times, theorists were concerned to maintain the philosophical project of inquiry into the realm of Truth and Reality was unrealizable and futile.¹ Thus Heraclitus of Ephesus, who “flourished” around 500 B.C.,² had little faith in the ordinary processes of human inquiry. “Though the inspired word is ever true, yet men are unable to understand it” (Fr. 2). “Of all the discourses I have heard, there is not one that achieves the understanding that Wisdom is something totally different” (Fr. 18). Heraclitus teaching of instability and all-pervasive flux invites the view that there cannot be definite and definitive truths about reality. Any philosophical doctrine claiming “the truth about reality” is foredoomed from the outset. There is no stable truth about the things of this world: “You cannot step twice into the same river, for fresh waters are ever flowing in upon you.” (Frs. 41–42). “Cold things become warm and what is warm cools; what is wet dries, and the dry becomes moist” (Fr. 39). For Heraclitus there is no fixity of fact and no stable reality that can be represented in unchanging truths accessible to man. Philosophy’s aspiration of understanding the world in in human terms developed on rational principles is foredoomed from the outset.³

19.2.2 *Eleatic Paradoxology*

Zeno of Elea flourished around 450 B.C. His mode of theorizing also exuded negativity. Focusing on the key contentions of his Pythagorean opponents, he proceeded to draw two contradictory conclusions from them, thus exhibiting their untenability via *reductio ad absurdum*. His special targets were the conceptions of unity and motion—the fundamentals of geometric construction. His classic “paradoxes of motion” have been a staple of mathematical deliberations since his day. A typical instance is...

Zeno of Elea also devised a series of ingenious arguments designed to show the impossibility of motion. One of these was the *Paradox of Achilles and the Tortoise*, based on the following narrative:

Notoriously fleet-footed Achilles has a race with a proverbially slow tortoise. Naturally enough, the tortoise demands a head start. But now by the time that Achilles reaches the tortoise’s starting point the tortoise will have moved on and will be somewhat ahead. And when Achilles reaches *that* position the tortoise will have moved on and will still be ahead a bit. And so on. Thus Achilles will never catch up with the tortoise.⁴

The paradox at issue here goes as follows:

- (1) At no stage of the endless sequence of positional catch-ups, will Achilles have succeeded reaching the tortoise. Therefore
- (2) Achilles will never pass the tortoise.

(3) But—as we know full well—Achilles will soon pass the tortoise.

Here theses (1) and (3) are perfectly in order. But the chain of inconsistency is broken at (2) which actually does not follow from (1) at all because the move from “at no stage of the sequence” to “never” is simply inappropriate.

The fact of it is that a deep underlying equivocation is at work in the paradox as between “the sequence of catch-ups is *unending* (limitless) *in steps*” and “the sequence of catch-ups is *unending* (limitless) *in time*.” The two expressions have a different sense. For the catch-up sequence just does not cover the whole of the future, seeing that it converges to a final limit even as $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$ converges to 1. Thus the steps of that sequence, even in endless totality, will only cover a finite timespan.

Zeno’s Achilles paradox is thus decisively resolvable through the recognition that an equivocation unravels the aproetic inconsistency through which it arises. (Whoever thinks that this unfavorable series does not sum up to one but always falls short of it just does not understand the function that those three little dots are serving here: they stand for *etcetera*, meaning and *all* the rest.)

Along these same lines, there is a cognate paradox already discussed by Aristotle (*Physics*, 239b 9–14). It runs essentially as follows:

Suppose you wish to move from point *A* to point *B*. Before you can accomplish this motion, you must first reach point *C*, the halfway point between *A* and *B*. But before you can reach point *C*, you must first reach point *D*, the halfway point between *A* and *C*. And so on. Thus before you can accomplish any motion you must first accomplish an infinity of prior motions, which means that you can never accomplish any motion at all.

The negativism of Zeno’s methodology is clear: if the concepts basic to philosophical theorizing are incoherent by leading to contradictions, then the entire project becomes unworkable.⁵

19.2.3 Protagorean Relativism

Protagoras of Abdera, who flourished around 440 B.C. was the anti-theoretician par excellence. (Indeed Plato characterized this position as “antilogic.”) For present purposes, two of his thesis are paramount:

- Man is the determining measure of all things, of what is that it is, and of what no not that it is not.
- All things can be disputed with equal merit on both sides—and even this very thesis itself can be disputed pro and con.

His man-measure doctrine was matched by a theory of divided reason, holding that there are always contradictory accounts (*logoi*) on basic issues and “two sides to every question” as it were. For every argument in favor of a thesis there is another to oppose it: opposite in substance and equal in category.

On this basis we are enplaced in a world of divergent: the wind is warm to one person, cold to another. The idea of impersonal fact is an illusion. And as Plato

represented him, Protagoras held this view not only with regret to sensory appraisals but to epistemic, moral, political and aesthetic qualities in general. All these are matters of personal opinion, and never matters of objective fact.

Protagoras accordingly rejected the whole idea of a Rationality able to adjudicate matters of impersonal fact.

19.2.4 *Socratic Negativism*

Plato's early dialogues repeatedly illustrates the "Socratic method" as an exercise in refuting various doctrinal contentions by showing that they lead to unacceptable, and often self-contradictory consequences. Deliberations of this sort were thus substantially destructive in bearing, demolishing various views without yielding any positive results.

Socrates rejection of the direct advocacy of positive doctrines left philosophizing as a potential exercise in futility. And this negativity squares with Socrates in view of his teaching and marked philosophical agnosticism as the course of wisdom. We have it from Plato that the Greek Oracle who asked if anyone was wiser than Socrates responded: "No one." Socrates himself interpreted this ironically, as meaning that since he himself adjured any special claims to deeper knowledge neither did anyone else possess it.⁶

19.2.5 *Empiricist Skepticism (Pyrrhonism)*

The academic skeptics, among whom Sextus Empiricus became the most prominent spokesman, argued against certitude in general (be it in mathematics, physics, or medicine)—let alone in philosophy.

Against the orthodox philosophers of the various schools, Sextus argued that their position requires criterial of certitude, and that this contention, which could not without vitiating circularity speak for itself, would always remain questionable.

Sextus Empiricus informs us:

That nothing is self-evident is plain, they the skeptics say, from the controversy which exists amongst the natural philosophers regarding, I imagine, all things, both sensibles and intelligibles; which controversy admits of no settlement because we can neither employ a sensible nor an intelligible criterion, since whatever criterion we may adopt is controverted and therefore discredited.⁷

Greek scepticism (*skepsis* = doubt) was set in train by Pyrrho (Fl. p. 320) whose doctrinal stance was that it is not possible to know the real nature of anything, Human inquiry is hamstrung by the fact that any contention can be opposed by its contradiction and can equally firm support built up another way. The only sensible reaction is that of suspension of judgment (*epochê*) that requires *aphasia* a non-commitment

sentence on the questions deflective of an uncaring indifference (*ataraxia*) regarding the issue. This radical and synoptic skepticism was followed by the more mitigated or limited skepticism of the later era of Plato's Academy who divided knowledge into two sectors—that of immediate experience and that of experience-transcending theorizing, and restricted skeptical systems to the latter experience-transcending speculative sector. But either way the entirety of speculative metaphysics was rejected by the skeptics.

19.2.6 Theological Fundamentalism

Various medieval theologians argued that while the arguments deployed against theorizing philosophers and philosophizing theologians by such theorists as Al-Ghazali (on the Muslim side), Yehudah Halevi (on the Jewish), and Nicholas of Cusa (on the Christian) were unfairly skeptical about philosophio-theological theorizing, holding that human rationality was unavailing in this domain and that if knowledge in these matters is available at all the divine cognition and revelation alone can provide it. As they saw it, the revelation-based thesis of religion are secure, but the speculation of secular philosophizing are so much smoke and mirrors.

19.2.7 Averroism

The Averroicist compromise. Two surveys; a divided field.

The reconciliation of Greek science with the history of Mediterranean Monotheism caused difficulty on all sides. (Reconnecting the Creation with the eternality of the universe; reconnecting natural processuality with miracles; reconnecting human free will with material causality). The result was a dual-truth theory that came to prominence under the name of Averroism (after...). The idea was a two-sided theory of knowledge and truth—the truth ascribing to human reason and the higher truth avoiding to revealed religion. On this basis philosophical understanding is pivotal and imperfect and must be substantiated to the decisive, firmer, and more fundamental truth of religion.

19.2.8 Humean Skepticism (Hume and Appearance/Reality Skepticism)

The everyday naïve view of how things work in the world—the world as a materialistic manifold of space, time, and causality falling under the aegis of diversity restricted laws. The view of the naïve and philosophically unsophisticated “man on

the street.” And the deeper, more sophisticated and cogent world view of the philosophical adepts who proceed on the realization that Reality has a nature altogether different from what we commonly think.

From our human point of view there is the everyday common sense world of every day thought and experience and the very different wall of reality to why we either have no access at all (Kant) or at which we can arrive only through a conjectural “reverse engineering” from familiar experience.

Given that no cognitive access-way to this experience-transcending Reality is open to us.

19.2.9 Positivism

Positivism’s founder, August Comte (1789–1857), divided the history of mankind’s cognitive endeavor into three successive phases, the theological, the philosophical, and the scientific. Each stage had its own characteristic modes of understanding and explanation: supernatural superstition in the theological stage; transcendental speculation in the metaphysical; and empirical inquiry in the scientific. Philosophy was accordingly to be consigned to the pre-scientific past—an era of unsophisticated outdated and out-moded speculation superseded among thinking people by a science based on inductive reason from observable fact.

The logical positivists of the post-WWI decade went on from where Comte left off with a doctrine not of cultural development but one of language and meaning. The meaning of discourse roots in its practical applications and logic, the methodology of reasoning, is a matter of the machinery of communication that simply annihilates whatever has no basis in empirical inquiry.

The logical positivists of the Vienna Circle school proposed to take a science-is-all line. As they saw it, the entire project of speculative philosophy—and metaphysics in particular—is a matter of fraud and delusion. Insofar as meaningful questions going beyond the formalities of logic and language are at issue, natural science can resolve them. A different approach was taken by cultural relativists, who were inspired by the social rather than natural sciences. They relegated all philosophizing to the level of mere opinion. As they saw it, there is no objectively determinable fact of the matter regarding issues of the sort with which philosophy has traditionally dealt. It is all simply a question of what people think. Seen from this angle traditional philosophy is not so much (as with earlier Comtean positivism) a futile venture in understanding the ways of the world as an exercise meaningless miscommunication by means of what is little more than vertical gibberish. Metaphysics on its traditional basis is not so much misinformation as meaningless babble.

On this basis any sort of original insight was denied to philosophy, leaving the discipline with the task only of expositing the methods and results of science. The early Rudolf Carnap and the late Hans Reichenbach and their “circles” of devotees in Berlin and Vienna were the main theoreticians of this venture into scientism, and A. J. Ayer in Britain was its prime publicist.

Again, adherents of the analytic school insisted that philosophy as traditionally conceived must be abandoned in favor of exegesis of the use of language. We should not—and sensibly cannot—investigate what such things as truth and justice and beauty inherently are and involve, but should instead investigate how the expressions such as “truth,” “justice,” “beauty” *are ordinarily used* when such matters are discussed.

19.2.10 *Pragmatic Skepticism (James)*

William James (1842–1910) was as much a psychologist as a philosopher and this was reflected in his view of philosophizing. Philosophy for him was less a venture in inquiry and rational question-resolution than as one in the formation of practical attitudes towards one’s place in the world. Purely specifiable philosophy serves no constructive function and theorizing is meaningful only when it can achieve in the way of practicable implementation. Thus purely speculative metaphysics of the sort common in the philosophical tradition is empty and presumptive—satisfying conduct of the business of life—in the only effectual antecedent of the traditional view to philosophizing as a field of rational inquiry is pointless and unproductive.

James held that ‘The true’ is only the expedient in our way of thinking, just as the right is only the expedient in our way of behaving.”⁸ He sought to replace “the truth” by a diversified plurality of truths.

For James, pragmatism was to be an instrument of change designed to revolutionize traditionally entrenched conceptions. It seeks to reject the construction of “high-fallutin” monolithic philosophical conceptions like truth, beauty, and justice, and to put utility, serviceability, efficiency, and effectiveness in their place. And pragmatic “success” is seen as a matter of getting things done in the setting of our everyday life affairs. As James put it, “ideas become true just insofar as they help us to get into satisfactory relation with other parts of our experience,” where those “other parts” reach beyond the range of theorizing, of inquiry and question-resolution. Turning away from matters of rational inquiry as such James rejected the idea that there is a “pure,” disinterestedly neutral and wholly impersonal realm of meaningful cognition. To his mind, philosophy central concepts like “knowledge,” “belief,” “truth,” “meaning,” are factors operating in a human practice of opinion formation. In his hands, pragmatism took on a personalistic and psychologistic orientation towards matters of affective and subjective satisfaction.⁹

After all, “Truth for us is simply a collective name for verification-processes... Truth is *made*, just as health, wealth, and strength are made, in the course of experience.”¹⁰ And at times James came perilously close to a “wishful thinking” view of truth that conflated the narrower *evidential* reasons for the substance of a belief with the broader *prudential* reasons why its adoption could prove advantageous.

19.2.11 Wittgensteinian Positivism

The Ludwig Wittgenstein (1889–1951) of his mature period was effectively a logical positivist, albeit one whose conical language was not that of technical science but that of the “language games” of normal, everyday communication. For him, philosophical problems do not pivot on meaningful questions but root in misunderstandings and misconceptions regarding the use of language. These misunderstandings issue in a puzzlement arising from unauthentic issues of language that do not constitute represent problems. They do not call for serious explanation but need to be explained away as products of misconception. The proper amount of philosophizing calls for a venture in linguistic clarifications that in effect demolishes anything resembling traditional philosophy, which is little but a morass of linguistic confusion.

When proper heed of the basic linguistic relativity is lacking and “language takes a holiday.” Misapprehension and misunderstanding arises when we take language outside its arena of proper use.

19.2.12 Heideggerian Indifference and Tranquility/*Gelassenheit*

Martin Heidegger (1884–1976) is yet another in the long chain of those who want to abolish philosophizing in the classical mode.

Heidegger’s position regarding the traditional issue of metaphysical experimentally transcendent philosophizing may be encapsulated in the injunction: “Don’t bother. View these matters with uncaring indifference. Preoccupation with them is pointless because no firm resolution is achievable. You cannot expect to realize anything instructive with regard to issues when many generations before you have tried and failed.”

Heidegger’s position results from a deep-rooted antipathy to rational inquiry as such. As he sees it, reflective thought in misapprehension. Authentic cognition is experiential. Reality only discloses itself to a passive receptivity open to the revelation of Being—of a reality outside the categorizations of human experience and rational thought.

Heidegger accordingly insisted that metaphysical issues of the traditional sort cannot be addressed meaningfully. We must, instead, limit our concerns to the prosaic matters of ordinary life and everyday affairs, and preserve a stance of relaxed and disdainful indifference (*gelassenheit*) towards those fruitless concerns with which speculative philosophers have traditionally dealt.

Heidegger held that the era of metaphysics has ended and that philosophical understating requisite a “new beginning” adumbrated in its own work. Like the positivists

he celebrated the demise—the “overcoming”—of metaphysics in negativity that abolishes all those traditional misconception and leaves in their place a somehow significant vacuity (“nothingness”) whose—rather mystical—disclosures of Reality constitute true wisdom. Authentic philosophizing, as he sees is, lies in the self-annihilation of everything that traditional philosophy it has historically developed.

19.2.13 Rorty

Richard Rorty (1931–2015[?]) too wants to see traditional philosophy consigned to the trash heap of historical obsolescence but follows John Dewey (and the earlier Marx) in wanting to replace its orthodox goal at deeper understanding by a project of societal consideration.

The Marxian dictum “Philosophy has heretofore sought to understand the condition of man, the real task, however, it to change it” gets at the crux of Rorty’s program—provided that a replica *change* by *improve*. The result is a quality-of-life oriented pragmatism that replaces traditional philosophizing with socio-cultural amelioration. (Whether individual culture or social well-being is the definitive aim of the enterprise remains somewhat unclear. Moreover, clarity of J. S. Mill’s thesis that a dissatisfied Socrates is preferable to a contented ignoramus is absent in Rorty.) Following Dewey, the principle task of philosophizing Rorty sees it is one of demolition—to claim the ground of the obsolete ideas and bad philosophizing of the first to ease the way from a liberal polity and politics designed to make people happier. Rorty would take philosophy not to promote clear thought but to motivate constructive (benevolent) action.

Accordingly, Rorty’s pragmatism insists “that one can be a philosopher precisely by being anti-Philosophical.”¹¹ The threads of pragmatic thought that he weaves together into his own so-called “neopragmatism” are substantially nihilistic in tenor and tendency. He has it that “truth is not the sort of thing one should expect to have a philosophically interesting theory about” (p. xiii). The pragmatist accordingly advocates the “post-Philosophical culture [of] the philosopher who has abandoned pretensions to [traditionalistic] Philosophy” (p. xl). Such a pragmatism abandons any idea of rational quality-control on the processes of inquirers and question-resolving deliberations.

It is clear that this position rests on a scepticism which—exactly in the manner of the ancient Sophists—proposes to abandon objectivity and impersonally rational standards and to rely on communal consensus alone, subordinating science to the social policy of shared opinion.

As Rorty sees it the traditional philosophical procedure of framing questions and devising answers is based on a mistaken commitment to the methodological unity of knowledge that assimilates philosophy to empirical inquiry. The main tasks of the enterprise is one of demolition—of exhibiting the vacuity and futility of the traditional ways of philosophizing and refuting the mistaken conceptions that have

got in their way of constructive investigating of the ways to inspire the well-being of people.

Rorty has the unique distinction of being the only prominent antiphilosopher to manifest the authenticity of abandoning the field as a domain of employment and shifting to earn his bread by being something other than a philosophy teacher.

19.3 A Survey of Positions

The available bases of apparently secure knowledge about the world would include:

- Ordinary experience
- Empirical science
- Philosophical speculation/Metaphysics
- Empirical Science.

With respect to these we have the following spectrum of philosophical positions:

- None actually succeed in providing secure knowledge: *Radical Scepticism*
- Secure knowledge is available throughout, except for metaphysics and religion: *Positivism*
- Only one provides secure knowledge, and that is
 - Ordinary experience: *Common sensism*
 - Empirical science: *Scientism*
 - Religious teaching: *Foundationalism*
- All—including metaphysics—provide access ways to include knowledge: *Liberal cognitivism*
- Secure knowledge is available throughout, except for religion: *Secularism*.

Accordingly, are the seven major alternative positions regarding the philosophy of knowledge. All the others reject them. And their proponents. But of these, only Liberal Cognitivism and Secularism allow some place to metaphysics in the cognitive scheme of things. Ironically, philosophers have often focused on rejecting the possibility of philosophical knowledge, thereby sawing off the very limb on which their own position hinges.

19.4 The Scandal of Philosophy

The strongest argument of philosophical nihilism lies in what is often called “the scandal of philosophy”—the fact that philosophers disagree, that despite...

Descartes, the founding father of modern philosophy, complained as follows:

I shall not say anything about philosophy, but that, seeing that it has been cultivated for many centuries by the best minds that have ever lived, and that nevertheless no single thing is to be found in it which is not subject of dispute, and in consequence which is not dubious, I had not enough presumption to hope to fare better there than other men had done. And also, considering how many conflicting opinions there may be regarding the self-same matter, all supported by learned people, while there can never be more than one which is true, I esteemed as well—nigh false all that only went so far as being probable.¹²

In the eighteenth century, David Hume deplored philosophy's chaotic lack of consensus in these terms:

Want of coherence in the parts, and of evidence in the whole, these are everywhere to be met with in the systems of the most eminent philosophers, and seem to have drawn disgrace upon philosophy itself....Even the rabble without doors may judge from the noise and clamour, which they hear, that all goes not well within. There is nothing which is not the subject of debate, and in which men of learning are not of contrary opinions. The most trivial question escapes not our controversy, and in the most momentous we are not able to give any certain decision. Disputes are multiplied, as if everything was uncertain; and these disputes are managed with the greatest warmth, as if everything was certain. Amidst all this bustle 'tis not reason, which carries the prize, but eloquence; and no man needs ever despair of gaining proselytes to the most extravagant hypothesis, who has art enough to represent it in any favourable colours....From hence in my opinion arises that common prejudice against metaphysical reasonings of all kinds, even amongst those, who profess themselves scholars.¹³

A century later Wilhelm Dilthey wrote (in 1867):

[Many think that] the development of philosophy encompasses through all those various systems a succession of systems which approaches a single perfected system in unending approximation. However in reality every age manifests the strife of all these systems among one another. This includes the present age, which shows no sign that this strife of systems is diminishing.¹⁴

In the late 1920s Moritz Schlick gave expression to a similarly discouraging view:

But it is just the ablest thinkers who most rarely have believed that the results of earlier philosophizing, including that of the classical models, remain unshakable. This is shown by the fact that basically every new system starts again from the beginning, that every thinker seeks his own foundation and does not wish to stand on the shoulders of his predecessors....This peculiar fate of philosophy has been so often described and bemoaned that it is indeed pointless to discuss it at all. Silent scepticism and resignation seem to be the only appropriate attitudes. Two thousand years of experience seem to teach that efforts to put an end to the chaos of systems and to change the fate of philosophy can no longer be taken seriously.¹⁵

The litany of dismay echoes through the ages: complaints regarding unsettled issues, unresolved controversy, unending disputes, and unachieved consensus. For more than two millennia, philosophers have grappled with “the big issues” of man and his place in the natural and social scheme of things without resolving anything. The consequence, as one observer remarks, is that “the history of philosophy has been, to a very great extent, a history of impressive failures, of large conceptions whose particular deficiencies have finally been laid bare for all to behold.¹⁶” We

look in vain for one consolidated and generally acknowledged item of philosophical “knowledge”—one “philosophical fact” on which the philosophical community at large has reached a settled consensus. As Husserl somewhere remarked, philosophy is in a state of conflict and confusion: “To be sure, we still have philosophical congresses. The philosophers meet but, unfortunately, not the philosophies.”

What Dilthey calls “the strife of systems” has always been a matter of dismay and embarrassment to philosophers. As one recent commentator puts it: “In Descartes, in Kant, in Hegel, in Husserl, in Wittgenstein...one finds the same disgust at the spectacle of philosophers quarreling endlessly over the same issue.”¹⁷ Doctrines are no sooner proposed than attacked, theories no sooner constructed than contested. Yet why should it be that—as one writer picturesquely puts it—“systems lie about us like the ruins of gigantic castles” without our being able to build one solid and defensible structure from this ample debris?¹⁸

More than any other factors, the strife of systems and the unattainability of consensus explain the recurrence of deep disillusionment with the discipline. There is little doubt that the proliferation of conflicting philosophical systems is viewed by many with deep discontent and dissatisfaction. Omar Khayyam long ago endowed this sentiment with poetis expression:

Myself when young did eagerly frequent
 Doctor and saint, and heard great argument
 About it and about; but evermore
 Came out by the same door as in I went.¹⁹

And yet in the end this sceptical perspective is profoundly problematic.

19.5 Overcoming the Scandal: Why Antiphilosophy Fails

The nihilistic tradition of philosophizing faces two decisively defeating obstacles: *metaphilosophical inconsistency* and *aporetic inescapability*. Let us briefly consider them in turn.

Metaphilosophical inconsistency roots in the fact that doctrinalizing about philosophy is itself a philosophical enterprise—that metaphilosophy is part of philosophy itself and that the question “How if at all is philosophizing possible and justifiable?” is itself a philosophical question. In this light espousing philosophical nihilism is already to engage in philosophizing.

As regards aporetic inevitability this is most readily shown by means of an example. Thus consider the following group of contentions:

- (1) Man often has control of his actions through deliberation and choice.
- (2) All human actions issue from causal processes.
- (3) The operations of causal processes leaves no room for deliberation and choice.

Here three considerations are crucial for present purposes.

1. Those preceding are mutually inconsistent. Rationality accordingly requires that at least one of them must be rejected.
2. Rationality also requires the rejection of any such thesis be given rational support: that some sort of rationale for abandoning such a contention be provided.
3. But providing a cogent case for rejecting any such thesis is in fact to engage in philosophy. To reason about such matters is to engage in philosophizing.

And just this same situation arises whenever we are confronted by any cluster of inconsistent propositions regarding philosophically relevant issues. In all such situations rationality leads us down the same path. In every such situation some of the collectively inconsistent theses has to go. But to show that one (or more) of them is inappropriate is to embark on a venture in philosophizing.

What makes philosophizing unavoidable for us is the fact that as *homo sapiens* we are caught up in what might be called the rational Coherence Predicament. The restoration of consistency within any body of incompatible philosophically relevant contentions is a rationally unavoidable and inherently philosophical endeavor.

The irony here is that as individuals have no real choice but to espouse some philosophical positions. To be sure, however, there need be not consensus because the experiential situation of different individuals is different and it is the course of their experience that provides different people with different bodies of data and different presuppositions.

Rationality requires the alignment of belief with evidence. And where the evidence differs the answers ought to reflect this: difference evidence must prepare us for different outcomes. And on this basis, that “scandal of philosophy” is not a scandal at all. People live in different manifolds, amidst different branches of knowledge and opinion, exposed to different courses of experience, with different personalities reflected in different attributes and properties. That they should think differently about matters large and small is in the circumstances only to be expected as rational and reasonable. Given that philosophy is the work of individuals who live in diverse thought-contexts of different cultural environments, the existence of philosophical diversity is wholly reasonable, and only to be expected.

Notes

1. On Protagoras see Kirk, Raven, and Schofield...
2. The Greeks often chronologized people by a single date—the “flourishing” age of forty.
3. A reliable and convenient overview of Heraclitus thought is Kirk, Raven, and Schofield (*op. cit.*).
4. Kirk, Raven, Schofield, *The Presocratic Philosophers*, 2nd ed. (Cambridge: Cambridge University Press, 1983), p. 272.
5. On Zeno see Kirk, Raven and Schofield (*op. cit.*), pp. xxx.
6. On Socrates and the Socratic Method see Gregory Vlastos, *Socrates: Ironist and Moral Philosopher* (Ithaca Cornell University Press, 1991).
7. Sextus Empiricus, *Outlines of Pyrrhonism*, bk. I, 178. The conflicting teachings of the philosophers were the very first of the five *fropoi* or sceptical arguments

- (rafioncs dubirmdi) by which Agrippa sought to support a sceptical position (ibid., 95).
8. William James, *Pragmatism* (1907), p. 34. A first approximation to a plausibly acceptable reformulation would make certain crucial additions, namely: ideas **GENERALLY COME TO BE RECOGNIZED AS** true just insofar as they are seen to stand in a harmonious relationship with other parts of our **COGNITIVE** experience. What such a recasting does is to make explicit the epistemic aspect of the situation, recognizing that truth as such is something rather different from its *recognition* as such.
 9. Often but not always. With certain factual statements what matters is, he suggests, “extraordinary fertility in consequences verifiable by sense” (*Pragmatism*, op. cit., p. 91)—that is successful prediction. The problem is that James is simply inconsistent on this point—for him “expediency is sometimes scientific success and sometimes personal satisfaction. On James’ cavalier attitude towards consistency, see Richard M. Gale,....
 10. William James, *Pragmatism* (op. cit.), p. 104.
 11. Richard Rorty, *Consequences of Pragmatism* (Minneapolis: University of Minnesota Press, 1982), p. xiii.
 12. Rene Descartes, *Discourse on Method*, pt. I; trans. E. S. Haldane and G. N. T. Ross (Cambridge: Cambridge University Press, 1911).
 13. David Hume, *A Treatise on Human Nature*, Introduction. When one considers the sorry history of the efforts of philosophers to produce recognizable truth, Hume’s sceptical confrere Montaigne writes, one must conclude that philosophy is little better than sophistical fable-making: “Et certes la philosophie n’est qu’une poésie sophistiquée” (Les Essais de Michel de Montaigne, ed. Pierre Villey, vol. III [Paris, 1922], p. 279).
 14. Wilhelm Dilthey, *Gesammelte Schriften*, vol. V (Stuttgart and Gottingen, 1960), p. 134. He observed that the tendency of his time was “to treat the systems of these great philosophers as a series of delusions, akin to a bad dream which, on awaking, one had best forget altogether” (p. 13), and he remarked that “the anarchy of philosophical systems is one of the most powerful supports from which scepticism draws ever-renewed nourishment. A contradiction arises between the historical realization of their boundless multiplicity and the claims of each to universal validity which supports the sceptical spirit more powerfully than any theoretical argumentation” (p. 75).
 15. Moritz Schlick, “The Turning Point in Philosophy,” *Logical Positivism* ed. A. J. Ayer (New York, 1959), pp. 53 ~ 54 (originally published as “Die Wende der Philosophie” in vol. 1 of *Erkenntnis* [1930]). Compare also Kant, CPHR, Preface to the First Edition, A vm—x; and Charles Sanders Peirce, “The Fixation of Belief,” *Collected Papers*, vol. V, ed. C. Hartshorne and P. Weiss (Cambridge, Mass., 1934), sec. 5.383.
 16. Joseph Margolis, ed., *An Introduction to Philosophical Inquiry* (New York, 1968), p. 10.
 17. Richard Rorty, *The Linguistic Turn* (Chicago, 1967), p. 1.

18. Karl Heim, *Das Weltbild der Zukunet* (Berlin, 1904), p. 1. This book is by no means the first, however, to have “the strife of systems” (*der Streit der Systeme*) as its central theme. Hegel aside, it was anticipated as of the 1860s in Wilhelm Dilthey’s unfinished masterpiece, *Weltanschauungslehre* (*Gesammelte Schriften*, vol. VIII. [Stuttgart and Gottingen, 1960]), where “the anarchy of philosophical systems” (*die Anarchie der philosophischen Systeme*) is the central issue. The problem-stage of his discussion is set by the fact that “a profusion of philosophical systems trails behind us and spreads out on every side, boundaryless and chaotic. In every time since their origin, they have excluded and contested one another, and there is no sign of hope that a decision among them can ever be realized” (p. 75). He goes on to speak of the “boundless expanse of ruins” of purportedly established philosophical systems (*ein anermessliches Truemmerfeld demonstrierter System*, p. 76).
19. Rubaiyat, trans. Edward Fitzgerald.

Chapter 20

The Rational Inescapability of Philosophizing



20.1 The Line of Reasoning

Rarely do substantive contentions *within* philosophy admit of decisive and categorical demonstration. But there is one contention *about* philosophy that plainly does so, namely the thesis that *there is no rational way to avoid philosophizing itself*.

The requisite demonstration is straightforward and can be set out as follows:

Consider some (i.e., any) set of collectively inconsistent philosophical contentions: $P_1, P_2, \dots P_n$. In view of their inconsistency, at least one of them must be abandoned—denied or at least modified. (The Annex provides several illustrations of this sort of situation.)

But now, two questions arise, namely:

1. Which of these contentions should give way to others? Which are the weaker links in the chain of inconsistency? Insofar as theses abandonments (or at least modifications and revisions) are required to restore consistency and coherence, how should this be achieved?
2. What is it that justifies this particular selection? How is a preferential determination of acceptability to be justified?

After all, when we undertake abandonments and/or revisions with regard to substantive claims, rationality requires that we be able to provide some justificatory account for this proceeding. And when the theses at issue are themselves philosophical in substance, there is simply no possible way of providing such a justificatory rationale without engaging in deliberations that are plainly and undeniably philosophical in nature. From the standpoint of rationality, we can only reject a contention (philosophical contentions included) by providing some sort of rationale of justifying considerations for so doing. But providing a ground for rejecting (abandoning, let alone denying) a philosophical contention is to engage in philosophizing.

To provide any sort of cogent explanation for why a philosophical contention is to be dismissed or relational in a situation of conflict is to engage in a philosophical discussion. In view of the very nature of the field, the explanation dismissing a philosophical contention from consideration is something that cannot but qualify in philosophical deliberation. One cannot address a philosophical thesis—be it positively or negatively—without philosophizing. After all, metaphilosophy—even the negative metaphilosophy of skepticism—is a branch of philosophy itself.

And so, in the end, the resort to philosophizing is an inescapable demand of reason.

20.2 Illustrative Instanced

This aspect of the situation is readily brought to view.

Let it be that *A*, *B*, *C* form an inconsistent triad of propositions. Then rationality demands that at least one be abandoned—or at least qualified. But if these propositions are philosophical contentions, then dismissing (or even just qualifying) one of these theses involves one in a philosophical endeavor.

Thus consider:

- (1) A choice whose outcome is causally predetermined is not free.
- (2) All human choices are casually predetermined.
- (3) Some human choices are made freely.

Clearly we cannot achieve a rational adjudication here without engaging in philosophical deliberation.

For present purposes the following triads provide further illustrations of the sort of inconsistency at the basis of the preceding discussion. In each case, the first two theses contradict the third.

I. A Free Will Paradox

- (1) Any decision whose outcome is securely predictable by others, even before the agent himself confronts it, is not free.
- (2) A person's choices are securely predictable on the basis of their established tastes, preferences, and objectives.
- (3) People's selective choices are generally free.

II. A Moral Obligation Paradox

- (1) It is never morally wrong to do what one cannot possibly help doing.
- (2) Sometimes—in some circumstances—breaking a promise is unavoidable.
- (3) Breaking a promise is always morally wrong.

III. An Ontological Paradox

- (1) We experience only the phenomena, the observable occurrences. The processes that produce, cause, and explain them lie outside the range of our observations.

- (2) To be is to be observable. Only that which is observable actually exists.
- (3) Those productive processes that produce the phenomena must exist, since effects require causes and cannot exist without them.

IV. The Philosophy Pointlessness Paradox

- (1) Philosophers do and cannot but disagree on virtually every issue within their field.
- (2) A field whose competent investigators cannot reach agreement is not just problematic but effectively pointless.
- (3) Philosophy is an instructive and significant domain of inquiry and deliberation.

Observe that in each case we have a cluster of inconsistent claims whose resolution calls for abandoning one or more of its component theses while retaining others—a step whose justification cannot avert in engaging in deliberations that qualify as philosophical. The irony here is that as long as our deliberations are rational, rejecting a philosophical thesis requires engaging in philosophical deliberations and thereby in the practice of philosophy itself.

20.3 The Socratic Discovery

Socrates was a convivial individual who loved conversation. In the course of his discussions with fellow Athenians he made an important discovery. He found that they had fixed opinions on many subjects, relating not only to personal conduct (“How should I deal with my difficult in-laws?”), public affairs (“whom should we elect to office?”), or natural proceedings (“Why so little rainfall today?”). But he also found in the course of discussion that they gave little if any thought to questions that could and should be raised about those initial questions themselves: questions about matters of question-management.

On this basis Socrates arrived at a critical realization:

- Questions engender further meta-questions with respect to the means and methods their resolution. In addressing a question one always encounters the correlative question: “What is it that makes that sort of proceeding into the right and proper way to address this sort of question?”

The regress of justificatory considerations becomes increasingly general, abstract, and fundamental until at last it issues in the sphere of philosophical considerations.

In general we begin with rather straightforward questions about matters of fact. Secondly there now arises the question of issue-resolute. How is one to resolve those first-order questions? What sorts of considerations are appropriate and what is to be their relative weight? And the important thing to note here is that these questions regarding the procedural properties of issue resolution are philosophical in nature.

And so he came to confront the problem: How is one to deal with questions about questioning procedures? And Socrates soon realized that people were disinclined to push matters that far.

So here we arrive at the crux of the Socratic Discovery: In elucidating the justificatory basis of opinions—even opinions about ordinary everyday matters—we are drawn to deliberate about philosophical issues.

And this sort of situation is typical.

What is legal? This, clearly, is not a question for philosophers. But what ought to be legal of legality is to make its proper contribution to the public good is something else again: a philosophically appropriate inquiry. What is the proper role of a legal system in the management of human affairs and what considerations function for and against the appropriate conduct of legal proceeding given the role that legal proceedings play in human affairs? Such second-order questions about the proper ways and means for securing answers takes us straightway into the philosophical arena.

So Socrates was right. Questions lie at the heart and core of philosophy and in addressing them we are inescapably drawn into a dual format of puzzlement and perplexity.

But Immanuel Kant was also right. The answer to any given question opens the door to others. In philosophy our answers always invite questions. “Do the ends justify the means?” If YES—then how, under what conditions, to what extent? If NO—then why not, and what of extreme cases (sacrificing one to save thousands, etc.)?

20.4 The Problem of Progress

How then is one to assess progress in this question management enterprise? It is tempting to think that one can assess philosophical progress by mere numeration, by simply assessing the statistical of question and answer. But this convenient resolution is impracticable—for many reasons, including three:

- The question domain is not finite, so progress cannot be assessed by the percentage of questions answered.
- The questions are not infinite in size. There are big ones and small ones, so that a counting comparison becomes a problem.
- A philosophical theory is itself arbiter of what questions are relevant and appropriate. For example, a materialist will not allow questions about spiritual issues to count.

All in all then, any sort of quantitative approach to philosophical progress is impracticable. Having answers to questions affords no objective benefit to a doctrine that is itself the arbiter of what sort of questions are meaningful and significant.

Philosophical progress thus cannot be assessed in quantitative terms of doing “more” than before. It will have to address doing better and proceed qualitative terms

of appraisal. And here lies the big problem. For there is no doctrine doctrinally neutral standard of adequacy in issue resolution that is itself independent of some basis of philosophical commitment. The adequacy of a philosophical question-resolution can only be assessed from the vantage point of a philosophical position. Theorizing can only address these issues from a theoretic point of view. And in the circumstances this will not do.

So in the end, theory can only be assessed via practice. Only the experience of having a life lived on its basis—seen in the full light of its practical consequences—can the adequacy of a philosophical position be properly assessed. In philosophy (as so often elsewhere) the proof of the pudding is in the eating.

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